Name:

Common Core Algebra II

Unit 2

Sketching Polynomials and Remainder Theorem

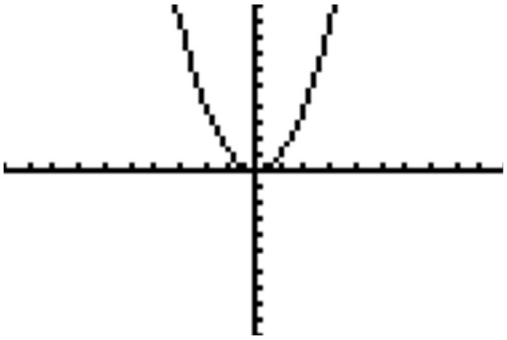
Mr. Schlansky

2025

Lesson 1: I can sketch polynomial graphs by its degree and sign. I can write end behavior by checking which way the graph is pointing as I look to the left and the right.
Sketching Polynomial Graphs

End Behavior

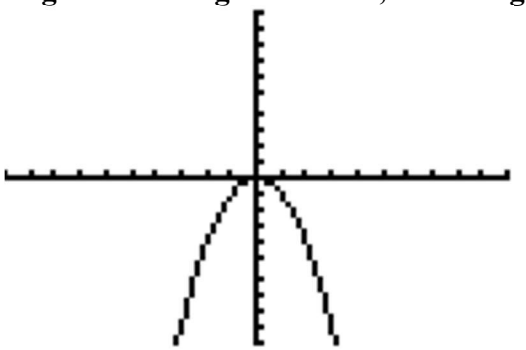
Positive leading coefficient, Even Degree



$x \to -\infty,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to \infty$

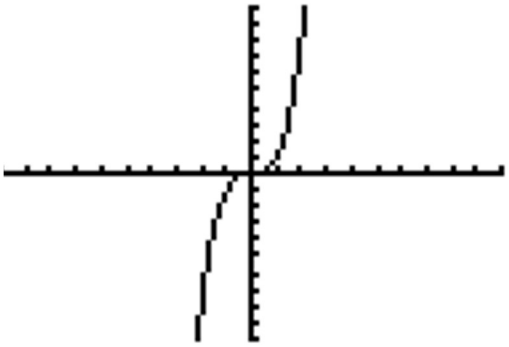
Negative leading coefficient, Even Degree



$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to -\infty$

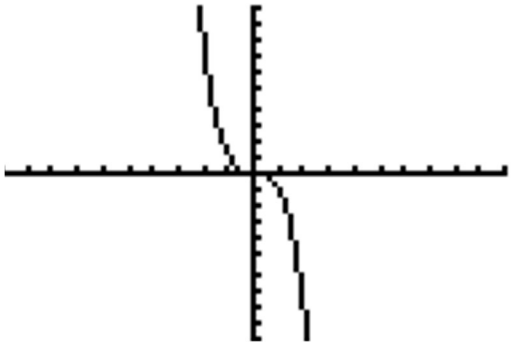
Positive leading coefficient, Odd Degree



$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to \infty$

Negative leading coefficient, Odd Degree



$x \to -\infty,\ f(x) \to \infty$

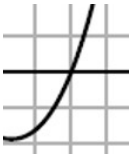
$x \to \infty,\ f(x) \to -\infty$

Even degree begins and ends pointing in same direction
Odd degree begins and ends point in opposite directions
Positive leading coefficient slopes up at the end
Negative leading coefficient slopes down at the end
Degree is the largest exponent.

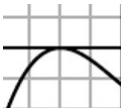
$x \to -\infty$	left
$x \to \infty$	right
$f(x) \to -\infty$	down
$f(x) \to \infty$	up

Lesson 2: I can sketch a polynomial graph by determining the zeros by negating the zeros.
If $x-a$ is a factor , then a is a zero (negate what comes after the x).
If x is a factor, then 0 is a zero.

Single roots pass through the x axis



Double roots bounce off the x axis



Lesson 3: I can sketch polynomial functions by finding the zeros by negating the factors.
Same notes as Lesson 2.
If given a trinomial factor, factor it for a double root.
The zeros hit the x-axis, the factors do not!

Lesson 4: I can determine if roots are imaginary by seeing if they hit the x-axis.
Real zeros hit the x-axis. Imaginary zeros do not hit the x-axis.

Lesson 5: I can find the remainder when a polynomial is divided using the remainder theorem. I can determine if a binomial is a factor by using remainder theorem to determine if the remainder is 0.
Remainder Theorem: Substitute the opposite of what you are dividing by in for x in order to find the remainder.

If given a graph, find the y value for the corresponding x value.

To determine if $x-a$ is a factor:
Use remainder theorem to find the remainder!
If the remainder is 0, it is a factor. If the remainder is not 0, it is not a factor.

Lesson 6: I can find k in a polynomial equation given a factor using remainder theorem.
Polynomial Equations Given a Factor
-Substitute the zero in for x and 0 in for f(x)
-Solve for k

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End Behavior and Shape of Polynomial Graphs

Sketch the shape and fill in the end behavior for each of the following polynomial equations

1. $f(x) = x^3 + 2x^2 - 9x - 18$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

2. $f(x) = x^4 - 10x^2 + 9$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

3. $p(x) = -x^3 - 3x^2 + 4x + 12$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

4. $f(x) = -x^4 + 3x^3 + 10x^2$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

5. $p(x) = x^3 - 3x^2 - 9x + 27$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

6. $h(x) = x^6 - 5x^4 + 4x^2$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

7. $g(x) = -\frac{1}{2}x^5 - 4x^2 + 3x^2 - 7$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

8. $f(x) = x^4 + 11x^3 + 15x^2 - 25x$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

9. $g(x) = -x^6 + 2x^3 + 4x^2 - 8x$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

10. $m(x) = 2x^3 + 4x^2 - 8x$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

11. $f(x) = -2x^4 - 2x^3 + 34x^2 + 42x - 72$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

12. $g(x) = -x^5 + 5x^4 + 8x^3 - 44x^2 - 32x + 64$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

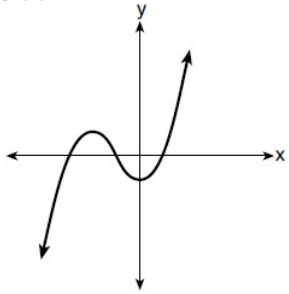
13. Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

Which function satisfies the given conditions?

