Name: Key

SAT Topical Review

Mr. Schlansky

Multiple Choice Strategy with Equations
Non-Calculator Section
Substitute each potential answer in for all x values
If the left hand side is equal to the right hand side, it is a solution to the equation.

Calculator Section
Store each potential answer (_____STO → X)
Type in left side
Type in right side
The answer is whichever answers match.

*Be sure to check all potential answers as most equations have multiple answers

1. The solution set of the equation $\sqrt{x+3} = 3-x$ is
1) {1} 3) {1, 6}
2) {0} 4) {2, 3}

$x=1$: $\sqrt{1+3} = 3-1$, $\sqrt{4} = 2$, $2=2$ ✓
$x=6$: $\sqrt{6+3} = 3-6$, $\sqrt{9} = -3$, $3 \neq -3$

2. What is the solution set for the equation $\sqrt{5x+29} = x+3$?
1) {4} 3) {4, 5}
2) {−5} 4) {−5, 4}

$x=4$: $\sqrt{5(4)+29} = 4+3$, $\sqrt{49} = 7$, $7=7$ ✓
$x=5$: $\sqrt{5(5)+29} \neq 5+3$, $\sqrt{54} \neq 8$ ✗
$x=-5$: $\sqrt{5(-5)+29} = -5+3$, $\sqrt{4} \neq -2$, $2 \neq -2$ ✗

3. The solution set of $\sqrt{3x+16} = x+2$ is
1) {−3, 4} 3) {3}
2) {−4, 3} 4) {−4}

$x=-4$: $\sqrt{3(-4)+16} = -4+2$, $\sqrt{4} \neq -2$, $2 \neq -2$
$x=3$: $\sqrt{3(3)+16} = 3+2$, $\sqrt{25} = 5$, $5=5$ ✓

4. The solution set of the equation $\sqrt{2x-4} = x-2$ is
1) {−2, −4} 3) {4}
2) {2, 4} 4) { }

$x=2$: $\sqrt{2(2)-4} = 2-2$, $\sqrt{0} = 0$, $0=0$ ✓
$x=4$: $\sqrt{2(4)-4} = 4-2$, $\sqrt{4} = 2$, $2=2$ ✓

5. The solution set for the equation $\sqrt{56-x} = x$ is
1) {−8, 7} 3) {7}
2) {−7, 8} 4) { }

$x=7$: $\sqrt{56-7} = 7$, $\sqrt{49} = 7$, $7=7$ ✓
$x=-8$: $\sqrt{56+8} \neq -8$, $\sqrt{64} \neq -8$, $8 \neq -8$ ✗

6. What is the solution set of $\frac{x}{2}+3 = 4$?
1) {-4} 3) {-3}
2) {2} 4) {6}

$x=2$ $\frac{2}{2}+3 = 4$
$1+3 = 4$
$4=4$

7. What is the solution set of $\frac{2}{x}+\frac{1}{2}=1$?

1) {4} 3) {-4}
2) {-2} 4) {0}

$x=4$
$\frac{2}{4}+\frac{1}{2}=1$
$\frac{1}{2}+\frac{1}{2}=1$
$1=1$ ✓

8. What is the solution set of $\frac{1}{x}+x+3=1$?

1) {1} 3) {}
2) {0} 4) {-1}

$x=-1$ $\frac{1}{-1}-1+3=1$
$-1-1+3=1$
$1=1$ ✓

9. What is the solution set of the equation $\frac{30}{x^2-9}+1=\frac{5}{x-3}$?

1) {2,3} 3) {3}
2) {2} 4) { }

$x=2$
$\frac{30}{(2)^2-9}+1=\frac{5}{2-3}$
$\frac{30}{-5}+1=\frac{5}{-1}$
$-6+1=-5$
$-5=-5$ ✓

