

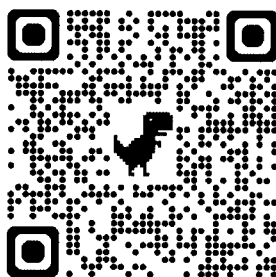
Name:

Common Core Algebra II

Unit 2

Sketching Polynomials and Remainder Theorem

Mr. Schlansky



Lesson 1: I can write intervals in both interval notation and set builder notation

Interval notation:

Where the graph starts and stops from left to right

Set builder notation:

If there is an infinity involved, x comes first.

If there is not an infinity involved, $a < x < b$

Lesson 2: I can determine the intervals where functions are increasing, decreasing, positive, and negative by finding the x value where each interval starts and stops.

Increasing: The function goes “uphill” from left to right

Decreasing: The function goes “downhill” from left to right

Positive: The function is above the x axis

Negative: The function is below the x axis

Lesson 3: I can determine the domain by finding where the graph starts and ends from left to right and determine the range by finding where the graph starts and ends from bottom to top.

Domain: Hold pen vertically and travel left to right to find where graph starts and ends.

Range: Hold pen horizontally and travel bottom to top to find where graph starts and ends.

Lesson 4: I can find key points of a graph using the 2nd Trace (Calc) Menu in my graphing calculator.

The zeros (x -intercepts) are where the graph hits the x axis. 2nd Trace (Calc), zeros, left bound (move cursor to the left of the point and hit enter), right bound (move cursor to the right of the point and hit enter), enter.

The y -intercept is when x is 0. Find this value in your table.

Local extrema are the turning points (minimum and maximum). 2nd Trace (Calc), minimum/maximum, left bound (move cursor to the left of the point and hit enter), right bound (move cursor to the right of the point and hit enter), enter.

Lesson 5: I can determine where functions are increasing/decreasing and positive/negative by stating the x value of where each piece starts and stops from left to right using my graphing calculator.

Increasing/Decreasing:

-Find the relative minimums and maximums using 2nd Trace Menu

-Use Lesson 3 to write the intervals where each piece starts and stops

Positive/Negative:

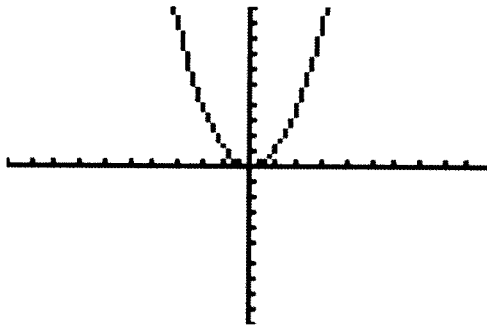
-Find the zeros using the 2nd Trace Menu

-Use Lesson 3 to write the intervals where each piece starts and stops

Lesson 6: I can sketch polynomial graphs by its degree and sign. I can write end behavior by checking which way the graph is pointing as I look to the left and the right.
Sketching Polynomial Graphs

End Behavior

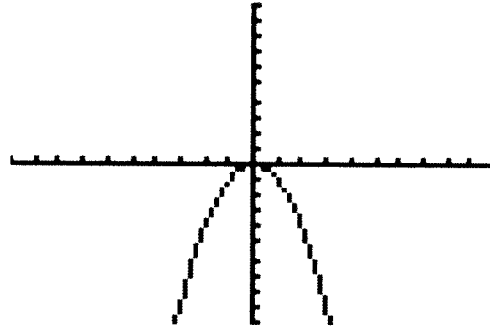
Positive leading coefficient, Even Degree



$$x \rightarrow -\infty, f(x) \rightarrow \infty$$

$$x \rightarrow \infty, f(x) \rightarrow \infty$$

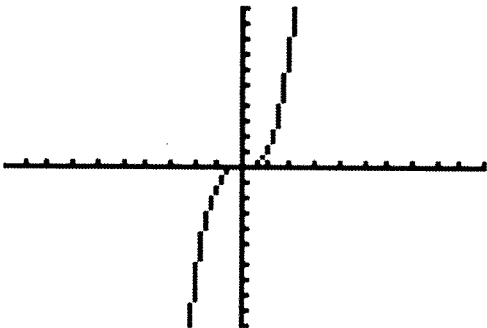
Negative leading coefficient, Even Degree



$$x \rightarrow -\infty, f(x) \rightarrow -\infty$$

$$x \rightarrow \infty, f(x) \rightarrow -\infty$$

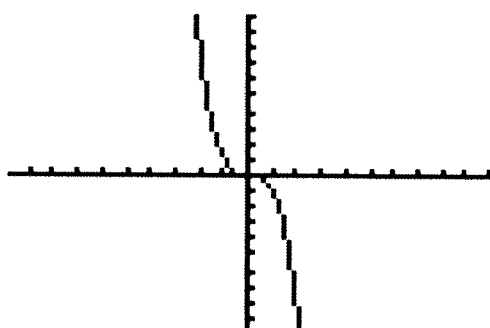
Positive leading coefficient, Odd Degree



$$x \rightarrow -\infty, f(x) \rightarrow -\infty$$

$$x \rightarrow \infty, f(x) \rightarrow \infty$$

Negative leading coefficient, Odd Degree



$$x \rightarrow -\infty, f(x) \rightarrow \infty$$

$$x \rightarrow \infty, f(x) \rightarrow -\infty$$

Even degree begins and ends pointing in same direction

Odd degree begins and ends point in opposite directions

Positive leading coefficient slopes up at the end

Negative leading coefficient slopes down at the end

Degree is the largest exponent.

$x \rightarrow -\infty$ left

$x \rightarrow \infty$ right

$f(x) \rightarrow -\infty$ down

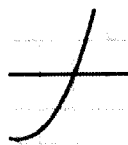
$f(x) \rightarrow \infty$ up

Lesson 7: I can sketch a polynomial graph by determining the zeros by negating the zeros.

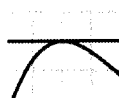
If $x - a$ is a factor, then a is a zero (negate what comes after the x).

If x is a factor, then 0 is a zero.

Single roots pass through the x axis



Double roots bounce off the x axis



Lesson 8: I can sketch polynomial functions by finding the zeros by negating the factors.

Same notes as Lesson 2.

If given a trinomial factor, factor it for a double root.

The zeros hit the x -axis, the factors do not!

Lesson 9: I can graph polynomial equations by typing the equation into $y=$ and plotting the points.

1) Type the equation into $y =$

2) Hit 2nd graph (table) and plot the points

-If a domain/interval is given, plot the points between those values and do NOT put arrows

-If a domain/interval is not given, plot all points that have x and y values between -10 and 10

Lesson 10: I can determine if roots are imaginary by seeing if they hit the x -axis.

Real zeros hit the x -axis. Imaginary zeros do not hit the x -axis.

Lesson 11: I can find the remainder when a polynomial is divided using the remainder theorem. I can determine if a binomial is a factor by using remainder theorem to determine if the remainder is 0.

Remainder Theorem: Substitute the opposite of what you are dividing by in for x in order to find the remainder.

If given a graph, find the y value for the corresponding x value.

To determine if $x - a$ is a factor:

Use remainder theorem to find the remainder!

If the remainder is 0 , it is a factor. If the remainder is not 0 , it is not a factor.

Lesson 12: I can find k in a polynomial equation given a factor by substituting the zero in for x and 0 in for y .

-Substitute the zero in for x and 0 in for $f(x)$

-Solve for k

Lesson 13: I can prepare for my test by practicing!

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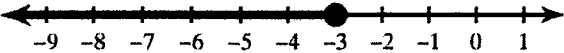
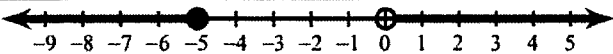
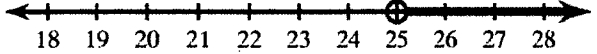
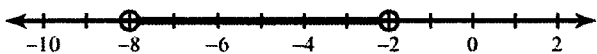
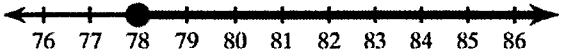
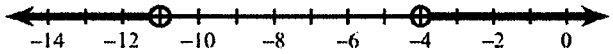
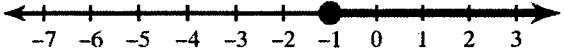
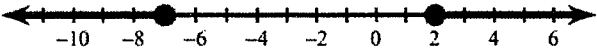
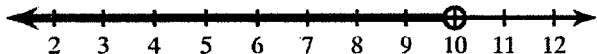


Interval Notation and Set Builder Notation

For each graph, write the appropriate interval in both set and interval notation.

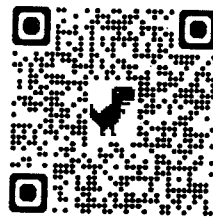
	Graph	Set Notation	Interval Notation
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

	Graph	Set Notation	Interval Notation
11			
12			
13			
14			
15			
16			
17			

	Graph	Set Notation	Interval Notation
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			

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Intervals

1. $f(x) = x^3 + 2x^2 - 9x - 18$
Shape: positive odd

y-intercept:

-18

x-intercepts (zeros):
 $\{-3, -2, 3\}$

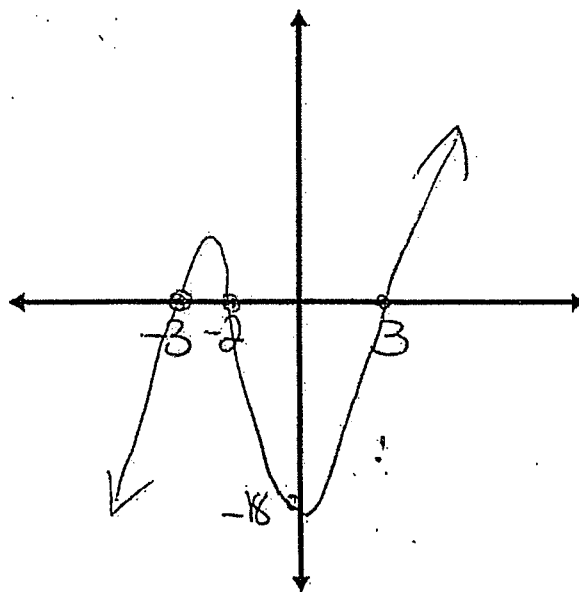
End Behavior: down left
 $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 $x \rightarrow \infty, f(x) \rightarrow \infty$
right up

increasing

decreasing

positive

negative



2. $f(x) = x^4 - 10x^2 + 9$
Shape: positive even

y-intercept: 9

x-intercepts (zeros):
 $\{-3, -1, 1, 3\}$

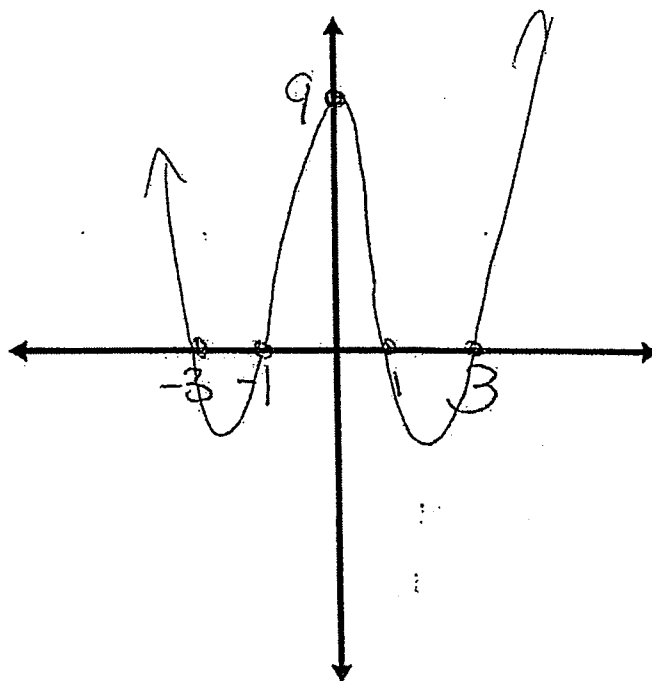
End Behavior: up left
 $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $x \rightarrow \infty, f(x) \rightarrow \infty$
right up

increasing

decreasing

positive

negative



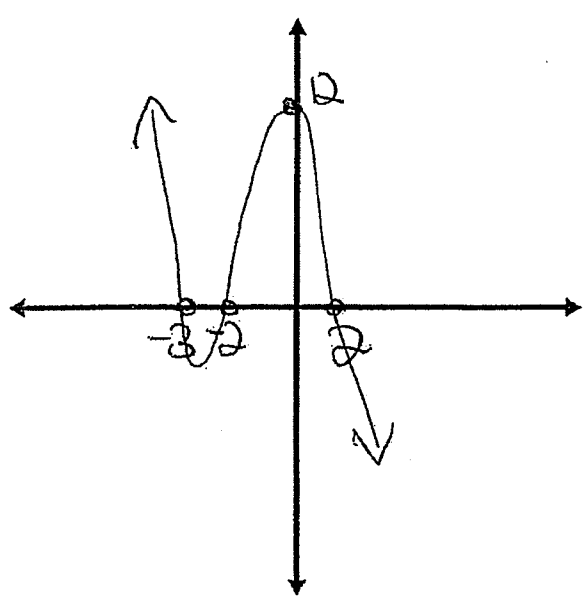
3. $p(x) = -x^3 - 3x^2 + 4x + 12$

Shape: negative odd increasing

y-intercept: 12 decreasing

x-intercepts (zeros): $\{-3, -2, 2\}$ positive

End Behavior:
 left up $x \rightarrow -\infty, f(x) \rightarrow \infty$
 right down $x \rightarrow \infty, f(x) \rightarrow -\infty$
 negative