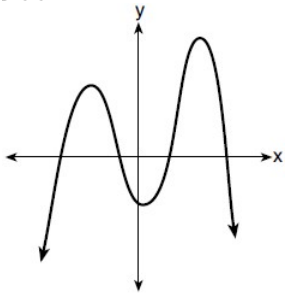
1) $f(x) = x^4 + 2x^2 + 1$

2)



3) $f(x) = -x^3 + 2x - 6$

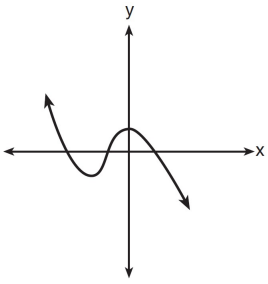
4)



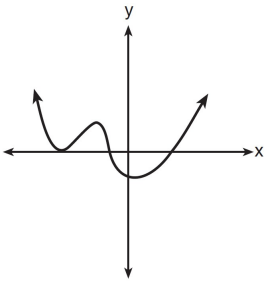
14. Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty, \ f(x) \to -\infty$
- as $x \to \infty, \ f(x) \to \infty$

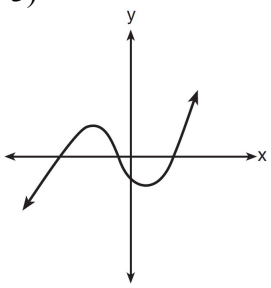
1)



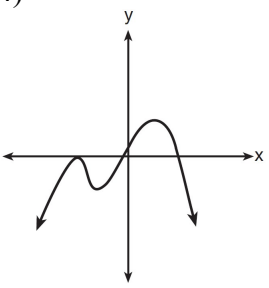
2)



3)



4)



15. Which description could represent the graph of $f(x) = 4x^2(x + a) - x - a$, if a is an integer?

1) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.
2) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.
3) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to -\infty$, and the graph has 4 x-intercepts.
4) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 4 x-intercepts.

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Writing Equations of Polynomial Equations

State the zeros for the following polynomials

1. $p(x) = (x+2)(x-4)(x+1)$

2. $p(x) = (x-6)(x+3)(x-8)$

3. $p(x) = x(x-6)(x+3)$

4. $p(x) = x(x-1)(x+1)$

5. $p(x) = x(x-7)(x+10)(x-3)$

6. $p(x) = (x-2)(x-3)(x+4)$

7. $p(x) = 2(x+1)(x-4)(x+6)$

8. $p(x) = -3x(x-2)(x-4)(x+7)$

Write a possible polynomial equation in factored for if the zeros are:

9. $\{-4,-2,3\}$

10. $\{6,-7,-2\}$

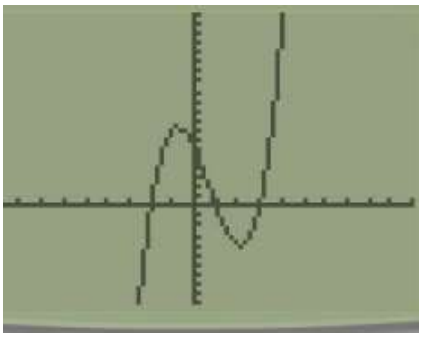
11. $\{0,1,-2,4\}$

12. $\{0,-2,-6,3\}$

13. $\{-7,8,3,\pm 5\}$

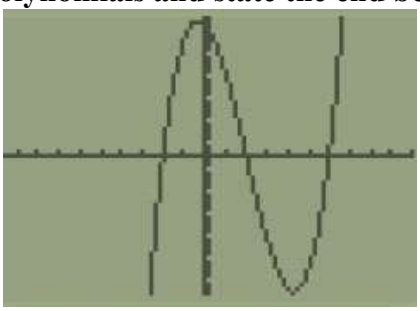
14. $\{0,\pm 4,7\}$

Write a possible equation for each of the following polynomials and state the end behavior

15. 

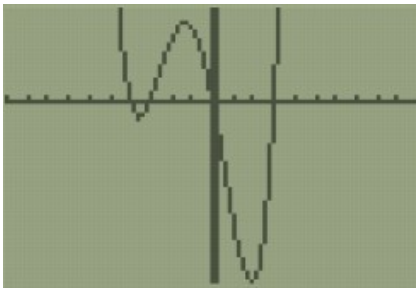
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

16. 

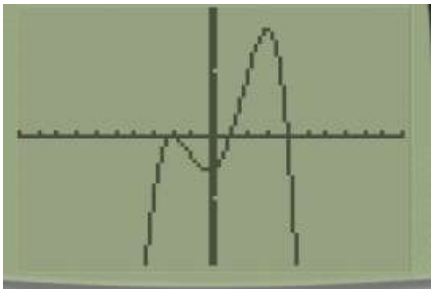
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

17. 

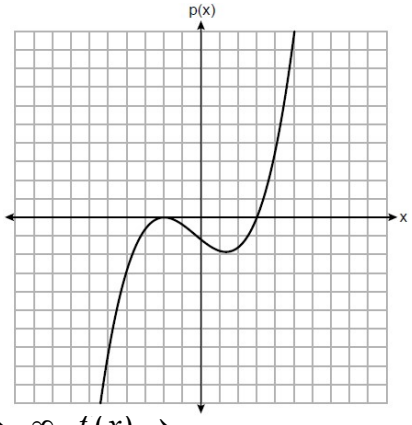
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

18. 

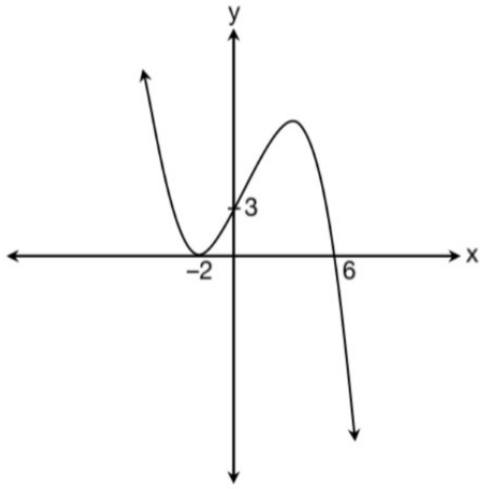
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

19. 

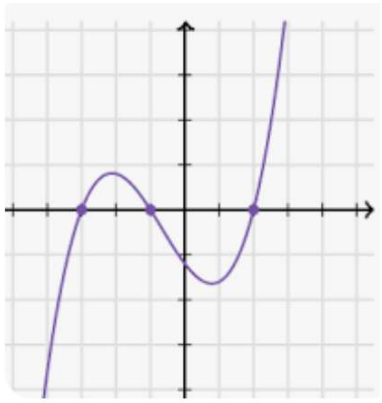
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

20. 