$x=3$
$\frac{30}{(3)^2-9}+1\neq\frac{5}{3-3}$
$\frac{30}{0}+1\neq\frac{5}{0}$

10. What is the solution set of $2|x+4|=8$?

1) {4} 3) {-8}
2) {-8,0} 4) {0}

$x=-8$
$2|-8+4|=8$
$2|-4|=8$
$2(4)=8$
$8=8$ ✓

$x=0$
$2|0+4|=8$
$2|4|=8$
$2(4)=8$
$8=8$ ✓

11. What is the solution set of $|x-4|=3x-6$?

1) {1} 3) {1,2.5}
2) {2.5} 4) {1,3}

$x=2.5$
$|2.5-4|=3(2.5)-6$
$|-1.5|=1.5$
$1.5=1.5$ ✓

$x=1$
$|1-4|=3(1)-6$
$|-3|=-3$
$3\neq-3$

12. What is the solution set of $|x-2|=3x+10$?

1) { } 3) {-6}
2) {-2} 4) {-2,-6}

$x=-2$ $|-2-2|=3(-2)+10$
$|-4|=4$
$4=4$ ✓

$x=-6$
$|-6-2|\neq3(-6)+10$
$|-8|\neq-8$
$8\neq-8$

13. What is the solution set of the equation $|4a+6|-4a=-10$?

1) No solution 3) $\left\{\frac{1}{2}\right\}$
2) {0} 4) $\left\{0,\frac{1}{2}\right\}$

$x=0$ $|4(0)+6|-4(0)=-10$
$|6|-0\neq-10$
$6\neq-10$

$x=\frac{1}{2}$
$|4(\frac{1}{2})+6|-4(\frac{1}{2})=-10$
$|8|-2=-10$
$8-2\neq-10$
$6\neq-10$

Solving Linear Equations

1) Get rid of fractions (Multiply by the LCD)
   Integer LCD: Find smallest common multiple of all integers in denominators
   Variable LCD: Put all factors in all denominators together
2) Get rid of parenthesis (Distribute)
3) Combine like terms on each side
4) Bring all variables to one side
5) Isolate variable (add/subtract first, divide last)

1. $6x+3-8x=13$

$-2x+3=13$
$-3 \quad -3$
$-2x=10$
$\frac{-2x}{-2}=\frac{10}{-2}$
$x=-5$

2. $17=3(p-5)+8$

$17=3p-15+8$
$17=3p-7$
$+7 \quad +7$
$\frac{24}{3}=\frac{3p}{3}$
$8=p$

3. $\frac{1}{3}\left(\frac{m}{7}\right)=\left(\frac{m+4}{7}\right)21$

$3+7m=3(m+4)$
$3+7m=3m+12$
$-3m \quad -3m$
$3+4m=12$
$-3 \quad -3$
$4m=9$
$\frac{4m}{4}=\frac{9}{4}$
$m=\frac{9}{4}$

4. $x+4-6x=6-8x-2$

$-5x+4=-8x+4$
$+8x \quad +8x$
$3x+4=4$
$-4 \quad -4$
$\frac{3x}{3}=\frac{0}{3}$
$x=0$

5. $5+3(q-4)=2(q+1)$

$5+3q-12=2q+2$
$3q-7=2q+2$
$-2q \quad -2q$
$q-7=2$
$+7 \quad +7$
$q=9$

6. $5-(t+3)=-1+2(t-3)$

$5-t-3=-1+2t-6$
$-t+2=2t-7$
$+t \quad +t$
$2=3t-7$
$+7 \quad +7$
$\frac{9}{3}=\frac{3t}{3}$
$3=t$

7. $2(x+4)=3(x-2)$

$2x+8=3x-6$
$-2x \quad -2x$
$8=x-6$
$+6 \quad +6$
$14=x$

8. $9x-8+4x=7x+16$

$13x-8=7x+16$
$-7x \quad -7x$
$6x-8=16$
$+8 \quad +8$
$\frac{6x}{6}=\frac{24}{6}$
$x=4$

9. $\frac{2}{5}\left(\frac{2}{5}x\right)=(10)\frac{5}{2}$

$x=25$

10. $\frac{3}{7}\left(\frac{3}{7}x\right)=(12)\frac{7}{3}$

$x=28$

11. $\frac{3}{4}\left(\frac{4}{3}x\right)=(24)\frac{3}{4}$

$x=18$

12. $\frac{9}{2}\left(\frac{2}{9}x\right)=(8)\frac{9}{2}$

$x=36$

13. $\left(\frac{m}{7}\right)=\left(\frac{m+4}{3}\right)21$

$7m=3(m+4)$

$7m=3m+12$

$-3m\ -3m$

$\frac{4m}{4}=\frac{12}{4}$

$m=3$

14. $\left(\frac{a}{28}\right)=\left(\frac{28}{30}\right)30$

$\frac{15a}{15}=\frac{28}{15}$

$a=\frac{28}{15}$

15. $\left(\frac{x}{3}\right)+\left(\frac{x+1}{2}\right)=(x)6$

$2x+3(x+1)=6x$

$2x+3x+3=6x$

$5x+3=6x$

$-5x\ \ -5x$

$3=x$

16. $\left(\frac{1}{7}\right)+\left(\frac{2x}{3}\right)=\left(\frac{15x-3}{21}\right)21$