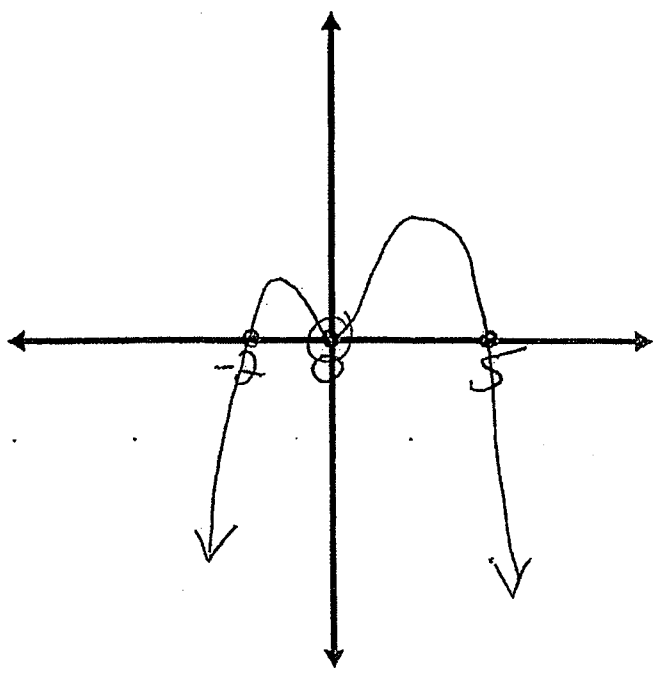
4. $f(x) = -x^4 + 3x^3 + 10x^2 + 0$

Shape: negative even increasing

y-intercept: 0 decreasing

x-intercepts (zeros): $\{-2, 0, 5\}$ positive
 double root
 bounces off

End Behavior:
 left down $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 right down $x \rightarrow \infty, f(x) \rightarrow -\infty$
 negative



5. $p(x) = x^3 - 3x^2 - 9x + 27$

Shape: positive odd increasing

y-intercept:

27

decreasing

x-intercepts (zeros):

$\{-3, 3, 3\}$

double root
bounces off

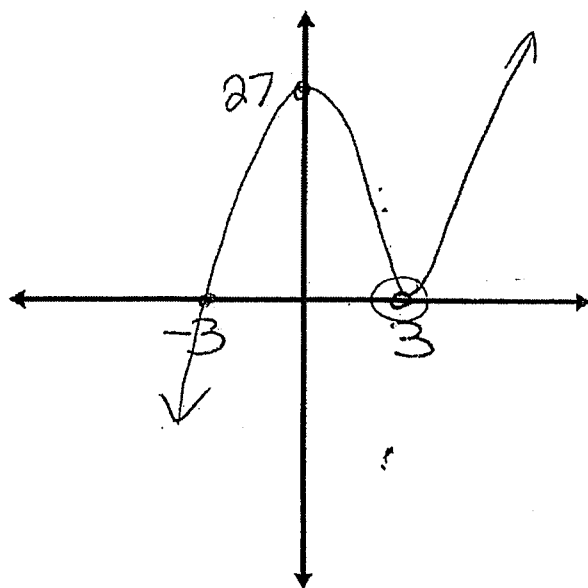
positive

End Behavior:

left $x \rightarrow -\infty, f(x) \rightarrow -\infty$ down

right $x \rightarrow \infty, f(x) \rightarrow \infty$ up

negative



6. $h(x) = x^6 - 5x^4 + 4x^2$

Shape:

positive even

increasing

y-intercept:

0

decreasing

x-intercepts (zeros):

$\{0, 0, 1, 4\}$

double root
bounces off

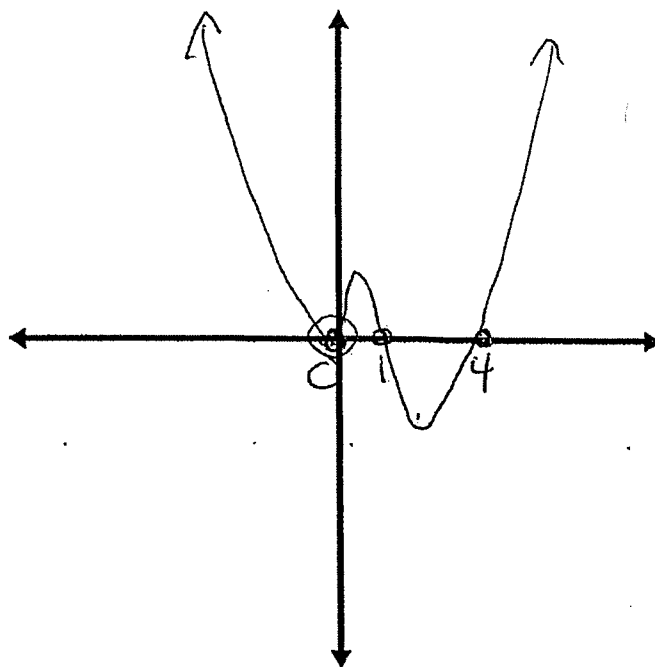
positive

End Behavior:

left $x \rightarrow -\infty, f(x) \rightarrow \infty$ up

right $x \rightarrow \infty, f(x) \rightarrow \infty$ up

negative



7. $f(x) = x^4 + 11x^3 + 15x^2 - 25x$

Shape: positive even
 ↗ increasing

y-intercept:

0

x-intercepts (zeros):

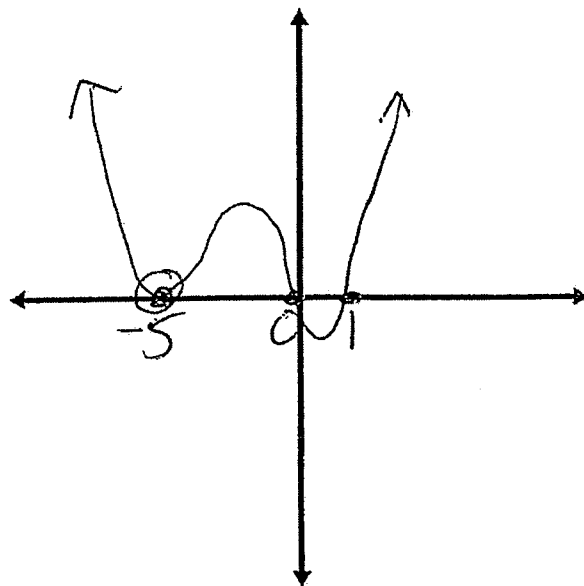
$\{-5, -5, 0, 1\}$

double root
 bounces off

End Behavior:

left $x \rightarrow -\infty, f(x) \rightarrow \infty$

right $x \rightarrow \infty, f(x) \rightarrow \infty$
 up positive
 up negative



8. $g(x) = -x^5 + 5x^4 + 8x^3 - 44x^2 - 32x + 64$

Shape: negative odd
 ↘ decreasing

y-intercept:

64

decreasing

x-intercepts (zeros):

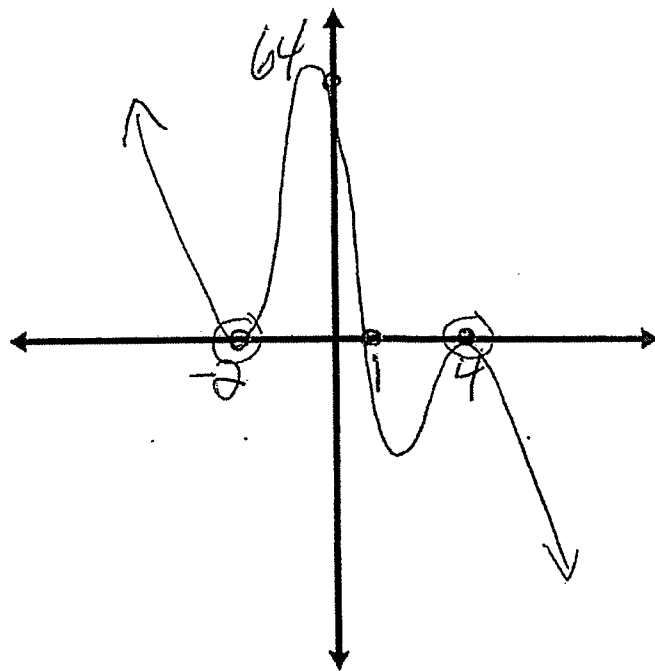
$\{-2, -2, 4, 4\}$

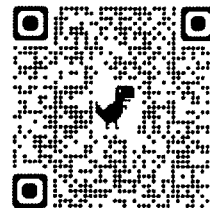
double roots
 bounce off

End Behavior:

left $x \rightarrow -\infty, f(x) \rightarrow \infty$

right $x \rightarrow \infty, f(x) \rightarrow -\infty$
 up positive
 down negative



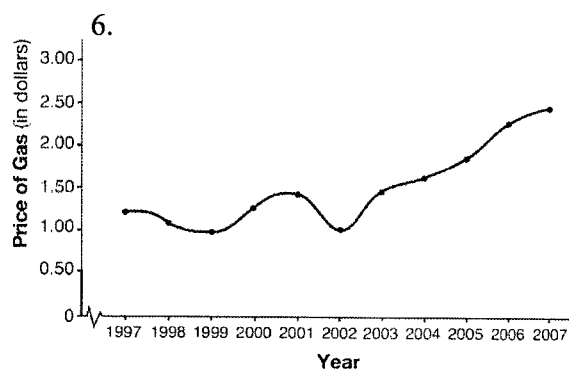
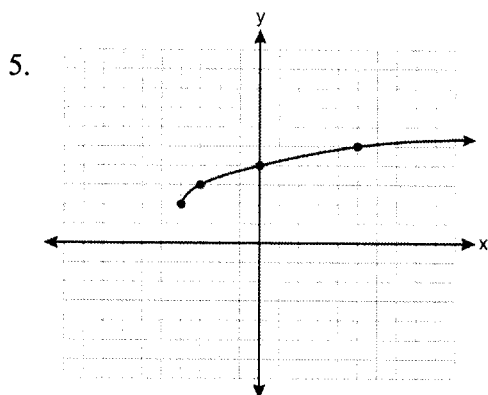
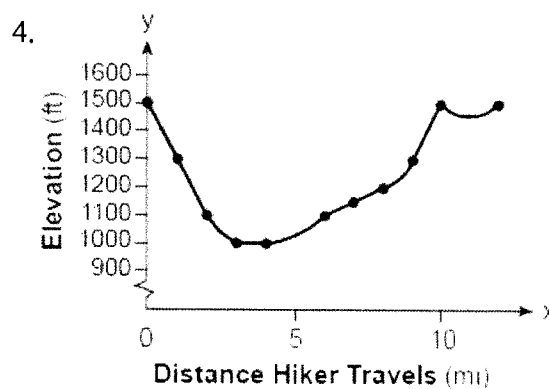
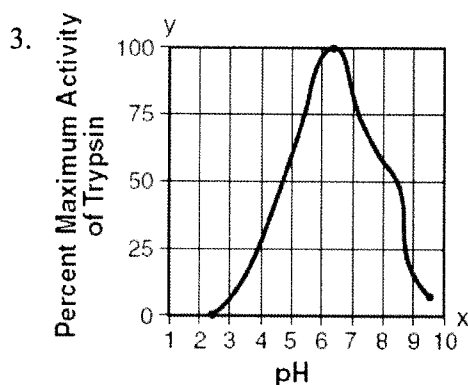
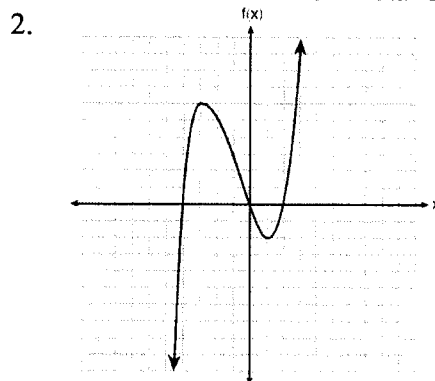
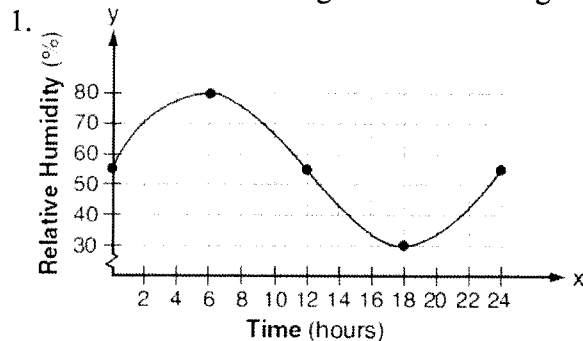