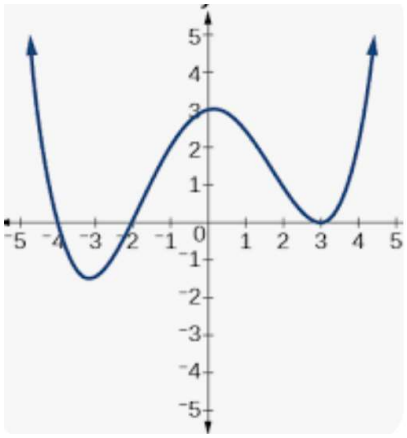
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

21. 

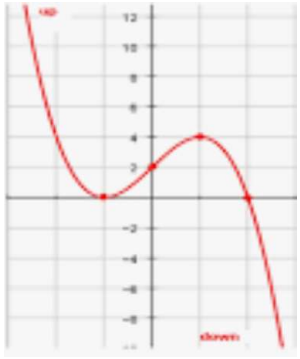
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

22. 

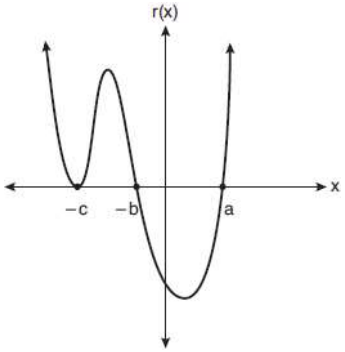
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

23. 

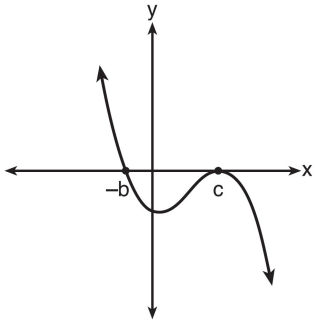
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

24. 

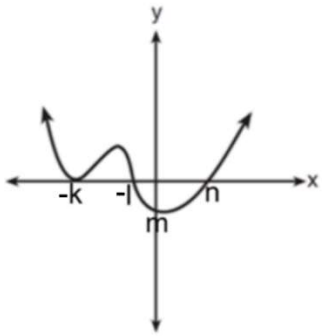
$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

25. 

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

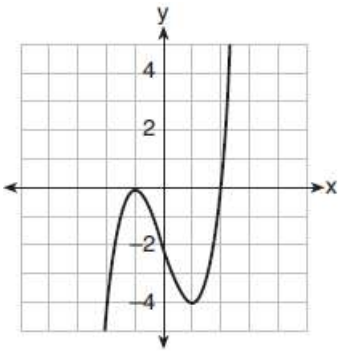
26. 

$x \to -\infty,\ f(x) \to$

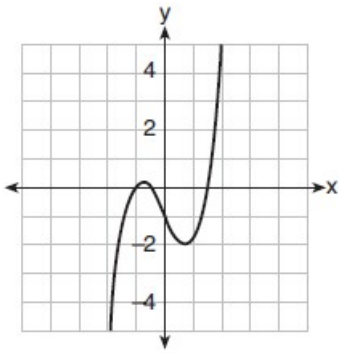
$x \to \infty,\ f(x) \to$

27. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

1)



3)



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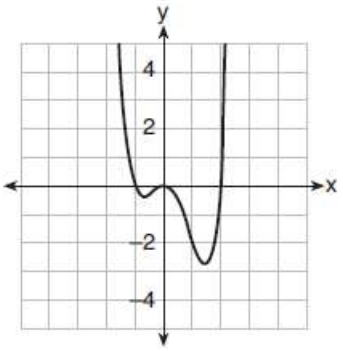
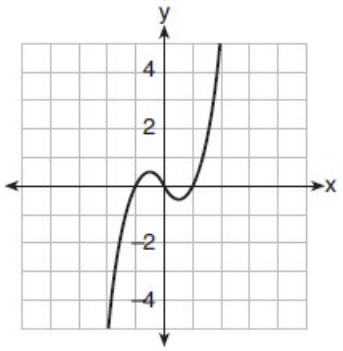
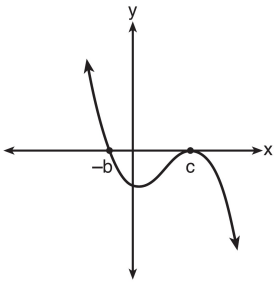


4)

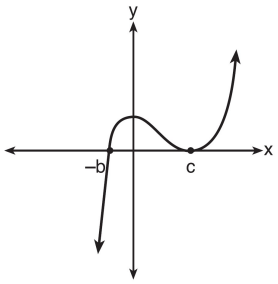


28. If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)\left(x^2 - 2cx + c^2\right)$?

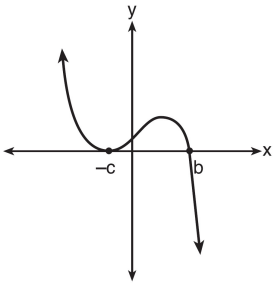
1)



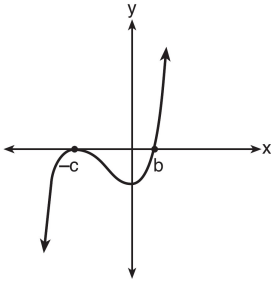
3)



2)



4)



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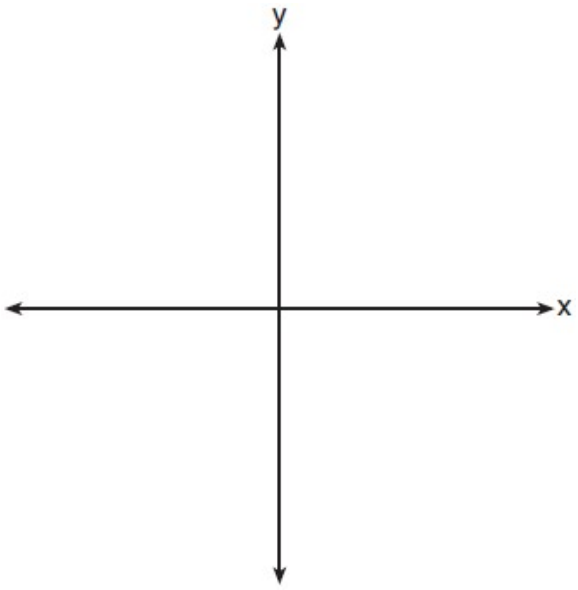
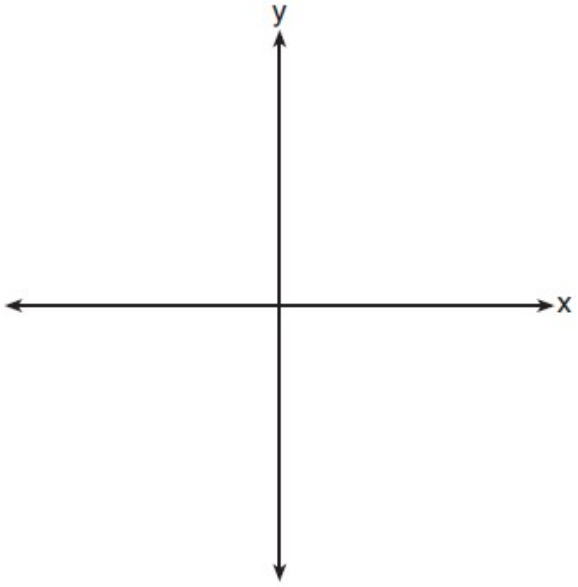
Sketching Polynomial Equations