$3+14x=15x-3$

$-14x\ -14x$

$3=x-3$

$+3\ \ +3$

$6=x$

17. $\left(\frac{3}{6x}\right)+\left(\frac{1}{2}\right)=\left(\frac{8}{x}\right)+\left(\frac{4}{3}\right)6x$

$3+3x=48+8x$

$-3x\ \ -3x$

$3=48+5x$

$-48\ -48$

$\frac{-45}{5}=\frac{5x}{5}$

$-9=x$

18. $\left(\frac{4x}{x-3}\right)=(2)+\left(\frac{12}{x-3}\right)x-3$

$4x=2(x-3)+12$

$4x=2x-6+12$

$4x=2x+6$

$-2x\ \ -2x$

$\frac{2x}{2}=\frac{6}{2}$

$x=3$

No solution (3 makes the denominator 0)

19. $\left(\frac{1}{m+10}\right)+\left(\frac{1}{5}\right)=\left(\frac{3}{m+10}\right)5(m+10)$

$5+m+10=15$

$m+15=15$

$-15\ -15$

$m=0$

20. $\frac{2}{x-1}=\frac{2}{x}+\frac{1}{x-1}$

$x(x+1)\quad x(x-1)$

$2x=2(x-1)+x$

$2x=2x-2+x$

$2x=3x-2$

$-3x\ \ -3x$

$\frac{-x}{-1}=\frac{-2}{-1}$

$x=2$

Solving Linear Systems
Elimination Method
(Variables are lined up):

1) Choose a variable to cancel and multiply each equation by the other's coefficient
*multiply by negative if they are the same sign
2) Add equations together
3) Solve equation for one variable
4) Substitute answer in to either equation to find the second variable

1. $-3(x - 2y = 14)$
$1(3x + 9y = 27)$
$-3x + 6y = -42$
$+\ 3x + 9y = 27$
$15y = -15$
$y = -1$
$x - 2y = 14$
$x - 2(-1) = 14$
$x + 2 = 14$
$x = 12$
$(12, -1)$

2. $1(3x - y = 3)$
$-3(x + 3y = 11)$
$3x - y = 3$
$+\ -3x - 9y = -33$
$-10y = -30$
$y = 3$
$3x - y = 3$
$3x - 3 = 3$
$+3 \quad +3$
$3x = 6$
$x = 2$
$(2, 3)$

3. $1(2x + y = 3)$
$2(-x + 3y = -12)$
$2x + y = 3$
$+\ -2x + 6y = -24$
$7y = -21$
$y = -3$
$2x + y = 3$
$2x - 3 = 3$
$+3 \quad +3$
$2x = 6$
$x = 3$
$(3, -3)$

4. $1(2x + 3y = 12)$
$3(3x - y = 13)$
$2x + 3y = 12$
$9x - 3y = 39$
$11x = 51$
$x = 3$
$2x + 3y = 12$
$2(3) + 3y = 12$
$6 + 3y = 12$
$-6 \quad -6$
$3y = 6$
$y = 2$
$(3, 2)$

5. $2(-3x + 4y = 12)$
$3(2x + y = -8)$
$-6x + 8y = 24$
$+\ 6x + 3y = -24$
$11y = 0$
$y = 0$
$2x + y = -8$
$2x + 0 = -8$
$2x = -8$
$x = -4$
$(-4, 0)$

6. $3(2x + 4y = -4)$
$-2(3x + 5y = -3)$
$6x + 12y = -12$
$+\ -6x - 10y = 6$
$2y = -6$
$y = -3$
$2x + 4y = -4$
$2x + 4(-3) = -4$
$2x - 12 = -4$
$+12 \quad +12$
$2x = 8$
$x = 4$
$(4, -3)$