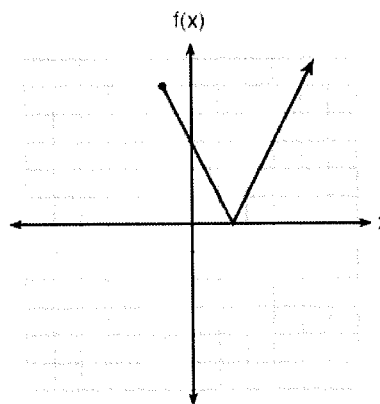
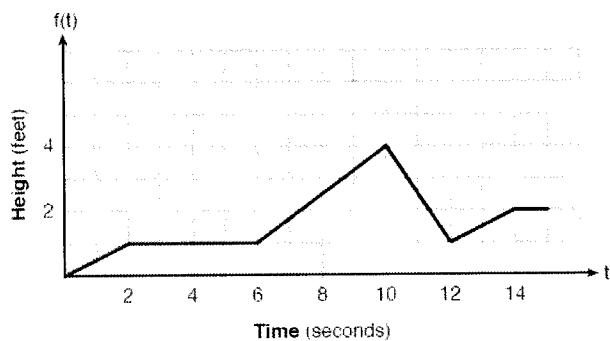
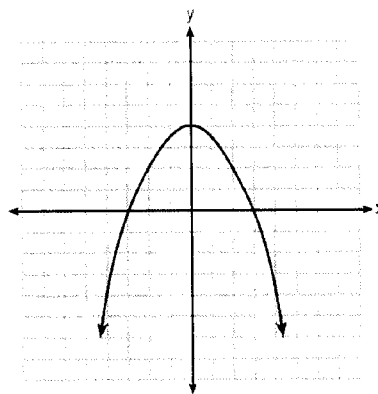
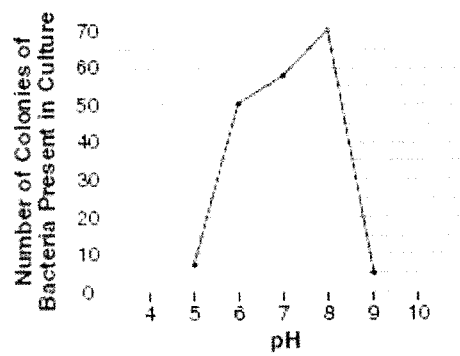
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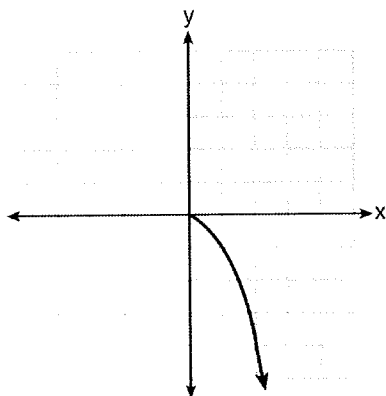
Domain and Range

State the domain and range of the following functions in interval and set builder notation

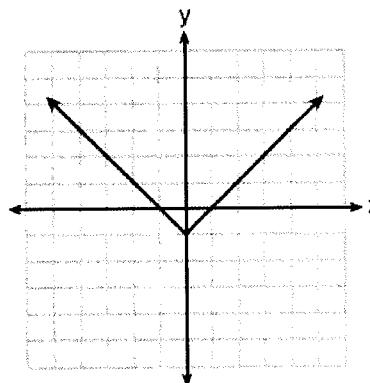




11.

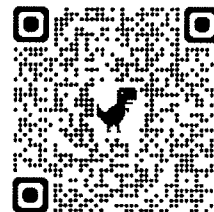


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Finding Key Points

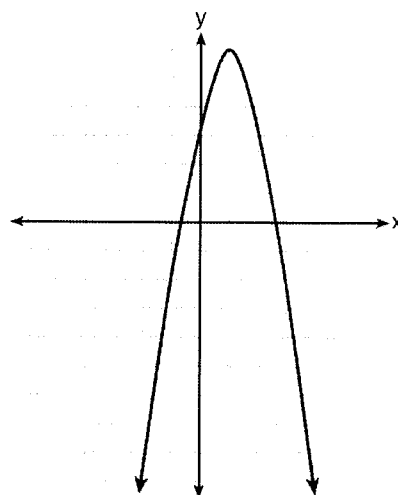
1. Given the function $f(x) = x^3 + 3x^2 - x - 2$, find the zeros and relative extrema to the *nearest tenth*.
2. Given the function $f(x) = -x^3 - 2x^2 + 2x + 3$, find the zeros and relative extrema to the *nearest tenth*.
3. Given the function $f(x) = x^4 - 8x^2 + x + 8$, find the zeros and relative extrema to the *nearest tenth*.
4. Given the function $f(x) = -x^4 + 3x^2 - 1$, find the zeros and relative extrema to the *nearest tenth*.

5. Given the function $f(x) = 2x^3 - 11x^2 - 14x + 26$, find the zeros and relative extrema to the *nearest tenth*
6. Given the function $f(x) = -3x^3 - 20x^2 - 10x + 8$, find the zeros and relative extrema to the *nearest tenth*.
7. Given the function $f(x) = -x^4 + 15x^2 - 7$, find the zeros and relative extrema to the *nearest tenth*.
8. Given the function $f(x) = x^3 + 8x^2 + 3x - 8$, find the zeros and relative extrema to the *nearest tenth*

9. Let f be the function represented by the graph below.

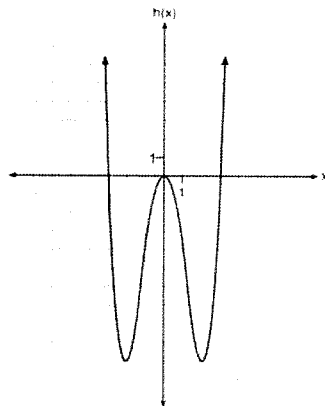
Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$.

Determine which function has the larger maximum value.
Justify your answer.



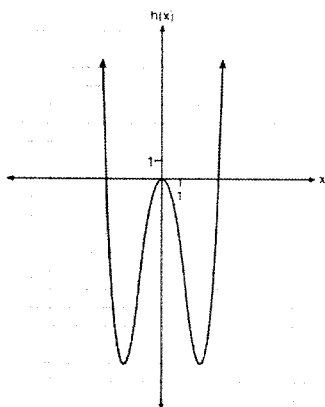
10. Which graph has a smaller relative minimum?

$$g(x) = x^3 + 4x^2 - 2x - 10$$



11. Which graph has the largest zero?

$$g(x) = x^3 + 4x^2 - 2x - 10$$



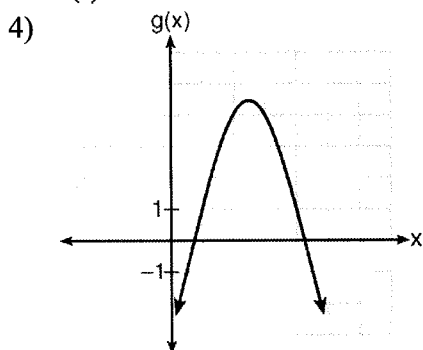
12. Which quadratic function has the largest maximum?

1) $h(x) = (3 - x)(2 + x)$

2)

x	f(x)
-1	-3
0	5
1	9
2	9
3	5
4	-3

3) $k(x) = -5x^2 - 12x + 4$



13. The graph representing a function is shown below.

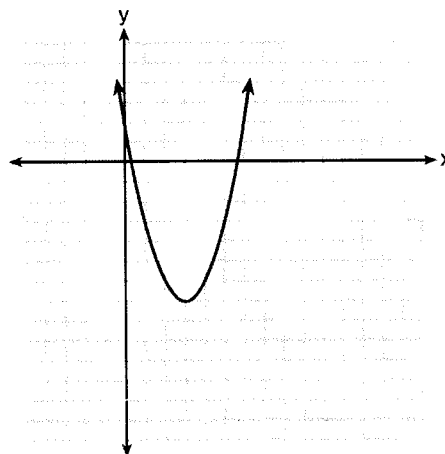
Which function has a minimum that is *less* than the one shown in the graph?

1) $y = x^2 - 6x + 7$

2) $y = |x + 3| - 6$

3) $y = x^2 - 2x - 10$

4) $y = |x - 8| + 2$



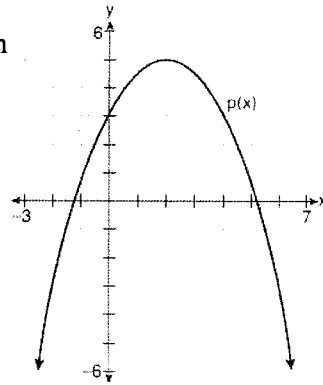
14. Which of the following functions has the greatest zero?

$f(x) = x^2 - 6x + 7$

$g(x) = x^2 - 2x - 10$

15. Consider $f(x) = 4x^2 + 6x - 3$, and $p(x)$ defined by the graph below. The difference between the values of the maximum of p and minimum of f is

- 1) 0.25 3) 3.25
2) 1.25 4) 10.25



16. The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \leq x \leq 3$. To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

17. A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x , in thousands. This function, p , is given below. Over the interval $0 \leq x \leq 9$, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit. Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

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Intervals with Key Points

1. Over what intervals are $f(x) = x^3 + 3x^2 - x - 2$:

Increasing

Decreasing

Positive

Negative

2. Over what intervals are $f(x) = -x^3 - 2x^2 + 2x + 3$:

Increasing

Decreasing

Positive

Negative

3. Over what intervals are $f(x) = -x^4 + 15x^2 - 7$:

Increasing

Decreasing

Positive

Negative

4. Over what intervals are $f(x) = x^3 + 8x^2 + 3x - 8$:

Increasing

Decreasing

Positive

Negative

5. Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?
- 1) $x < -2$, only
 - 2) $x < -2$ or $x > 3$
 - 3) $x < -2$ or $0 \leq x \leq 3$
 - 4) $x > 3$, only

6. At which x value is the graph of $f(x) = 2x^3 - 11x^2 - 14x + 26$ *not* decreasing?
- 1) -5
 - 2) 3.9
 - 3) 1.7
 - 4) 4.3

7. The graph of $y = 2^x - 4$ is positive on which interval?
- 1) $(-\infty, \infty)$
 - 2) $(2, \infty)$
 - 3) $(0, \infty)$
 - 4) $(-4, \infty)$

8. An estimate of the number of milligrams of a medication in the bloodstream t hours after 400 mg has been taken can be modeled by the function below.

$$I(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t,$$

where $0 \leq t \leq 6$

Over what time interval does the amount of medication in the bloodstream strictly increase?