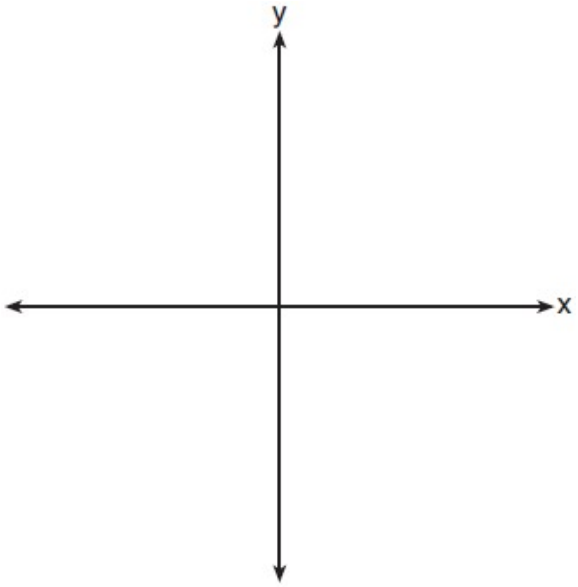
1. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.



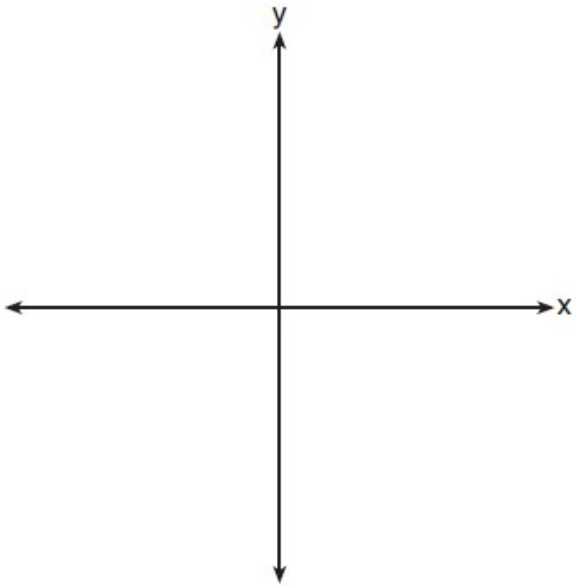
2. The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.



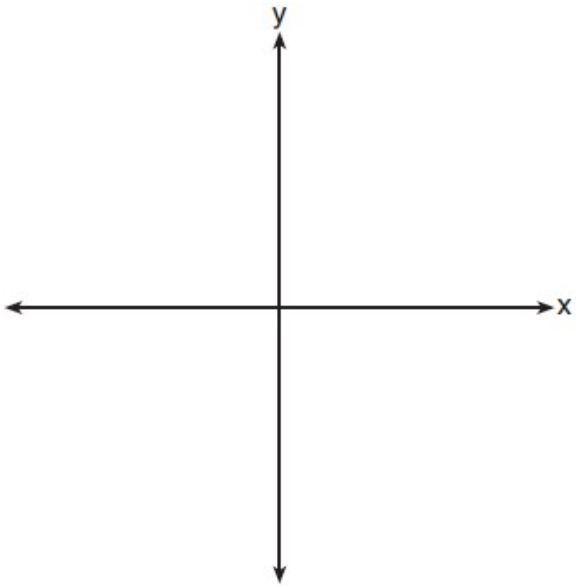
3. The zeros of a quartic polynomial function *h* are -2, 1, 1, and 3. Sketch a graph of $y = h(x)$ on the grid below.



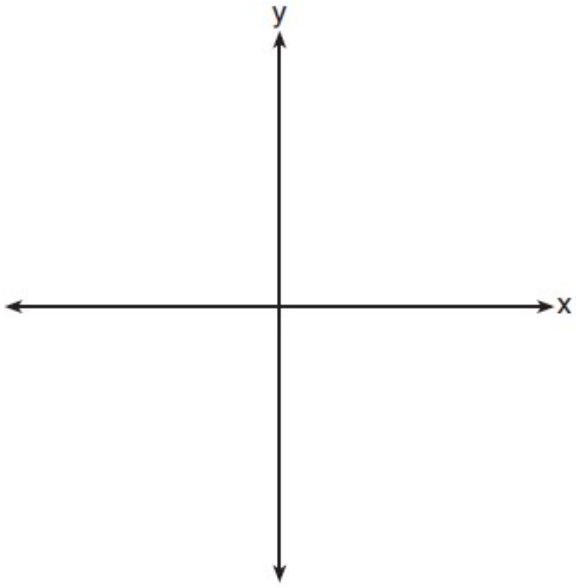
4. The zeros of a polynomial function are $-5, -5, \pm 2,$ *and* 0. Sketch a graph of the polynomial functions on the grid below.



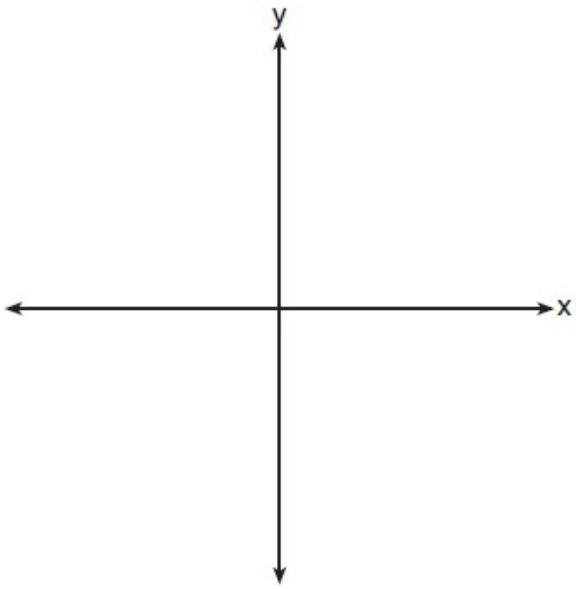
5. On the grid below, sketch a cubic polynomial whose factors are $x-3$, $x+4$, and $x+2$.