7. $2(2x+4y=-4)$
$4(3x-2y=-14)$

$+4x+8y=-8$
$12x-8y=-56$

$16x=-64$
$16 \quad 16$

$x=-4$

$2x+4y=-4$
$2(-4)+4y=-4$
$-8+4y=-4$
$+8 \quad +8$
$4y=4$
$4 \quad 4$
$y=1$

$(-4,1)$

8. $-2(5x-y=-17)$
$5(2x-3y=-12)$

$+-10x+2y=34$
$10x-15y=-60$

$-13y=-26$
$-13 \quad -13$

$y=2$

$5x-y=-17$
$5x-2=-17$
$+2 \quad +2$
$5x=-15$
$5 \quad 5$
$x=-3$

$(-3,2)$

9. $2(4x+y=10)$
$1(-3x-2y=0)$

$+8x+2y=20$
$-3x-2y=0$

$5x=20$
$5 \quad 5$
$x=4$

$4x+y=10$
$4(4)+y=10$
$16+y=10$
$-16 \quad -16$
$y=-6$

$(4,-6)$

10. $2(5x+5y=15)$
$5(-2x+3y=-21)$

$+10x+10y=30$
$-10x+15y=-105$

$25y=-75$
$25 \quad 25$

$y=-3$

$5x+5y=15$
$5x+5(-3)=15$
$5x-15=15$
$+15 \quad +15$
$5x=30$
$5 \quad 5$
$x=6$

$(6,-3)$

11. $1(7x+2y=-1)$
$2(x-y=5)$

$+7x+2y=-1$
$2x-2y=10$

$9x=9$
$9 \quad 9$

$x=1$

$x-y=5$
$1-y=5$
$-1 \quad -1$
$-y=4$
$-1 \quad -1$
$y=-4$

12. $1(-3x-2y=12)$
$2(5x+y=-13)$

$-3x-2y=12$
$+10x+2y=-26$

$7x=-14$
$7 \quad 7$

$x=-2$

$5x+y=-13$
$5(-2)+y=-13$
$-10+y=-13$
$+10 \quad +10$
$y=-3$

Solving Linear Systems

Substitution Method (One of the variables is isolated):

1) Substitute one equation into the other
2) Solve equation for one variable
3) Substitute answer in to either equation to find the second variable