- 1) 0 to 2 hours
- 2) 0 to 3 hours
- 3) 2 to 6 hours
- 4) 3 to 6 hours

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End Behavior and Shape of Polynomial Graphs

Sketch the shape and fill in the end behavior for each of the following polynomial equations

1. $f(x) = x^3 + 2x^2 - 9x - 18$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

2. $f(x) = x^4 - 10x^2 + 9$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

3. $p(x) = -x^3 - 3x^2 + 4x + 12$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

4. $f(x) = -x^4 + 3x^3 + 10x^2$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

5. $p(x) = x^3 - 3x^2 - 9x + 27$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

6. $h(x) = x^6 - 5x^4 + 4x^2$

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

$$7. g(x) = -\frac{1}{2}x^5 - 4x^2 + 3x^2 - 7$$

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

$$8. f(x) = x^4 + 11x^3 + 15x^2 - 25x$$

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

$$9. g(x) = -x^6 + 2x^3 + 4x^2 - 8x$$

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

$$10. m(x) = 2x^3 + 4x^2 - 8x$$

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

$$11. f(x) = -2x^4 - 2x^3 + 34x^2 + 42x - 72$$

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

$$12. g(x) = -x^5 + 5x^4 + 8x^3 - 44x^2 - 32x + 64$$

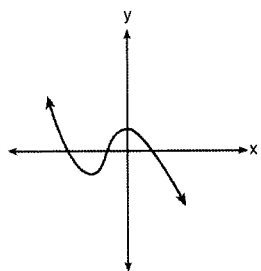
$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

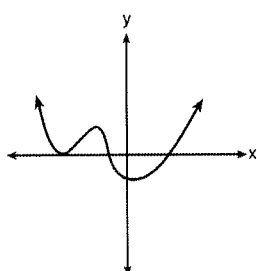
13. Which graph has the following characteristics?

- as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$
- as $x \rightarrow \infty$, $f(x) \rightarrow \infty$

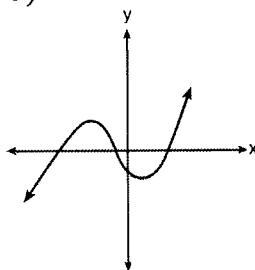
1)



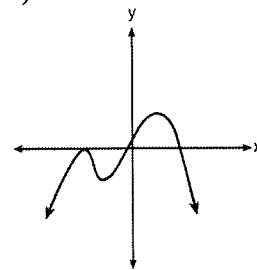
2)



3)



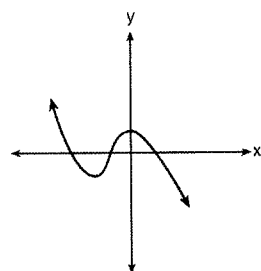
4)



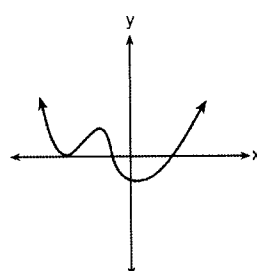
14. Which graph has the following characteristics?

- $x \rightarrow -\infty$, $f(x) \rightarrow \infty$
- $x \rightarrow \infty$, $f(x) \rightarrow \infty$

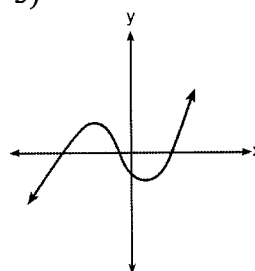
1)



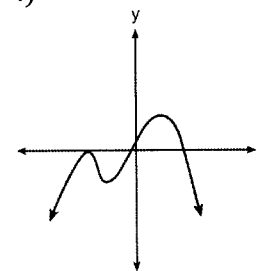
2)



3)



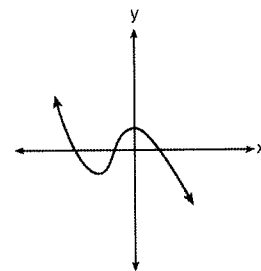
4)



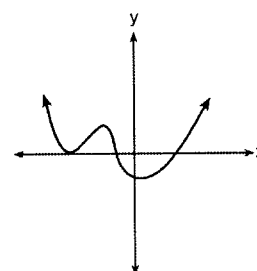
15. Which graph has the following characteristics?

- $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$
- $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

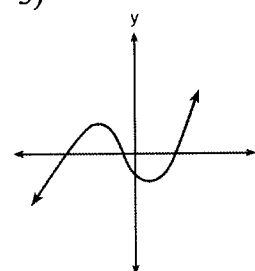
1)



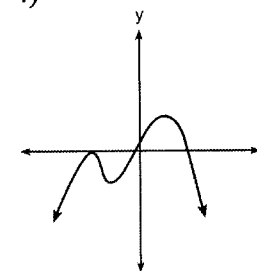
2)



3)



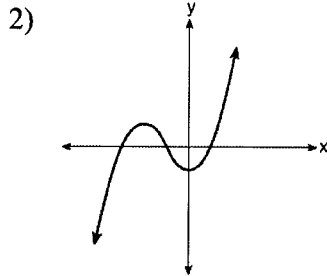
4)



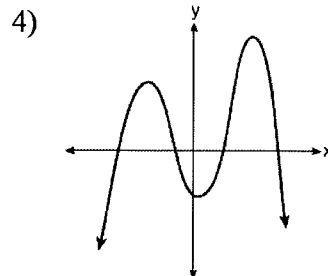
16. Consider the end behavior description below.

- as $x \rightarrow -\infty, f(x) \rightarrow \infty$
- as $x \rightarrow \infty, f(x) \rightarrow -\infty$

1) $f(x) = x^4 + 2x^2 + 1$



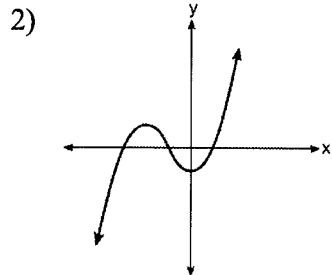
3) $f(x) = -x^3 + 2x - 6$



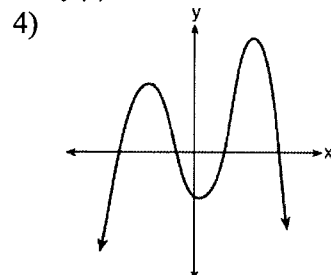
17. Consider the end behavior description below.

- as $x \rightarrow -\infty, f(x) \rightarrow \infty$
- as $x \rightarrow \infty, f(x) \rightarrow \infty$

1) $f(x) = x^4 + 2x^2 + 1$



3) $f(x) = -x^3 + 2x - 6$



18. Consider the end behavior description below.

- as $x \rightarrow -\infty, f(x) \rightarrow \infty$
- as $x \rightarrow \infty, f(x) \rightarrow -\infty$

Which function satisfies the given conditions?

1) $f(x) = -x^4 + 3x^3 + 2x^2 - 1$

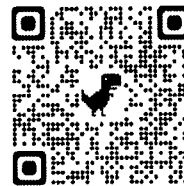
2) $f(x) = 2x^3 - 7x + 5$

3) $f(x) = -7x^5 + 5x^4 + 8x^2 - 6$

4) $f(x) = -8x^7 + 5x^5 - 11x^2 + 2x - 7$

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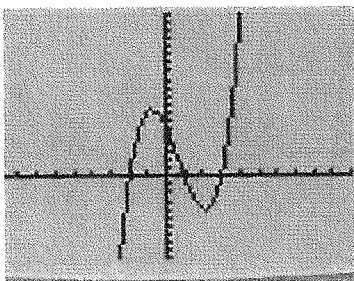
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Writing Equations of Polynomial Equations

Write a possible equation for each of the following polynomials and state the end behavior

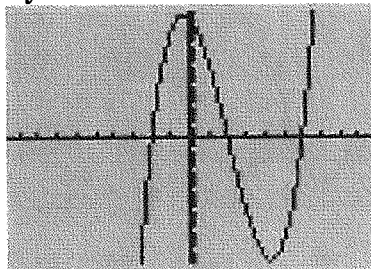
1.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

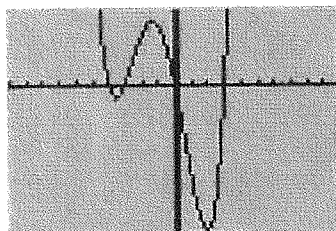
2.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

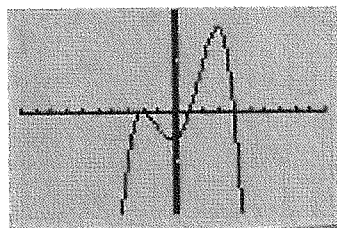
3.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

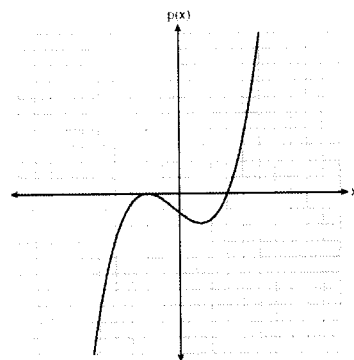
4.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

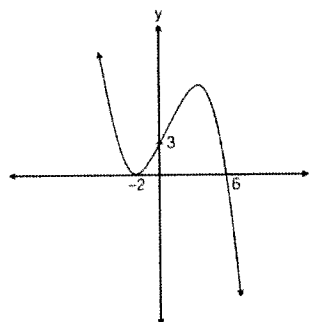
5.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

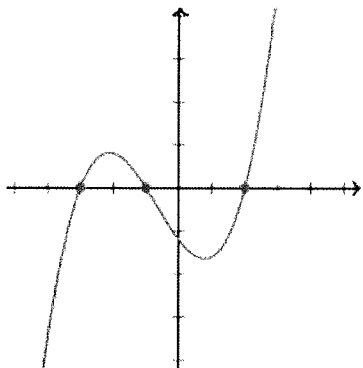
6.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

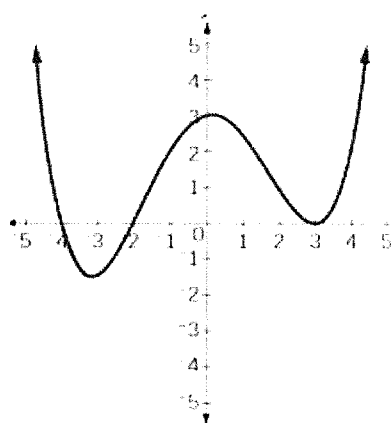
7.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

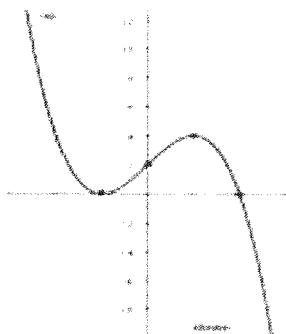
8.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

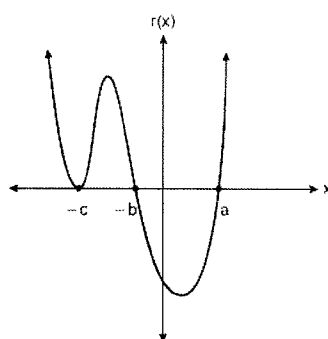
9.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

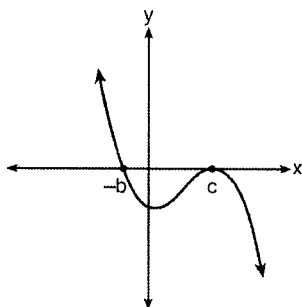
10.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

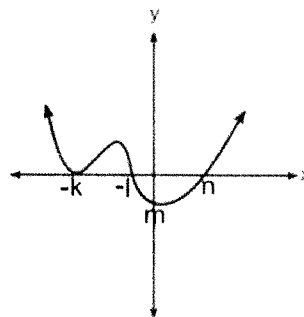
11.



$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$

12.