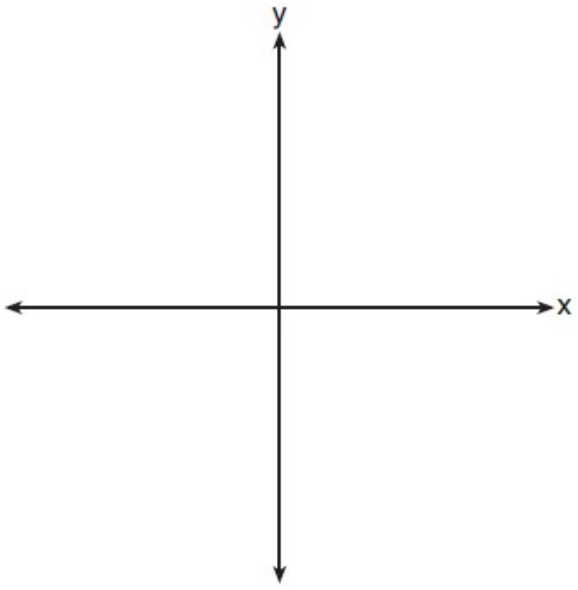
6. On the grid below, sketch a quartic polynomial whose factors are $x+5$, $x+2$, $x+2$, and $x-4$.



7. On the grid below, sketch a cubic polynomial whose factors are $x-3$ and $x^2+8x+16$.



8. On the grid below, sketch a quartic polynomial whose factors are x^2-4x+4 and $x^2+10x+25$.



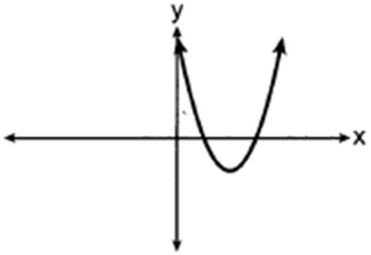
Name _____ Date _____

Mr. Schlansky Algebra II

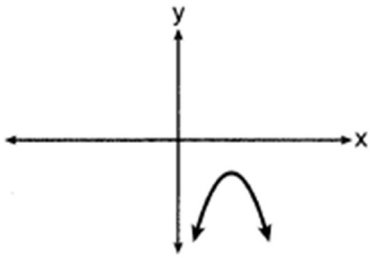
Imaginary Zeros

1. Which graph has imaginary roots?

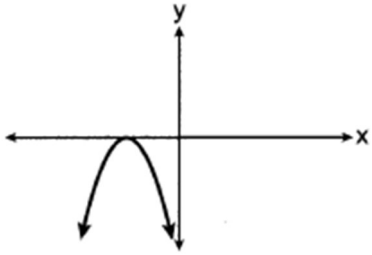
1)



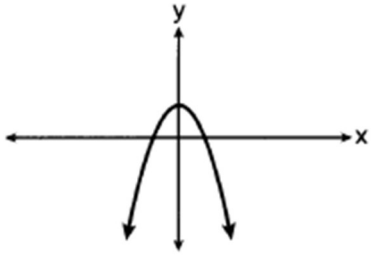
2)



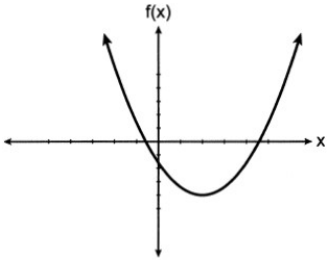
3)



4)

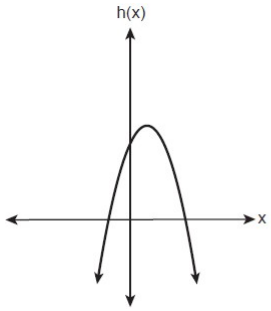


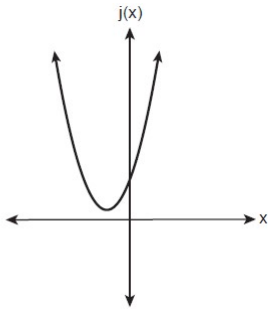
2. If $f(x)$ is represented by the graph below, Does $f(x)$ have imaginary roots? Explain your answer.



3. Which quadratic functions have imaginary roots?

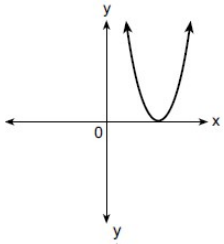
1) $h(x)$ only
2) $j(x)$ only
3) Both $j(x)$ and $h(x)$
4) Neither $j(x)$ or $h(x)$

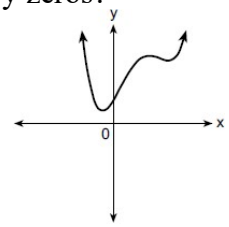


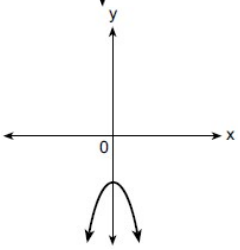


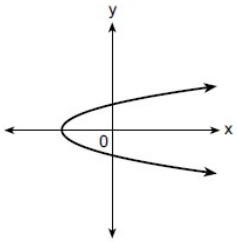
4. Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

5. Which of the following graphs have imaginary zeros?

I 

II 

III 

IV 

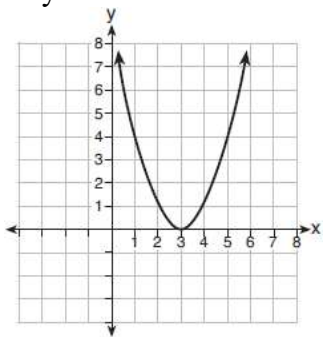
1) I and IV
2) II and III
3) II only
4) III and IV

6. Which representation of a quadratic has imaginary roots?

1)

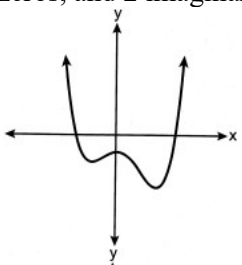
x	y
–2.5	2
–2.0	0
–1.5	–1
–1.0	–1
–0.5	0
0.0	2

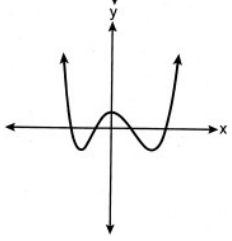
2) $2(x+3)^2 = 64$

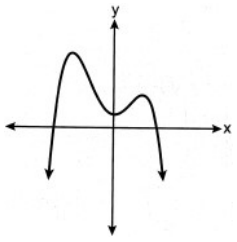
3) 

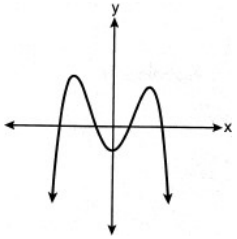
4) $2x^2 + 32 = 0$

7. Which graph could represent a 4th degree polynomial function with a positive leading coefficient, 2 real zeros, and 2 imaginary zeros?