1. Solve the following systems of equations:

$a = -2b$

$5a - 3b = 13$

$5(-2b) - 3b = 13$

$-10b - 3b = 13$

$\frac{-13b}{-13} = \frac{13}{-13}$

$b = -1$

$a = -2b$

$a = -2(-1)$

$a = 2$

2. Solve the following systems of equations:

$y = x + 3$

$3x + 2y = 26$

$3x + 2(x+3) = 26$

$3x + 2x + 6 = 26$

$5x + 6 = 26$

$-6 \quad -6$

$\frac{5x}{5} = \frac{20}{5}$

$x = 4$

$y = x + 3$

$y = 4 + 3$

$y = 7$

3. Solve the following systems of equations:

$y = 2x + 1$

$2x + 2y = 14$

$2x + 2(2x+1) = 14$

$2x + 4x + 2 = 14$

$6x + 2 = 14$

$-2 \quad -2$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$

$y = 2x + 1$

$y = 2(2) + 1$

$y = 5$

4. Solve the following systems of equations:

$a = 3b + 1$

$5b - 2a = 1$

$5b - 2(3b+1) = 1$

$5b - 6b - 2 = 1$

$-b - 2 = 1$

$+2 \quad +2$

$\frac{-b}{-1} = \frac{3}{-1}$

$b = -3$

$a = 3b + 1$

$a = 3(-3) + 1$

$a = -8$

5. Solve the following systems of equations:

$x+2y=9$

$\frac{2y}{2}=\frac{8}{2}$

$y=4$

$x+2(4)=9$

$x+8=9$

$-8\ -8$

$x=1$

$(1,4)$

6. Solve the following systems of equations:

$8x-y=-10$

$y=3x+5$

$8x-(3x+5)=-10$

$8x-3x-5=-10$

$5x-5=-10$

$+5\ \ +5$

$\frac{5x}{5}=\frac{-5}{5}$

$x=-1$

$(-1,2)$

$y=3x+5$

$y=3(-1)+5$

$y=2$

7. Solve the following systems of equations:

$3x+2y=7$

$x+7=4y$

$-7\ -7$

$x=4y-7$

$3(4y-7)+2y=7$

$12y-21+2y=7$

$14y-21=7$

$+21\ +21$

$\frac{14y}{14}=\frac{28}{14}$

$y=2$

$x=4y-7$

$x=4(2)-7$

$x=1$

$(1,2)$

8. Solve the following systems of equations:

$\frac{y}{2}+x=4$

$y+x=2$

$-x\ -x$

$y=2-x$

$2\left(\frac{2-x}{2}\right)+x=(4)2$

$2-x+2x=8$

$2+x=8$

$-2\ \ -2$

$x=6$

$y=2-x$

$y=2-6$

$y=-4$

$(6,-4)$

Solving Linear Equations/Systems Followed by Evaluating Expressions
Solve the equation/system and then substitute that answer into the given expression

1. If $2(8 + p) = 22$, find the value of $3p - 7$

$16 + 2p = 22$
$-16 \quad -16$
$\frac{2p}{2} = \frac{6}{2}$
$p = 3$

$3p - 7$
$3(3) - 7$
$9 - 7$
$\boxed{2}$

2. If $2(4z + 3) = 7z + 4$, find the value of $3z^2$

$8z + 6 = 7z + 4$
$-7z \quad -7z$
$z + 6 = 4$
$-6 \quad -6$
$z = -2$

$3z^2$
$3(-2)^2$
$\boxed{12}$

3. If $-a - 6 = 8 - 2(9 + a)$, find the value of $-2a - 7$

$-a - 6 = 8 - 18 - 2a$
$-a - 6 = -2a - 10$
$+2a \quad +2a$
$a - 6 = -10$
$+6 \quad +6$
$a = -4$

$-2a - 7$
$-2(-4) - 7$
$\boxed{1}$

4. If $2x + 3(2x - 1) = 6x - 6$, find the value of $\frac{2x}{3}$

$2x + 6x - 3 = 6x - 6$
$8x - 3 = 6x - 6$
$-6x \quad -6x$
$2x - 3 = -6$
$+3 \quad +3$
$\frac{2x}{2} = \frac{-3}{2}$
$x = -1.5$

$\frac{2x}{3}$
$\frac{2(-1.5)}{3}$
$\boxed{-1}$

For #5-6, if (x,y) is the solution to the following systems, what is the value of x + y?

5. $3(-2x+y=5)$ $2(3x+2y=3)$

$-6x+3y=15$

$+6x+4y=6$

$7y=21$

$y=3$

$3x+2y=3$

$3x+2(3)=3$

$3x+6=3$

$-6 \quad -6$

$3x=-3$

$x=-1$

$x+y$

$-1+3$

2

6. $3(5x-2y=13)$ $2(2x+3y=9)$

$15x-6y=39$

$+4x+6y=18$

$19x=57$

$x=3$

$2x+3y=9$

$2(3)+3y=9$

$6+3y=9$

$-6 \quad -6$

$3y=3$

$y=1$

(3,1)

$x+y$

$3+1$

4

For #7-8, if (x,y) is the solution to the following systems, what is the value of x - y?

7. $3(4x+2y=16)$ $-2(x+3y=14)$

$+12x+6y=48$

$-2x-6y=-28$

$10x=20$

$x=2$

$x+3y=14$

$2+3y=14$

$-2 \quad -2$

$3y=12$

$y=4$

$x-y$

$2-4$

-2

8. $-2(x+2y=-1)$ $1(2x+y=7)$

$-2x-4y=2$

$+2x+y=7$

$-3y=9$

$y=-3$

$x+2y=-1$

$x+2(-3)=-1$

$x-6=-1$

$+6 \quad +6$

$x=5$

$x-y$

$5-3$

8

For #9-10, if (x,y) is the solution to the following systems, what is the value of 2x + y?

9. $2x+y=7$

$y=x+1$

$2x+x+1=7$

$3x+1=7$

$-1 \quad -1$

$3x=6$

$x=2$

$y=x+1$

$y=2+1$

$y=3$

$2x+y$

$2(2)+3$

7

10. $\frac{2y=4x+12}{2} \quad \frac{}{2}$

$-x+3y=18$

$y=2x+6$

$-x+3(2x+6)=18$

$-x+6x+18=18$

$5x+18=18$

$-18 \quad -18$

$5x=0$

$x=0$

$y=2x+6$

$y=2(0)+6$

$y=6$

$2x+y$

$2(0)+6$

6

Systems of Linear Equations/Inequalities Word Problems

The first equation is almost always just $x+y=$ for an amount. The second equation is usually a money equation.