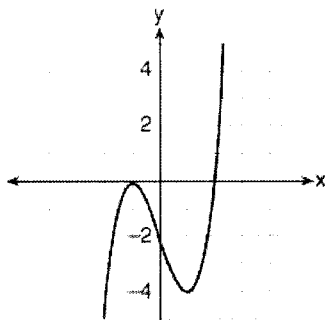


$$x \rightarrow -\infty, f(x) \rightarrow$$

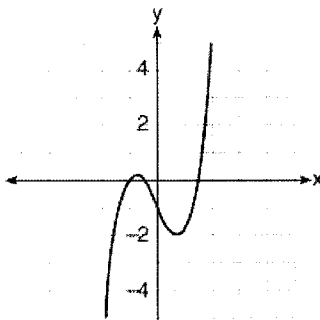
$$x \rightarrow \infty, f(x) \rightarrow$$

13. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

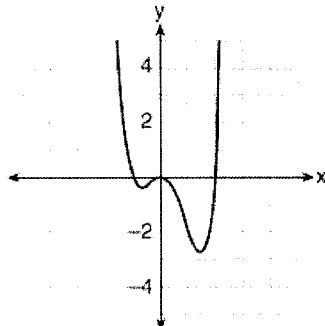
1)



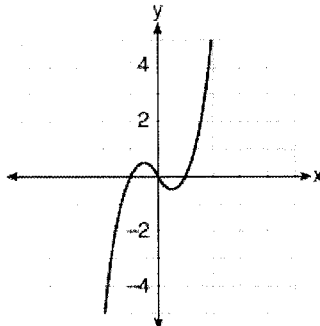
3)



2)

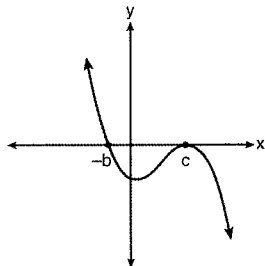


4)

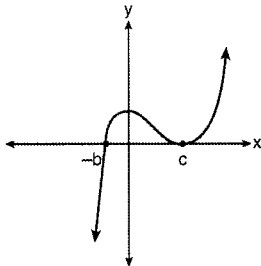


14. If a , b , and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)(x^2 - 2cx + c^2)$?

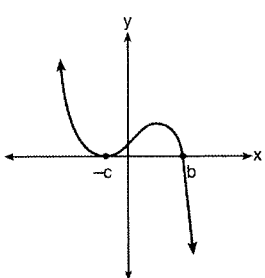
1)



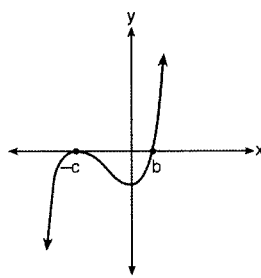
3)



2)



4)



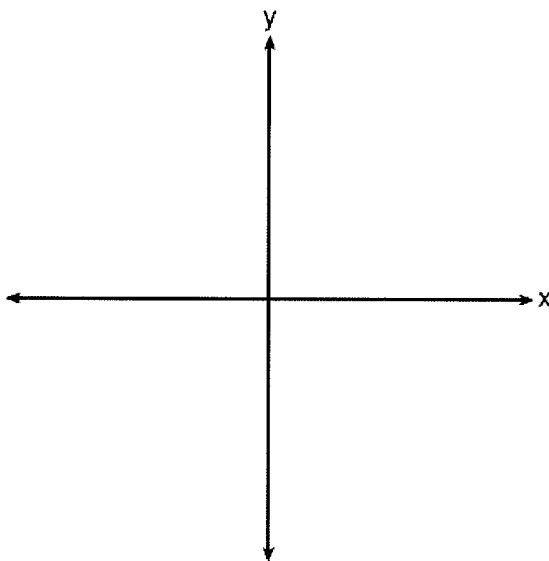
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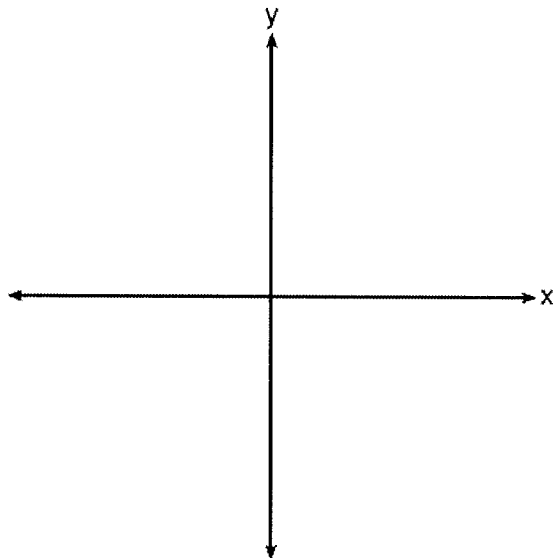


Sketching Polynomial Equations

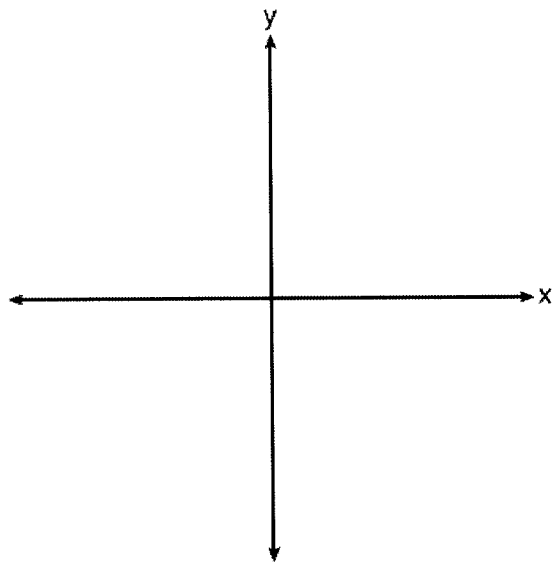
1. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.



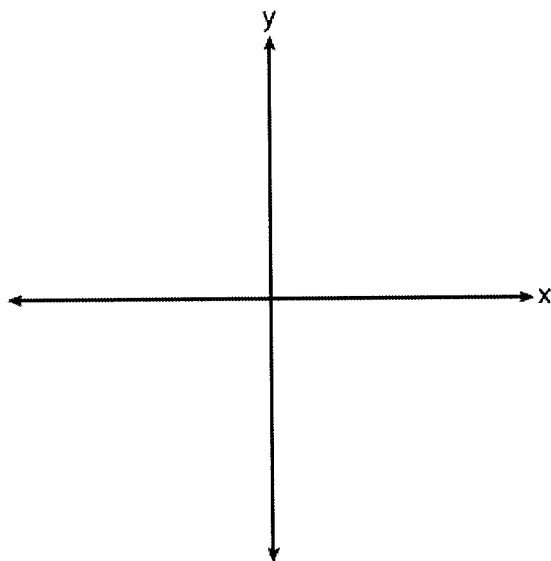
2. The zeros of a quartic polynomial function are 2, -2, 4, and -4. Use the zeros to construct a possible sketch of the function, on the set of axes below.



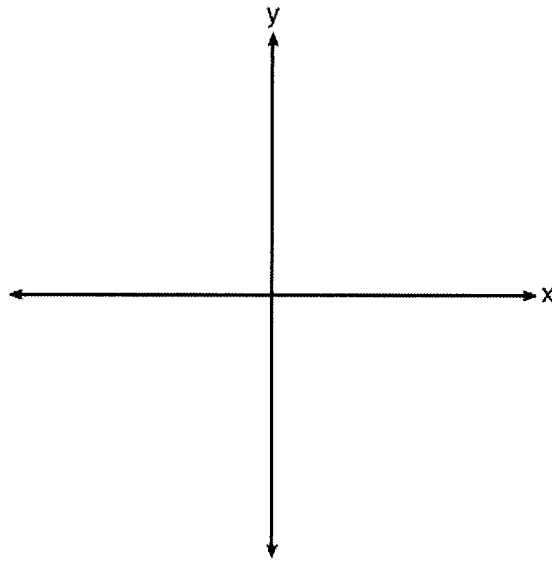
3. The zeros of a quartic polynomial function h are $-2, 1, 1,$ and 3 . Sketch a graph of $y = h(x)$ on the grid below.



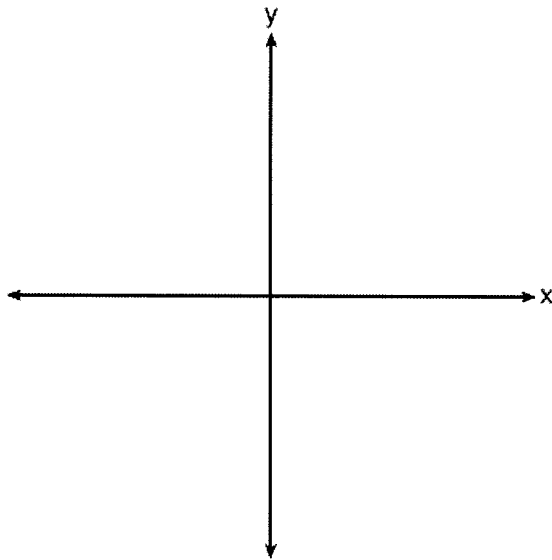
4. The zeros of a polynomial function are $-5, -5, \pm 2,$ and 0 . Sketch a graph of the polynomial functions on the grid below.



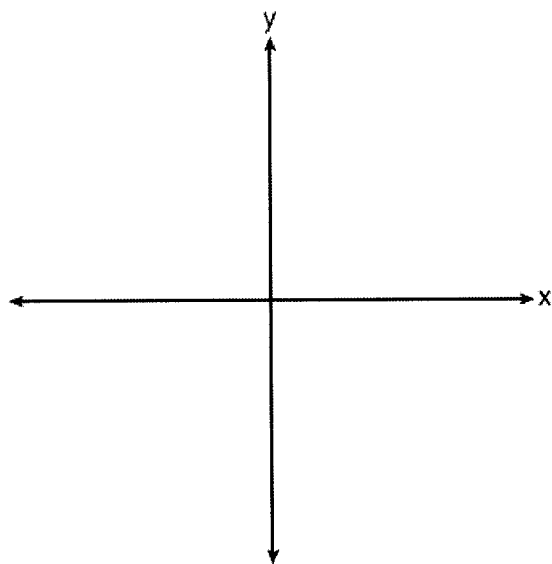
5. On the grid below, sketch a cubic polynomial whose factors are $x-3$, $x+4$, and $x+2$.



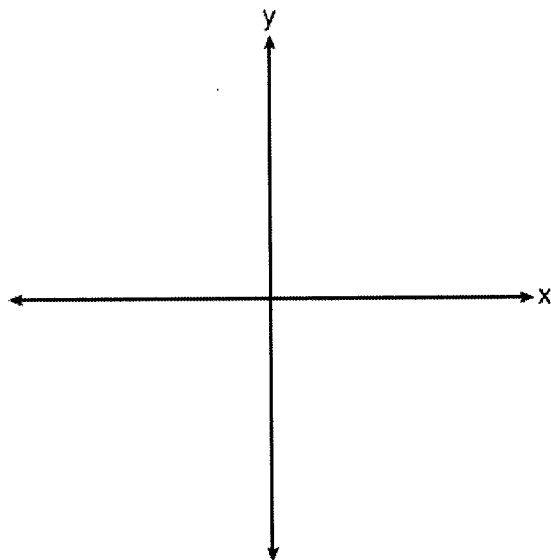
6. On the grid below, sketch a quartic polynomial whose factors are $x+5$, $x+2$, $x+2$, and $x-4$.



7. On the grid below, sketch a cubic polynomial whose factors are $x - 3$ and $x^2 + 8x + 16$.



8. On the grid below, sketch a quartic polynomial whose factors are $x^2 - 4x + 4$ and $x^2 + 10x + 25$.



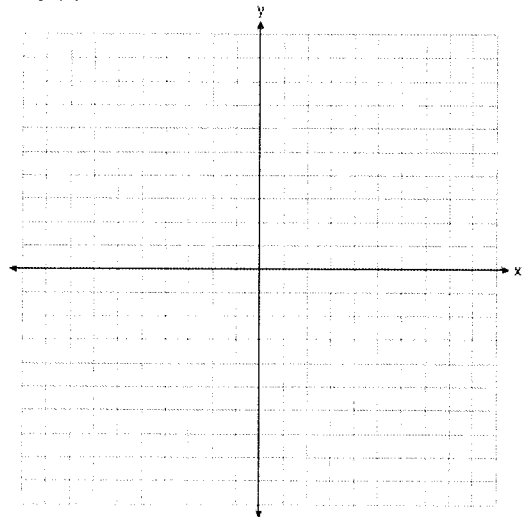
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Graphing Polynomial Equations

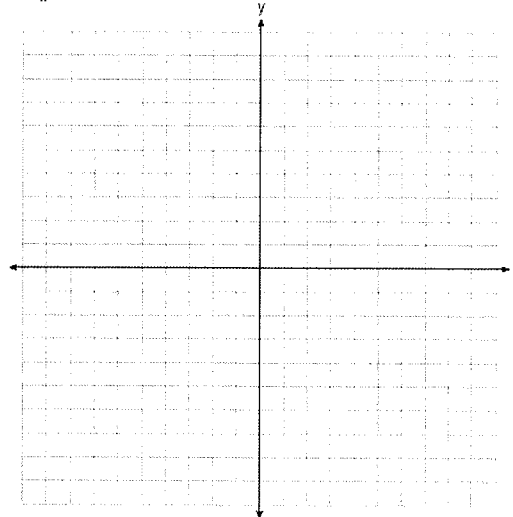
1. $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$.