1) 

2) 

3) 

4) 

Name _____ Date _____

Mr. Schlansky Algebra II

The Remainder Theorem

1. What is the remainder when $p(x) = x^3 - 9x^2 + 21x - 5$ is divided by $x - 5$? Is $x - 5$ a factor of $p(x)$? Explain your answer.

2. What is the remainder when $p(x) = x^4 - 8x^2 + 3x$ is divided by $x + 4$? Is $x + 4$ a factor of $p(x)$? Explain your answer.

3. What is the remainder when $p(x) = x^3 - 2x^2 + 6x - 2$ is divided by $x - 3$? Is $x - 3$ a factor of $p(x)$? Explain your answer.

4. What is the remainder when $p(x) = x^3 - 5x^2 - 5x + 25$ is divided by $x + 2$? Is $x + 2$ a factor of $p(x)$? Explain your answer.

5. What is the remainder when $p(x) = x^3 - 6x^2 + 4x - 1$ is divided by $x - 6$? Is $x - 6$ a factor of $p(x)$? Explain your answer.

6. What is the remainder when $p(x) = x^3 - 3x^2 - 8x + 4$ is divided by $x + 2$? Is $x + 2$ a factor of $p(x)$? Explain your answer.

7. What is the remainder when $p(x) = 2x^3 + 5x + 2$ is divided by $2x + 1$. Is $2x + 1$ a factor of $p(x)$? Explain your answer.

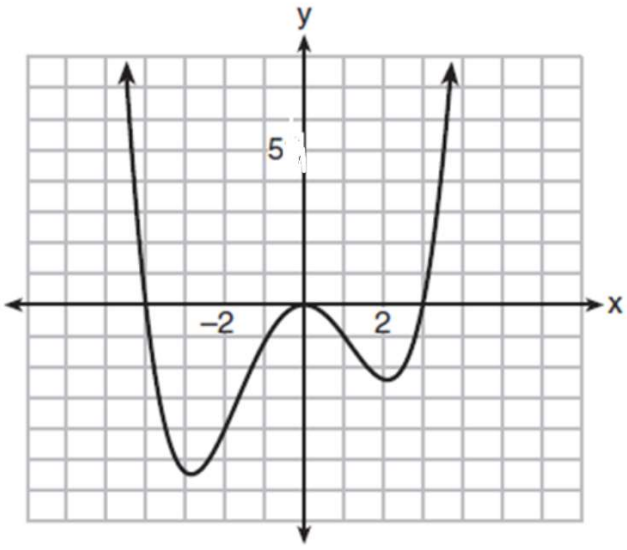
8. What is the remainder when $p(x) = 3x^3 - 2x^2 - 27x + 18$ is divided by $3x - 2$. Is $3x - 2$ a factor of $p(x)$? Explain your answer.

9. What is the remainder when $p(x) = 2x^3 - 2x^2 + 3$ is divided by $4x + 1$. Is $4x + 1$ a factor of $p(x)$? Explain your answer.

Use the graph below to the right to answer the following two questions.

10. What is the remainder when the following polynomial is divided by $x - 1$? Is $x - 1$ a factor? Explain your answer.

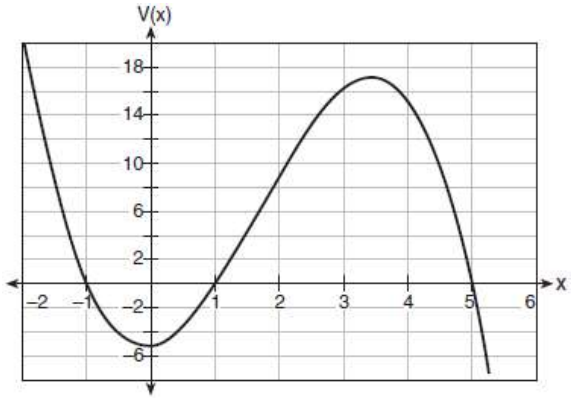
11. What is the remainder when the following polynomial is divided by $x + 3$? Is $x + 3$ a factor? Explain your answer.



Use the graph below to the right to answer the following two questions.

12. What is the remainder when the following polynomial is divided by $x-1$? Is $x-1$ a factor? Explain your answer.

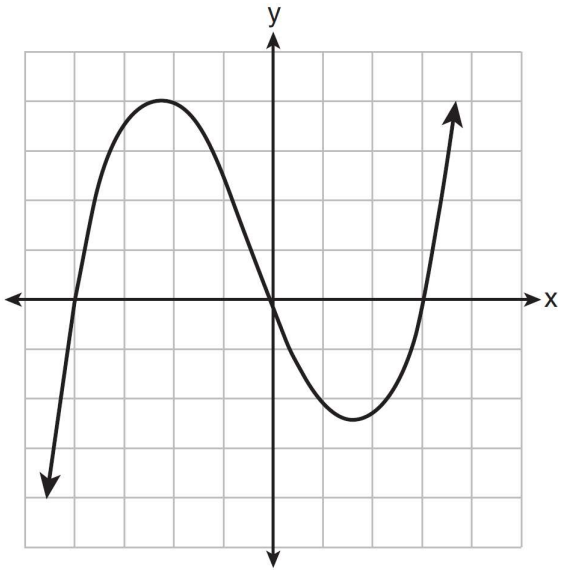
13. What is the remainder when the following polynomial is divided by $x+2$? Is $x+2$ a factor? Explain your answer.



Use the graph below to the right to answer the following two questions.