*For coins, the money equation is $.01p$, $.05n$, $.10d$, *or* $.25q$

At least/more than/no less than/minimum is $\geq$

At most/less than/no more than is/maximum is $\leq$

To solve the system, use elimination method. Steps are listed in a previous lesson.

1. Alicia purchased H half-gallons of ice cream for \$3.50 each and P packages of ice cream cones for \$2.50 each. She purchased 14 items and spent \$43. Which system of equations could be used to determine how many of each item Alicia purchased?

3.50H + 2.50P = 43
H + P = 14

1) $3.50H+2.50P=43$
$H+P=14$

2) $3.50P+2.50H=43$
$P+H=14$

3) $3.50H+2.50P=14$
$H+P=43$

4) $3.50P+2.50H=14$
$P+H=43$

a+c=150

2. The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost \$7.75 and an adult ticket cost \$10.25. If the cinema sold \$1470 worth of tickets, which system of equations could be used to determine how many adult tickets, a, and how many child tickets, c, were sold?

7.75c + 10.25a = 1470

1) $a+c=150$
$10.25a+7.75c=1470$

2) $a+c=1470$
$10.25a+7.75c=150$

3) $a+c=150$
$7.75a+10.25c=1470$

4) $a+c=1470$
$7.75a+10.25c=150$

b+t=15 2b+3t=38

3. A recreation center ordered a total of 15 tricycles and bicycles from a sporting goods store. The number of wheels for all the tricycles and bicycles totaled 38. Write a linear system of equations that models this scenario, where t represents the number of tricycles and b represents the number of bicycles ordered.

1) $b+t=38$
$2b+3t=15$

2) $b+t=15$
$2b+3t=38$

3) $b+t=38$
$3b+2t=15$

4) $b+t=15$
$3b+2t=38$

D+Q=30 .10D+.25Q=4.80 .10 .25

4. Lizzy has 30 coins that total \$4.80. All of her coins are dimes, D, and quarters, Q. Which system of equations models this situation?

1) $D+Q=4.80$
$.10D+.25Q=30$

2) $D+Q=30$
$.10D+.25Q=4.80$

3) $D+Q=30$
$.25D+.10Q=4.80$

4) $D+Q=4.80$
$.25D+.10Q=30$

5. Jordan works for a landscape company during his summer vacation. He is paid \$12 per hour for mowing lawns and \$14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least \$250 this week. If m represents the number of hours mowing lawns and g represents the number of hours planting gardens, which system of inequalities could be used to represent the given conditions?

1) $m+g \leq 40$
$12m+14g \geq 250$

2) $m+g \geq 40$
$12m+14g \leq 250$

3) $m+g \leq 40$
$12m+14g \leq 250$

4) $m+g \geq 40$
$12m+14g \geq 250$

$m+g \leq 40$
$12m+14g \geq 250$

6. Edith babysits for a hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an hour as a library assistant working l hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least \$80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation.

1) $a+l \geq 15$
$4a+8l \leq 80$

2) $a+l \leq 15$
$4a+8l \geq 80$

3) $a+l \geq 80$
$4a+8l \leq 15$

4) $a+l \leq 15$
$4a+8l \geq 80$

$a+l \leq 15$
$4a+8l \geq 80$

7. An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer, p, costs \$50 and each computer, c, costs \$500. The store can ship a maximum of 15 items per day. Which of the following is a correct system of inequalities to model this situation?

1) $p+c \leq 15$
$50p+500c \geq 2500$

2) $p+c \geq 2500$
$50p+500c \leq 15$

3) $p+c \geq 15$
$50p+500c \leq 2500$

4) $p+c \leq 2500$
$50p+500c \geq 15$

$p+c \leq 15$
$50p+500c \geq 2500$

8. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater. Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, a, and child tickets, c, that would satisfy the cinema's goal.

1) $12.50a+6.25c \leq 200$
$a+c \geq 1500$

2) $12.50a+6.25c \geq 200$
$a+c \leq 1500$

3) $a+c \leq 200$
$12.50a+6.25c \geq 1500$

4) $a+c \geq 200$
$12.50a+6.25c \leq 1500$

$a+c \leq 200$
$12.50a+6.25c \geq 1500$

$a+p=165$

9. Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for $1.75 per pound and peaches for $2.50 per pound. If she made $337.50, how many pounds of peaches did she sell?