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

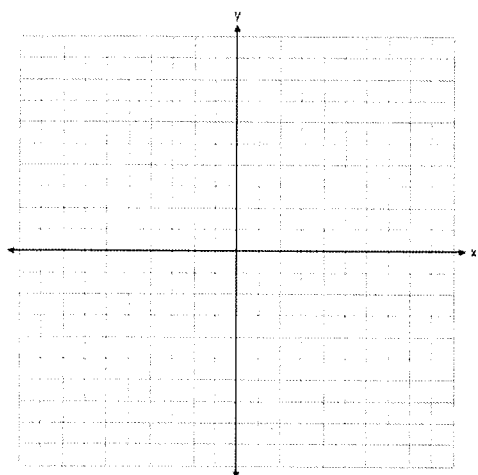
2. $y = x^3 - 4x^2 + 2x + 7$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

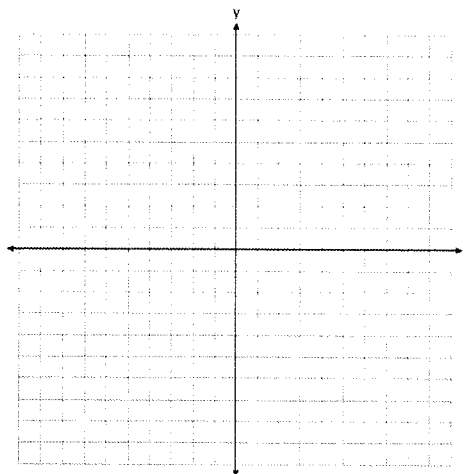
3. $p(x) = -x^3 - x^2 + 4x + 4$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

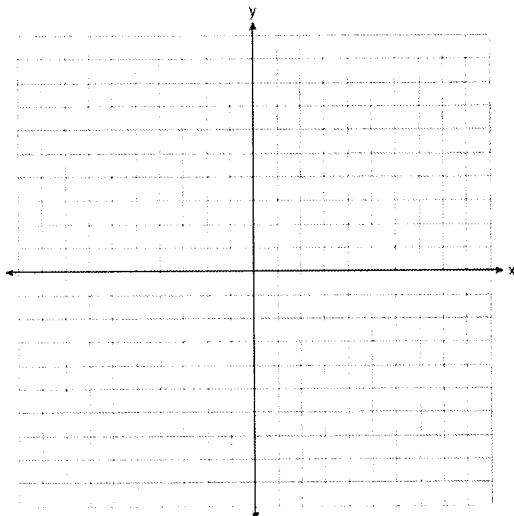
4. $p(x) = x^3 + 2x^2 - x - 2$ from $-2 \leq x \leq 1$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

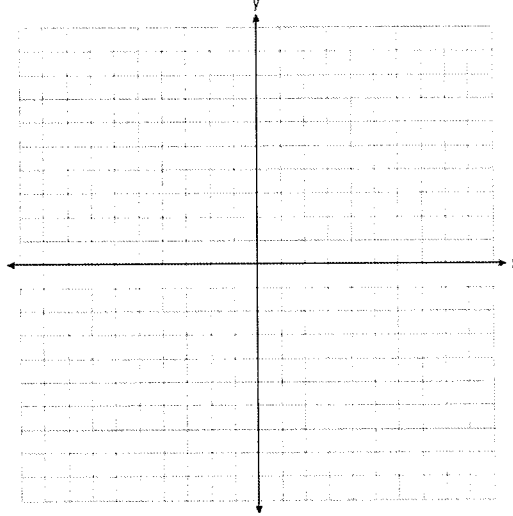
5. $p(x) = -x^3 + 4x^2 - 2x - 4$ from $-1 \leq x \leq 3$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

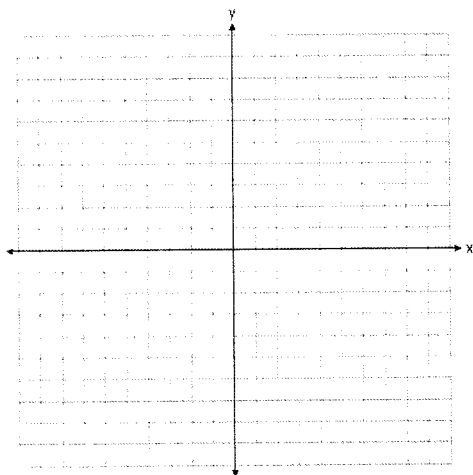
6. $p(x) = x^3 - 3x^2 - 5x + 1$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

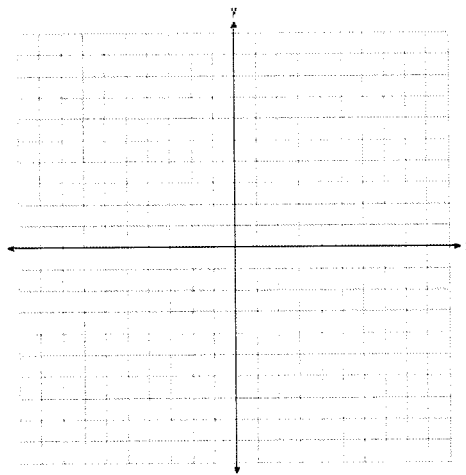
7. $p(x) = x^3 - 2x^2 + 3x - 8$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$

8. $p(x) = -x^4 + x^2 - 7x + 2$ from $-2 \leq x \leq 1$



$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



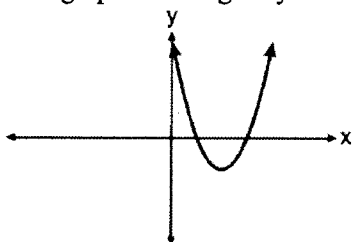
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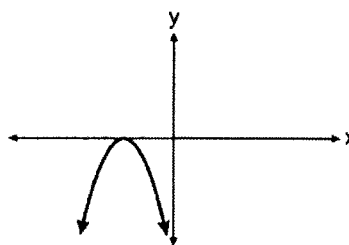
Imaginary Zeros

1. Which graph has imaginary roots?

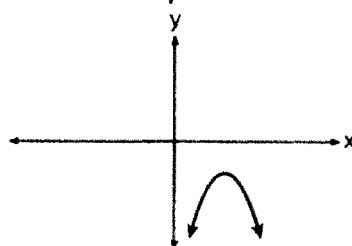
1)



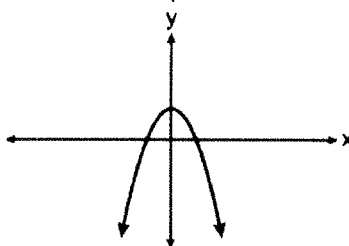
3)



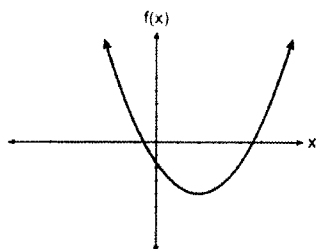
2)



4)



2. If $f(x)$ is represented by the graph below, Does $f(x)$ have imaginary roots? Explain your answer.



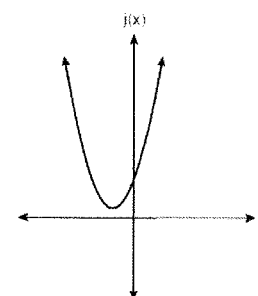
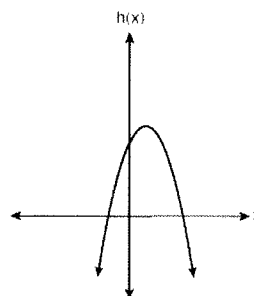
3. Which quadratic functions have imaginary roots?

1) $h(x)$ only

2) $j(x)$ only

3) Both $j(x)$ and $h(x)$

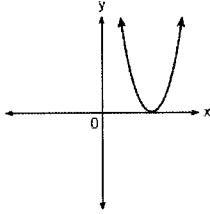
4) Neither $j(x)$ or $h(x)$



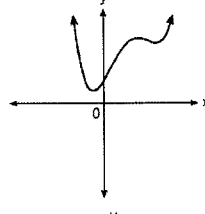
4. Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

5. Which of the following graphs have imaginary zeros?

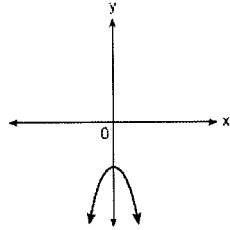
I



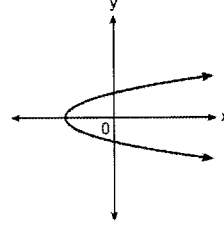
II



III



IV



1) I and IV

2) II and III

3) II only

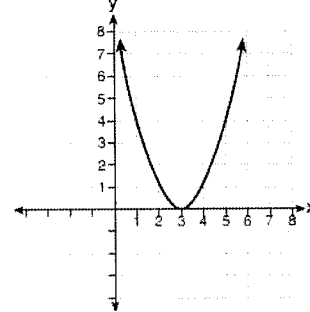
4) III and IV

6. Which representation of a quadratic has imaginary roots?

1)

x	y
-2.5	2
-2.0	0
-1.5	-1
-1.0	-1
-0.5	0
0.0	2

3)

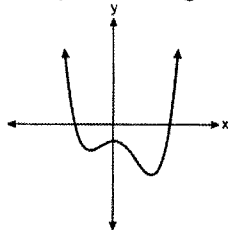


2) $2(x+3)^2 = 64$

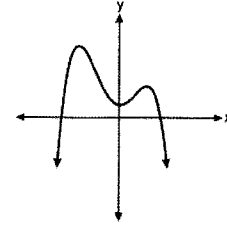
4) $2x^2 + 32 = 0$

7. Which graph could represent a 4th degree polynomial function with a positive leading coefficient, 2 real zeros, and 2 imaginary zeros?

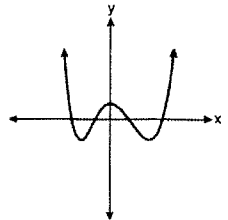
1)



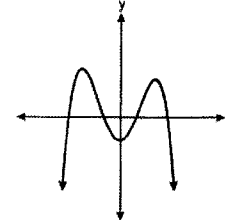
3)



2)



4)



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The Remainder Theorem

1. What is the remainder when $p(x) = x^3 - 9x^2 + 21x - 5$ is divided by $x - 5$? Is $x - 5$ a factor of $p(x)$? Explain your answer.
2. What is the remainder when $p(x) = x^4 - 8x^2 + 3x$ is divided by $x + 4$? Is $x + 4$ a factor of $p(x)$? Explain your answer.
3. What is the remainder when $p(x) = x^3 - 2x^2 + 6x - 2$ is divided by $x - 3$? Is $x - 3$ a factor of $p(x)$? Explain your answer.
4. What is the remainder when $p(x) = x^3 - 5x^2 - 5x + 25$ is divided by $x + 2$? Is $x + 2$ a factor of $p(x)$? Explain your answer.
5. What is the remainder when $p(x) = x^3 - 6x^2 + 4x - 1$ is divided by $x - 6$? Is $x - 6$ a factor of $p(x)$? Explain your answer.

6. What is the remainder when $p(x) = x^3 - 3x^2 - 8x + 4$ is divided by $x + 2$? Is $x + 2$ a factor of $p(x)$? Explain your answer.

7. What is the remainder when $p(x) = 2x^3 + 5x + 2$ is divided by $2x + 1$. Is $2x + 1$ a factor of $p(x)$? Explain your answer.