14. What is the remainder when the following polynomial is divided by $x-1$? Is $x-1$ a factor? Explain your answer.

15. What is the remainder when the following polynomial is divided by $x-3$? Is $x-3$ a factor? Explain your answer.



16. Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

1) $x-2$
2) $x+2$
3) $x-4$
4) $x+4$

17. Which binomial is *not* a factor of the expression $x^3 - 11x^2 + 16x + 84$?

1) $x+2$
2) $x+4$
3) $x-6$
4) $x-7$

18. Which binomial is *not* a factor of the expression $x^3 - 6x^2 - 49x - 66$?

1) $x-11$
2) $x+2$
3) $x+6$
4) $x+3$

19. Which binomial is a factor of the expression $x^3 - 7x - 6$?

1) $x + 3$
2) $x - 1$
3) $x - 2$
4) $x + 2$

20. Which binomial is *not* a factor of the expression $x^3 - 4x^2 - 25x + 28$?

1) $x + 6$
2) $x - 7$
3) $x - 1$
4) $x + 4$

21. Which binomial is not a factor of $p(x) = 2x^3 + 7x^2 - 5x - 4$?

1) $x + 4$
2) $x + 1$
3) $x - 1$
4) $2x + 1$

22. Which binomial is not a factor of $p(x) = 2x^3 - 5x^2 + 6x - 2$?

1) $x - 1$
2) $x - 2$
3) $2x - 1$
4) $2x + 1$

23. Given $P(x) = x^3 - 3x^2 - 2x + 4$, which statement is true?

1) $(x - 1)$ is a factor because $P(-1) = 2$.
2) $(x + 1)$ is a factor because $P(-1) = 2$.
3) $(x + 1)$ is a factor because $P(1) = 0$.
4) $(x - 1)$ is a factor because $P(1) = 0$.

24. If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$

1) equals 0 and $2x + 1$ is a factor of $f(x)$
2) equals 0 and $2x - 1$ is a factor of $f(x)$
3) does not equal 0 and $2x + 1$ is not a factor of $f(x)$
4) does not equal 0 and $2x - 1$ is a factor of $f(x)$

25. Consider the function $f(x) = 2x^3 + x^2 - 18x - 9$. Which statement is true?

1) $2x - 1$ is a factor of $f(x)$.
2) $x - 3$ is a factor of $f(x)$.
3) $f(3) \neq f\left(-\frac{1}{2}\right)$
4) $f\left(\frac{1}{2}\right) = 0$

Name _____ Date _____

Mr. Schlansky Algebra II

Finding k in a Polynomial Equation

1. Consider the polynomial $p(x) = x^3 + kx^2 + x + 6$. Find a value of k so that $x + 1$ is a factor of P.

2. Consider the polynomial $p(x) = x^3 + kx - 30$. Find a value of k so that $x + 3$ is a factor of P.

3. Given $p(x) = 6x^3 + 31x^2 + kx - 12$, and $x + 4$ is a factor, find the value of k.

4. Given $z(x) = 6x^3 + bx^2 - 52x + 15$, and $x + 5$ is a factor, find the value of *b*.

5. Given $p(x) = x^3 + 5x^2 + kx - 24$, and $x + 3$ is a factor, find the value of *k*.

6. If $x - 1$ is a factor of $x^3 - kx^2 + 2x$, what is the value of *k*?

7. The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of *a*.

8. Consider the polynomial $p(x) = x^3 + kx + 2$. Find a value of k so that $x - 2$ is a factor of P.

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Mr. Schlansky Algebra II

Polynomial Graphs/Remainder Theorem Review Sheet

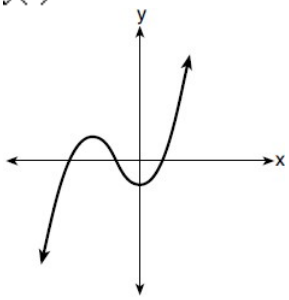
1. Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

Which function satisfies the given conditions?

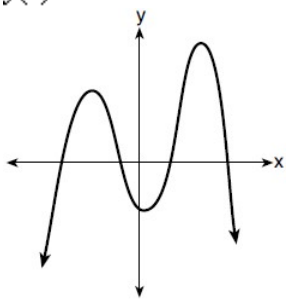
1) $f(x) = x^4 + 2x^2 + 1$

2)



3) $f(x) = -x^3 + 2x - 6$

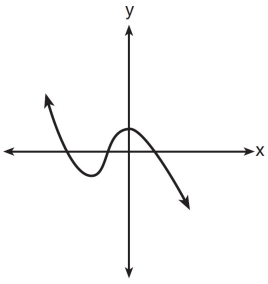
4)



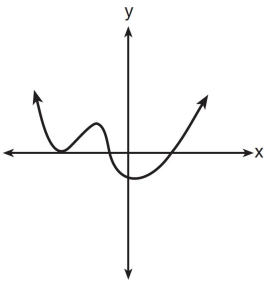
2. Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty,\ f(x) \to -\infty$
- as $x \to \infty,\ f(x) \to \infty$

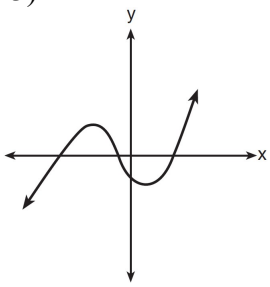
1)



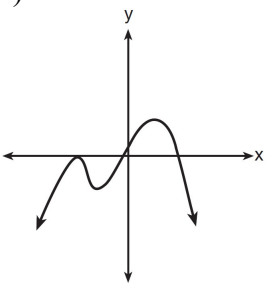
2)



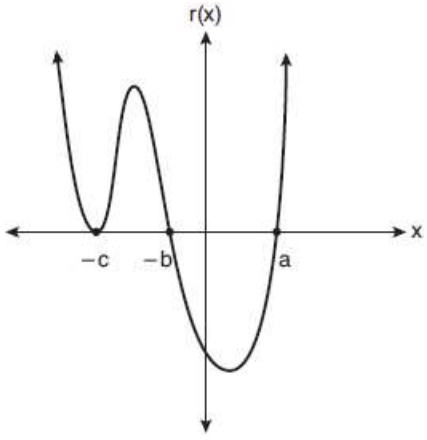
3)



4)



3. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$

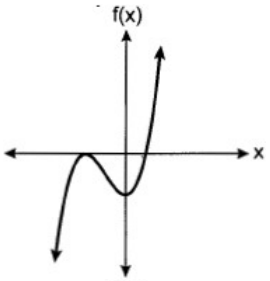
2) $r(x) = (x+a)(x-b)(x-c)^2$

3) $r(x) = (x+a)(x-b)(x-c)$

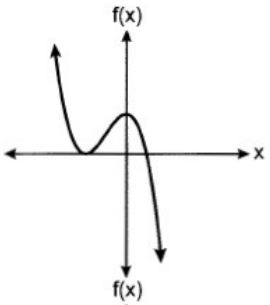
4) $r(x) = (x-a)(x+b)(x+c)^2$

4. Which graph best represents the graph of $f(x) = (x+a)^2(x-b)$, where a and b are positive real numbers?

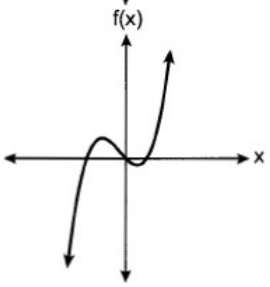
1)