$-1.75(a+p=165)$
$1(1.75a+2.50p=337.50)$
$-1.75a-1.75p=-288.75$
$+1.75a+2.50p=337.50$
$.75p=48.75$
$.75 \quad .75$
$p=65$

↓ get a to cancel

$a+o=108$

10. During its first week of business, a market sold a total of 108 apples and oranges. The second week, five times the number of apples and three times the number of oranges were sold. A total of 452 apples and oranges were sold during the second week. Determine how many apples and how many oranges were sold the first week.

$-5(a+o=108)$
$1(5a+3o=452)$
$-5a-5o=-540$
$+5a+3o=452$
$-2o=-88$
$-2 \quad -2$
$o=44$

$a+o=108$
$a+44=108$
$-44 \quad -44$
$a=64$

$s+l=20$

11. Last week, a candle store received $355.60 for selling 20 candles. Small candles sell for $10.98 and large candles sell for $27.98. How many large candles did the store sell?

$-10.98(s+l=20)$
$1(10.98s+27.98l=355.60)$

↓ get s to cancel

$-10.98s-10.98l=-219.6$
$+10.98s+27.98l=355.6$
$17l=136$
$17 \quad 17$
$l=8$

12. Byron has 72 coins in his piggy bank. The piggy bank contains only dimes and quarters. If he has $14.70 in his piggy bank, how many dimes does he have in his piggy bank?

$.10d$ $.25q$

$-.10(d+q=72)$
$1(.10d+.25q=14.70)$

↓ cancel d

$-.10d-.10q=-7.2$
$+.10d+.25q=14.70$
$.15q=7.5$
$.15 \quad .15$
$q=50$

$d+q=72$
$d+50=72$
$-50 \quad -50$
$d=22$

Infinite Solutions and No Solutions
Linear Equations and Systems of Linear Equations
When the coefficients are the same, you will end up with either infinite solutions (constant terms are the same) or no solutions (constant term are different)

Infinite Solutions	No solutions
Coefficients of x and y are the same **Constant terms are the same**	**Coefficients of x and y are the same** **Constant terms are different**
$2x+4=2x+4$	$2x+4=2x+5$
$2x+3y=5$ $2x+3y=5$	$2x+3y=4$ $2x+3y=5$

To determine when an equation or a system has infinite/no solutions:
For a system of equations, you may have to multiply the equation(s) by a constant so that the coefficients are the same.

1. What is the solution to the equation $3x+4=3x-8$?

No Solution

the constants are different

2. What is the solution to the equation $3x+4=3x+4$?

Infinite Solutions

The constants are the same

3. What is the solution to the equation $2(x+2)+4x=6x+2$?

$2x+4+4x=6x+2$

$6x+4=6x+2$

No Solution

the constants are different

4. What is the solution to the equation $3(x-2)+2x=-8+5x+2$?

$3x-6+2x=5x-6$

$5x-6=5x-6$

Infinite Solutions

the constants are the same

5. For what value of a does the equation $6x-1=ax-2$ have no solution?

a=6

the coefficients must be the same.

6. For what value of a does the equation $ax+1=\frac{1}{2}x+1$ have infinitely many solutions?

$a=\frac{1}{2}$ The coefficients must be the same

7. For what value of a does the equation $10x+7=ax-2$ have no solution?

$a=10$ the coefficients must be the same

8. For what value of a does the equation $-2x+3=ax+3$ have infinitely many solutions?

$a=-2$ The coefficients must be the same

9. For what value of a does the equation $3x-7=ax+2$ have no solutions?

$a=3$ the coefficients must be the same

10. For what value of a does the equation $5x+2=5x+a$ have infinitely many solutions?

$a=2$ the constants must be the same

11. State a value of a such that the equation $5x+2=5x+a$ has no solutions.

$a=1$ any value except 2

The constants must be different

12. For what value of a does the equation $\frac{2}{3}x-1=\frac{2}{3}x+a$ have infinitely many solutions?

$a=-1$

The constants must be the same

13. State a value for a such that the equation $\frac{2}{3}x-1=\frac{2}{3}x+a$ has no solutions.

$a=5$ any value except -1

The constants must be different

14. In the following system of equations, for what value of a does the system of equations have infinitely many solutions?