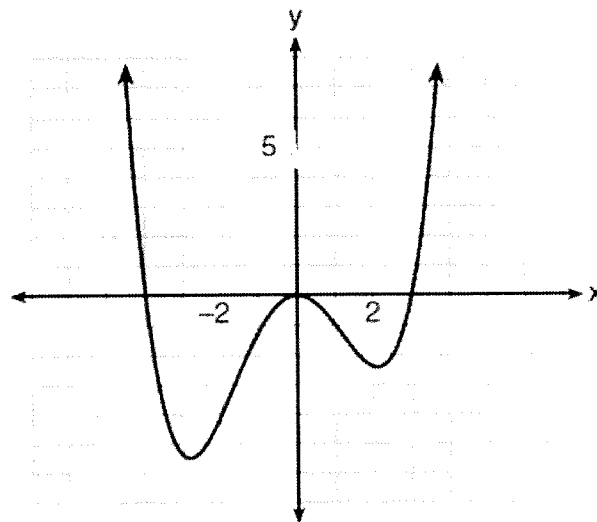
8. What is the remainder when $p(x) = 3x^3 - 2x^2 - 27x + 18$ is divided by $3x - 2$. Is $3x - 2$ a factor of $p(x)$? Explain your answer.

9. What is the remainder when $p(x) = 2x^3 - 2x^2 + 3$ is divided by $4x + 1$. Is $4x + 1$ a factor of $p(x)$? Explain your answer.

Use the graph below to the right to answer the following two questions.

10. What is the remainder when the following polynomial is divided by $x - 1$? Is $x - 1$ a factor? Explain your answer.

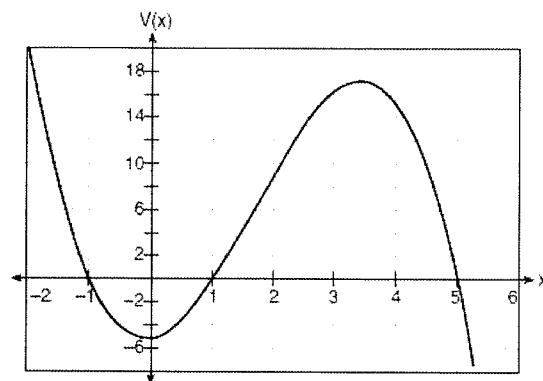
11. What is the remainder when the following polynomial is divided by $x - 3$? Is $x - 3$ a factor? Explain your answer.



Use the graph below to the right to answer the following two questions.

12. What is the remainder when the following polynomial is divided by $x-1$? Is $x-1$ a factor? Explain your answer.

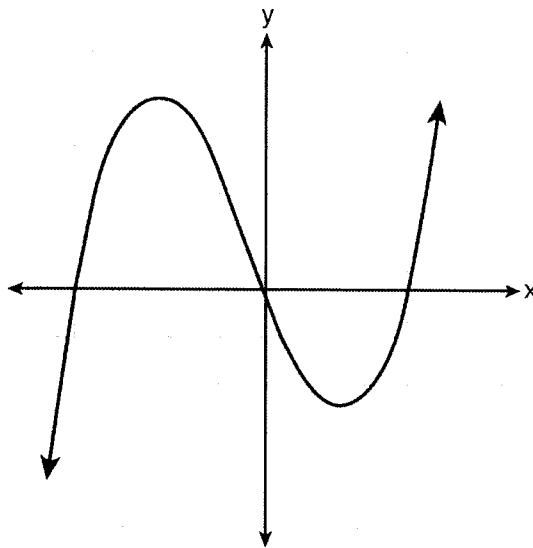
13. What is the remainder when the following polynomial is divided by $x+2$? Is $x+2$ a factor? Explain your answer.



Use the graph below to the right to answer the following two questions.

14. What is the remainder when the following polynomial is divided by $x-1$? Is $x-1$ a factor? Explain your answer.

15. What is the remainder when the following polynomial is divided by $x-3$? Is $x-3$ a factor? Explain your answer.



16. Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

- | | |
|----------|----------|
| 1) $x-2$ | 3) $x-4$ |
| 2) $x+2$ | 4) $x+4$ |

17. Which binomial is *not* a factor of the expression $x^3 - 11x^2 + 16x + 84$?

- | | |
|----------|----------|
| 1) $x+2$ | 3) $x-6$ |
| 2) $x+4$ | 4) $x-7$ |

18. Which binomial is *not* a factor of the expression $x^3 - 6x^2 - 49x - 66$?

- | | |
|-----------|----------|
| 1) $x-11$ | 3) $x+6$ |
| 2) $x+2$ | 4) $x+3$ |

19. Which binomial is a factor of the expression $x^3 - 7x - 6$?

- | | |
|------------|------------|
| 1) $x + 3$ | 3) $x - 2$ |
| 2) $x - 1$ | 4) $x + 2$ |

20. Which binomial is *not* a factor of the expression $x^3 - 4x^2 - 25x + 28$?

- | | |
|------------|------------|
| 1) $x + 6$ | 3) $x - 1$ |
| 2) $x - 7$ | 4) $x + 4$ |

21. Which binomial is not a factor of $p(x) = 2x^3 + 7x^2 - 5x - 4$?

- | | |
|------------|-------------|
| 1) $x + 4$ | 3) $x - 1$ |
| 2) $x + 1$ | 4) $2x + 1$ |

22. Which binomial is not a factor of $p(x) = 2x^3 - 5x^2 + 6x - 2$?

- | | |
|------------|-------------|
| 1) $x - 1$ | 3) $2x - 1$ |
| 2) $x - 2$ | 4) $2x + 1$ |

23. Given $P(x) = x^3 - 3x^2 - 2x + 4$, which statement is true?

- | | |
|--|---|
| 1) $(x - 1)$ is a factor because $P(-1) = 2$. | 3) $(x + 1)$ is a factor because $P(1) = 0$. |
| 2) $(x + 1)$ is a factor because $P(-1) = 2$. | 4) $(x - 1)$ is a factor because $P(1) = 0$. |

24. If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$

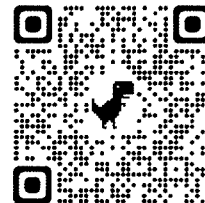
- | | |
|--|--|
| 1) equals 0 and $2x + 1$ is a factor of $f(x)$ | 3) does not equal 0 and $2x + 1$ is not a factor of $f(x)$ |
| 2) equals 0 and $2x - 1$ is a factor of $f(x)$ | 4) does not equal 0 and $2x - 1$ is a factor of $f(x)$ |

25. Consider the function $f(x) = 2x^3 + x^2 - 18x - 9$. Which statement is true?

- | | |
|-------------------------------------|---|
| 1) $2x - 1$ is a factor of $f(x)$. | 3) $f(3) \neq f\left(-\frac{1}{2}\right)$ |
| 2) $x - 3$ is a factor of $f(x)$. | 4) $f\left(\frac{1}{2}\right) = 0$ |

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Finding k in a Polynomial Equation

1. Consider the polynomial $p(x) = x^3 + kx^2 + x + 6$. Find a value of k so that $x + 1$ is a factor of P .

2. Consider the polynomial $p(x) = x^3 + kx - 30$. Find a value of k so that $x + 3$ is a factor of P .

3. Given $p(x) = 6x^3 + 31x^2 + kx - 12$, and $x + 4$ is a factor, find the value of k .

4. Given $z(x) = 6x^3 + bx^2 - 52x + 15$, and $x + 5$ is a factor, find the value of b .

5. Given $p(x) = x^3 + 5x^2 + kx - 24$, and $x + 3$ is a factor, find the value of k .

6. If $x - 1$ is a factor of $x^3 - kx^2 + 2x$, what is the value of k ?

7. The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

8. Consider the polynomial $p(x) = x^3 + kx + 2$. Find a value of k so that $x - 2$ is a factor of P .

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Polynomial Graphs/Remainder Theorem Review Sheet

1. The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \leq x \leq 3$. To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

2. A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x , in thousands. This function, p , is given below. Over the interval $0 \leq x \leq 9$, state the maximum profit and round to the *nearest integer*.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

3. Over which intervals is the graph of $f(x) = -x^4 + 15x^2 - 7$ strictly decreasing?

- 1) $(-2.7, 0)$
- 2) $(-\infty, -2.5)$
- 3) $(2.5, \infty)$
- 4) $(-1.4, 1.2)$

4. Over which intervals is the graph of $f(x) = x^3 + 8x^2 + 3x - 8$ strictly decreasing?

- 1) $(-6, 0)$
- 2) $(-\infty, -6)$
- 3) $(-.2, \infty)$
- 4) $(-5.1, -.2)$

5. Consider the end behavior description below.

- as $x \rightarrow -\infty, f(x) \rightarrow \infty$
- as $x \rightarrow \infty, f(x) \rightarrow -\infty$

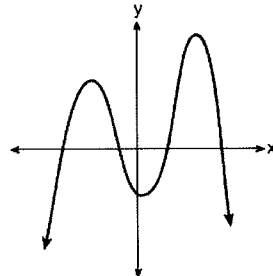
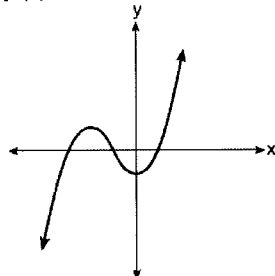
Which function satisfies the given conditions?

1) $f(x) = x^4 + 2x^2 + 1$

3) $f(x) = -x^3 + 2x - 6$

2)

4)



6. Which graph has the following characteristics?

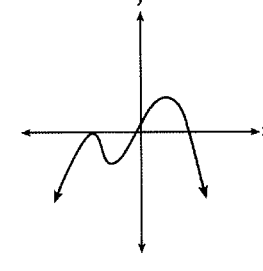
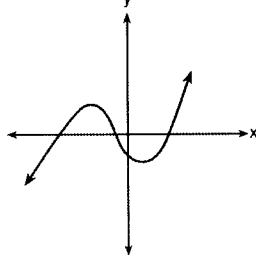
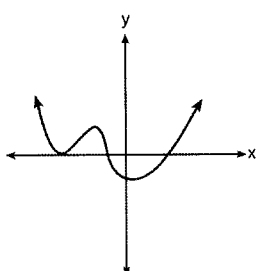
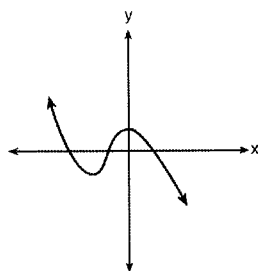
- three real zeros
- as $x \rightarrow -\infty, f(x) \rightarrow -\infty$
- as $x \rightarrow \infty, f(x) \rightarrow \infty$

1)

2)

3)

4)



7. A sketch of $r(x)$ is shown below.

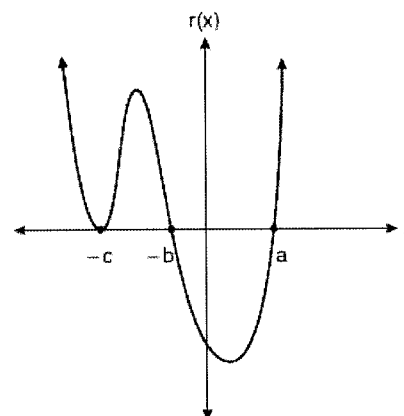
An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$

3) $r(x) = (x+a)(x-b)(x-c)$

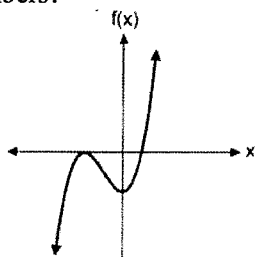
2) $r(x) = (x+a)(x-b)(x-c)^2$

4) $r(x) = (x-a)(x+b)(x+c)^2$

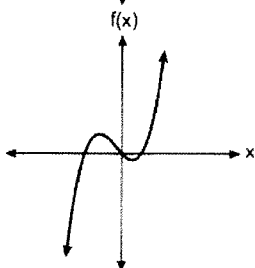


8. Which graph best represents the graph of $f(x) = (x + a)^2(x - b)$, where a and b are positive real numbers?

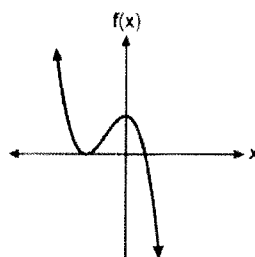
1)



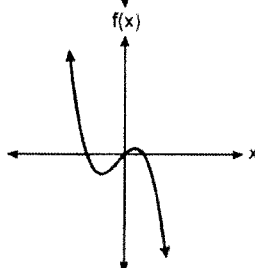
2)



3)