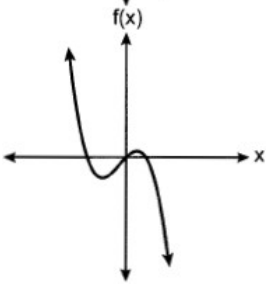
3)



2)

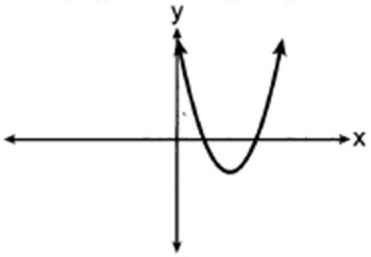


4)

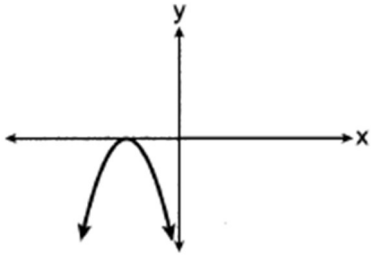


5. Which graph has imaginary roots?

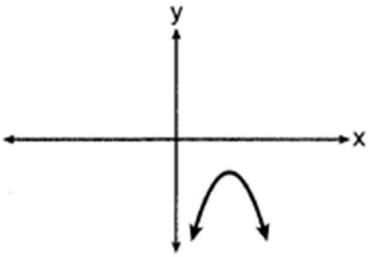
1)



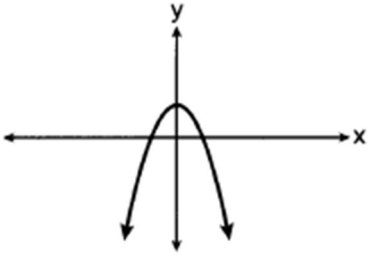
3)



2)

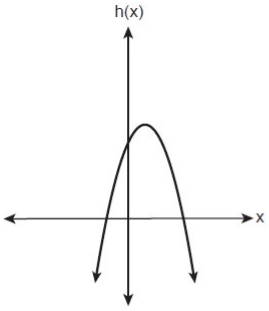


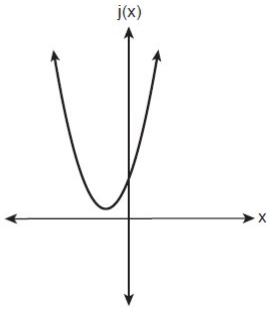
4)



6. Which quadratic functions have imaginary roots?

1) $h(x)$ only
2) $j(x)$ only
3) Both $j(x)$ and $h(x)$
4) Neither $j(x)$ or $h(x)$





7. Is $x-6$ a factor of x^3-6x^2+4x-1? Explain your answer.

8. Is $x+2$ a factor of $p(x)=x^3-3x^2-8x+4$? Explain your answer.

9. Which binomial is *not* a factor of the expression $x^3-6x^2-49x-66$?

1) $x-11$
2) $x+2$
3) $x+6$
4) $x+3$

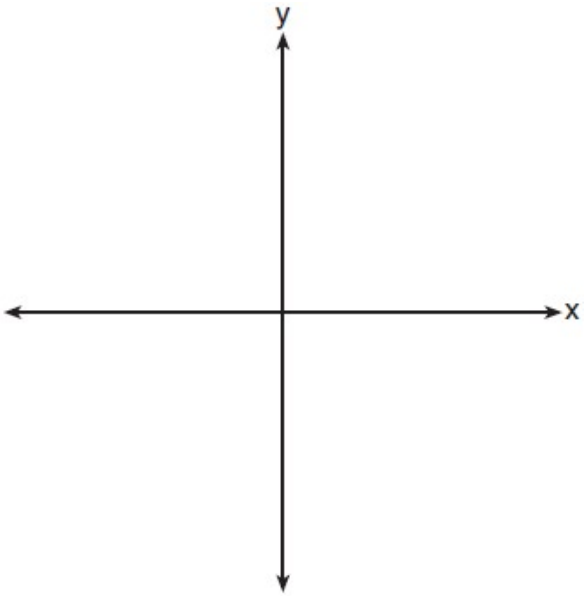
10. Which binomial is a factor of the expression x^3-7x-6?

1) $x+3$
2) $x-1$
3) $x-2$
4) $x+2$

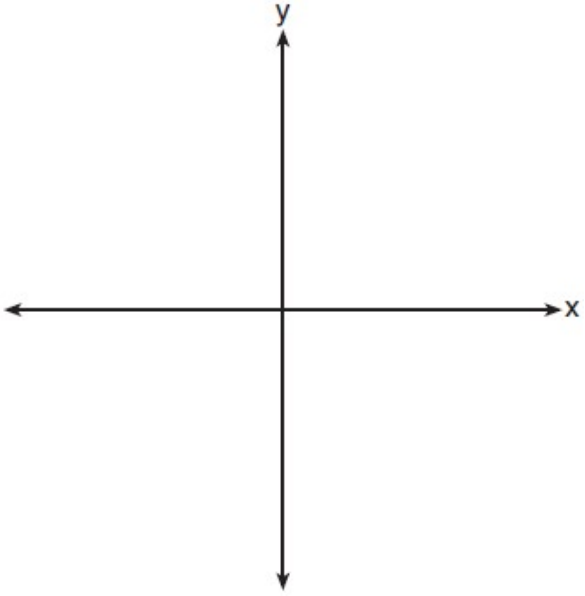
11. Given $p(x)=6x^3+31x^2+kx-12$, and $x+4$ is a factor, find the value of *k*.

12. Consider the polynomial $p(x)=x^3+kx-30$. Find a value of k so that $x+3$ is a factor of P.

13. Sketch the graph of a polynomial function whose factors are $(x+1)$, $(x-4)^2$, and $(x+2)$.



14. Sketch the graph of a polynomial functions whose zeros are -5, -2, -2, and 6.



15. Solve for x:
$3x^2 - 4x - 4 = 0$

16. Solve for x:
$6x^2 - 11x - 2 = 0$

17. Solve for x:
$x^3 + 6x^2 = 4x + 24$

18. Solve for x:
$x^3 - 2x^2 = 9x - 18$