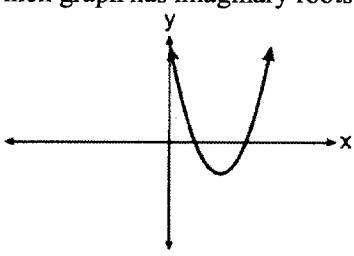


4)

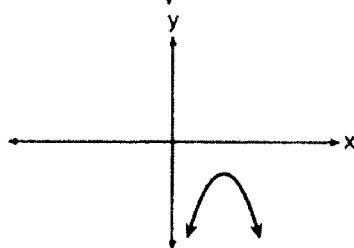


9. Which graph has imaginary roots?

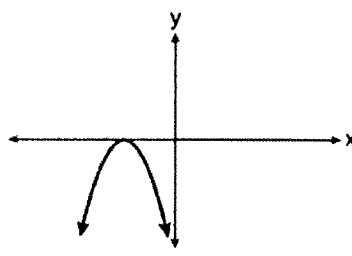
1)



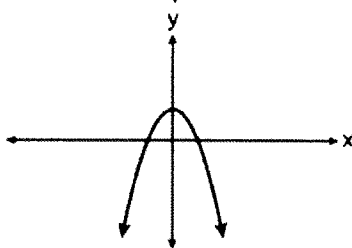
2)



3)



4)



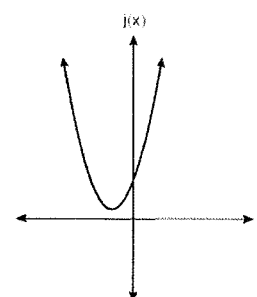
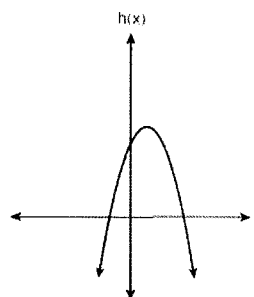
10. Which quadratic functions have imaginary roots?

1) $h(x)$ only

2) $j(x)$ only

3) Both $j(x)$ and $h(x)$

4) Neither $j(x)$ or $h(x)$



11. Is $x-6$ a factor of $x^3 - 6x^2 + 4x - 1$? Explain your answer.

12. Is $x+2$ a factor of $p(x) = x^3 - 3x^2 - 8x + 4$? Explain your answer.

13. Which binomial is *not* a factor of the expression $x^3 - 6x^2 - 49x - 66$?

- | | |
|-----------|----------|
| 1) $x-11$ | 3) $x+6$ |
| 2) $x+2$ | 4) $x+3$ |

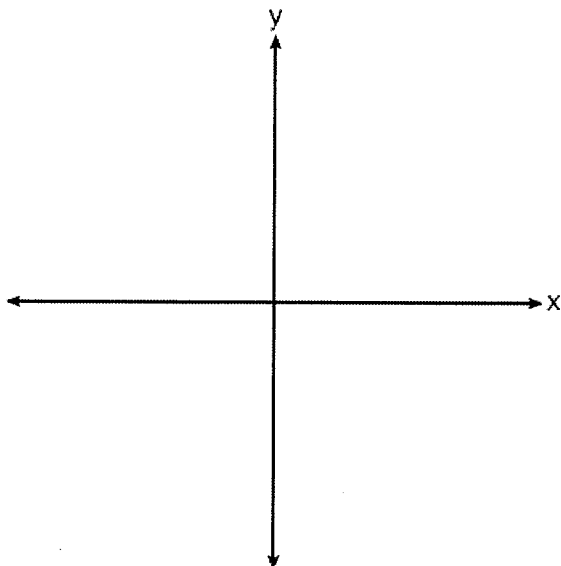
14. Which binomial is a factor of the expression $x^3 - 7x - 6$?

- | | |
|----------|----------|
| 1) $x+3$ | 3) $x-2$ |
| 2) $x-1$ | 4) $x+2$ |

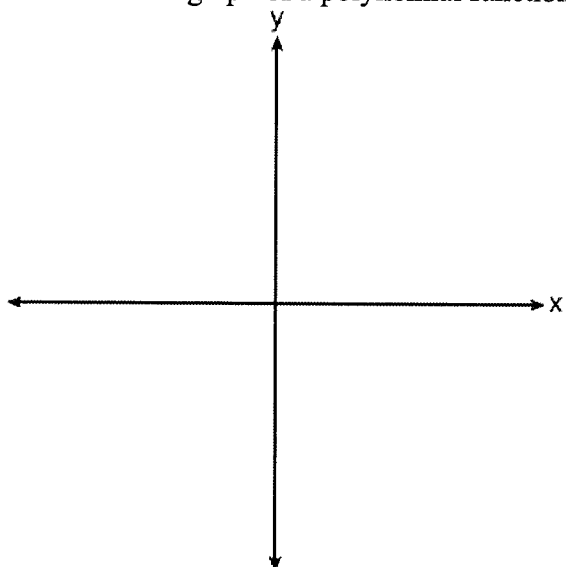
15. Given $p(x) = 6x^3 + 31x^2 + kx - 12$, and $x+4$ is a factor, find the value of k .

16. Consider the polynomial $p(x) = x^3 + kx - 30$. Find a value of k so that $x+3$ is a factor of P .

17. Sketch the graph of a polynomial function whose factors are $(x+1)$, $(x-4)^2$, and $(x+2)$.



18. Sketch the graph of a polynomial functions whose zeros are -5, -2, -2, and 6.



19. Solve for x:

$$3x^2 - 4x - 4 = 0$$

20. Solve for x:

$$6x^2 - 11x - 2 = 0$$

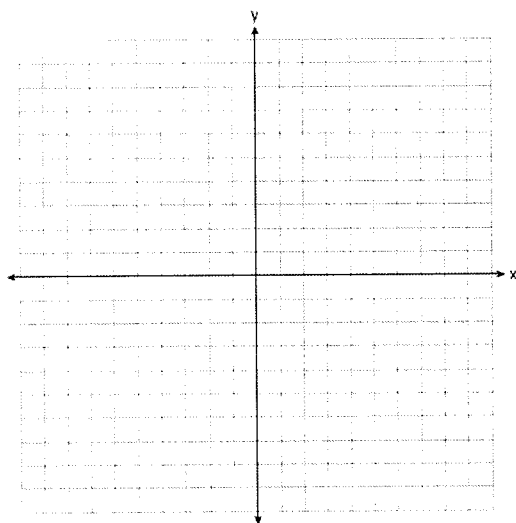
21. Solve for x:

$$x^3 + 6x^2 = 4x + 24$$

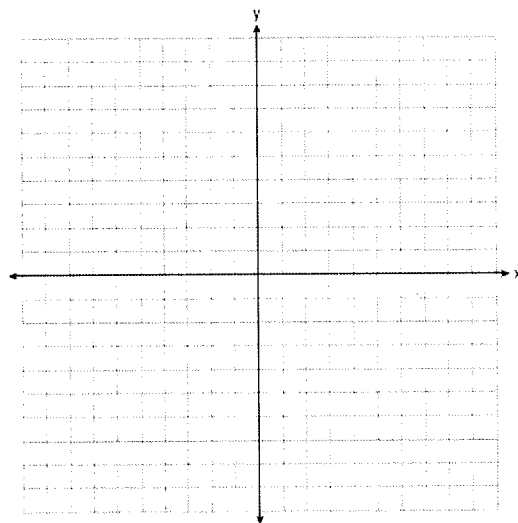
22. Solve for x:

$$x^3 - 2x^2 = 9x - 18$$

23. Graph $p(x) = -x^3 + 4x^2 - 2x - 4$
from $-1 \leq x \leq 3$



24. $p(x) = x^3 - 3x^2 - 5x + 1$



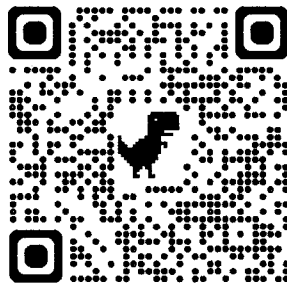
Name:

Common Core Algebra II

Unit 4

Functions and Key Points

Mr. Schlansky



Lesson 1: I can determine the domain by finding where the graph starts and ends from left to right and determine the range by finding where the graph starts and ends from bottom to top.

Domain: Hold pen vertically and travel left to right to find where graph starts and ends.

Range: Hold pen horizontally and travel bottom to top to find where graph starts and ends.

Lesson 2: I can write intervals in both interval notation and set builder notation

Interval notation:

Where the graph starts and stops from left to right

Set builder notation:

If there is an infinity involved, x comes first.

If there is not an infinity involved, $a < x < b$

Lesson 3: I can determine the intervals where functions are increasing, decreasing, positive, and negative by finding the x value where each interval starts and stops.

Increasing: The function goes “uphill” from left to right

Decreasing: The function goes “downhill” from left to right

Positive: The function is above the x axis

Negative: The function is below the x axis

Lesson 4: I can find key points of a graph using the 2nd Trace (Calc) Menu in my graphing calculator.

The zeros (x -intercepts) are where the graph hits the x axis. 2nd Trace (Calc), zeros, left bound (move cursor to the left of the point and hit enter), right bound (move cursor to the right of the point and hit enter), enter.

The y -intercept is when x is 0. Find this value in your table.

Local extrema are the turning points (minimum and maximum). 2nd Trace (Calc), minimum/maximum, left bound (move cursor to the left of the point and hit enter), right bound (move cursor to the right of the point and hit enter), enter.

Lesson 5: I can determine where functions are increasing/decreasing and positive/negative by stating the x value of where each piece starts and stops from left to right using my graphing calculator.

Increasing/Decreasing:

-Find the relative minimums and maximums using 2nd Trace Menu

-Use Lesson 3 to write the intervals where each piece starts and stops

Positive/Negative:

-Find the zeros using the 2nd Trace Menu

-Use Lesson 3 to write the intervals where each piece starts and stops

Lesson 6: I can solve systems of equations graphically using my graphing calculator to find the point(s) of intersection.

Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

- 1) Type equations into Y_1 and Y_2
- 2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
- 3) 2nd Trace (Calc), 5 (Intersect)
- 4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

*The solutions to the system of equations are the x values of the intersections.

Lesson 7: I can transform functions using the translation and reflection rules.

Translations (+ or -)

If adding to $f(x)$, the graph moves up or down

If adding to x , the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

Reflection (-)

Reflect over the axis that you are *not* negating.

$y = -f(x)$, reflection over the x - axis (negate the y , reflect over the x)

$y = f(-x)$, reflection over the y - axis (negate the x , reflect over the y)

Lesson 8: I can find the average rate of change of a function using $\frac{y_2 - y_1}{x_2 - x_1}$.

Average rate of change: $\frac{y_2 - y_1}{x_2 - x_1}$

ALWAYS CREATE A TABLE

If given a table: circle the values in the table and do $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$

If given a graph: create a table and pull the y values from the graph. Then circle the values in the table and do $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$.

If given an equation: create a table by typing into $Y=$ and going to 2nd Graph (Table). Then circle the values in the table and do $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$.

Lesson 9: I can find the average rate of change and write a context sentence using $\frac{y_2 - y_1}{x_2 - x_1}$

and the given script.

Average rate of change: $\frac{y_2 - y_1}{x_2 - x_1}$

“On average, from x to x, the y topic is increasing/decreasing by AROC y units per x unit”

Lesson 10: I can determine which intervals have the fastest and slowest average rate of change by looking at the slopes (graph) or calculating the AROC for each interval (table).

Average rate of change: $\frac{y_2 - y_1}{x_2 - x_1}$

Graphs: The steeper the slope, the greater the average rate of change. The flatter the slope, the slower the average rate of change.

Tables: Calculate the average rate of change for each interval.

Lesson 11: I can find the inverse of a function by switching x and y and solving for y.

Inverse of a function $f^{-1}(x)$:

Switch x and y, solve for y

If multiple choice:

Type the original equation into y = and pull a few nice points from the table

Switch x and y in your table

Type in each choice and see which has the switched points in its table

Lesson 12: I can determine if functions are even, odd, or neither by determining if it is symmetric to the y axis or the origin.

IF GIVEN AN EQUATION, TYPE INTO Y=!!!

Even Functions: Symmetric to the y-axis

Odd Functions: Symmetry to the origin (Turn it upside down and see the exact same thing)

Lesson 13: I can prepare for my exam by practicing!