$ax+3y=10$
$2x+3y=10$

$a=2$ the coefficients must be the same

15. In the following system of equations, for what values of a and b does the system of equations have infinitely many solutions?

$ax+by=6$
$4x-5y=6$

$a=4$
$b=-5$ the coefficients must be the same

16. In the following system of equations, for what values of a and b does the system of equations have no solutions?

$ax+by=2$
$2x+3y=4$

$a=2$
$b=3$ the coefficients must be the same

17. In the following system of equations, for what values of a and b does the system of equations have infinitely many solutions?

$\frac{1}{2}x+3y=12$
$ax+by=12$

$a=\frac{1}{2}$
$b=3$ the coefficients must be the same

18. In the following system of equations, for what values of a and b does the system of equations have no solutions?

$ax+by=8$
$.7x+2.3y=7$

$a=.7$
$b=2.3$ the coefficients must be the same.

19. In the xy-plane, the system of equations $x+4y=8$, $2x+8y=a$ has infinitely many solutions. What is the value of a?

Since the coefficients must be the same, I have to multiply one of the equations by something to make them the same

$2(x+4y=8)$
$2x+8y=a$

$2x+8y=16$
$2x+8y=a$

$a=16$

The constants must be the same

20. In the xy-plane, the system of equations $\begin{cases} 2x-3y=6 \\ -6x+9y=a \end{cases}$ has infinitely many solutions. What is the value of a?

$-3(2x-3y=6)$

$-6x+9y=-18$

$-6x+9y=9$

$a=-18$

the constants must be the same.

21. In the following system of equations, what for what value of a and b does the system of equations have infinitely many solutions?

$2(ax+by=4)$

$2x-6y=8$

$2ax+2by=8$

$2x-6y=8$

Since the coefficients must be the same, I set them equal to each other.

Since the constants must be the same, I multiply.

$\frac{2a}{2}=\frac{2}{2}$ $\quad \frac{2b}{2}=\frac{-6}{2}$

$a=1$ $\quad b=-3$

22. In the following system of equations, what for what value of a and b does the system of equations have infinitely many solutions?

$4(ax+by=2)$

$4x-8y=8$

$4ax+4by=8$

$4x-8y=8$

$\frac{4a}{4}=\frac{4}{4}$ $\quad \frac{4b}{4}=\frac{-8}{4}$

$a=1$ $\quad b=-2$

23. In the following system of equations, k is a constant and x and y are variables. For what value of k will the system of equation have no solution?

$2(kx+y=5)$

$8x+2y=6$

$2kx+2y=10$

$8x+2y=6$

$\frac{2k}{2}=\frac{8}{2}$

$k=4$

Since the coefficients must match, I multiply one of the equations

24. In the following system of equations, k is a constant and x and y are variables. For what value of k will the system of equation have no solution?

$5(kx-2y=3)$

$2(3x-5y=6)$

$5kx-10y=15$

$6x-10y=12$

$\frac{5k}{5}=\frac{6}{5}$

$k=\frac{6}{5}$

Literal Equations Solving for a Variable

Follow same steps as equation solving. Don't combine unlike terms.
When isolating, add or subtract first, divide last
To get rid of fractions, multiply by the LCD

1. If $abx - 5 = 0$, what is x in terms of a and b?

1) $x = \frac{5}{ab}$
2) $x = -\frac{5}{ab}$
3) $x = 5 - ab$
4) $x = ab - 5$

$abx - 5 = 0$
$+5 \quad +5$
$\frac{abx}{ab} = \frac{5}{ab}$
$x = \frac{5}{ab}$

2. In the equation $A = p + prt$, t is equivalent to

1) $\frac{A - pr}{p}$
2) $\frac{A - p}{pr}$
3) $\frac{A}{pr} - p$
4) $\frac{A}{p} - pr$

$A = p + prt$
$-p \quad -p$
$\frac{A-p}{pr} = \frac{prt}{pr}$
$\frac{A-p}{pr} = t$

3. The formula for potential energy is $P = mgh$, where P is potential energy, m is mass, g is gravity, and h is height. Which expression can be used to represent g?

(1) $P - m - h$
(2) $P - mh$
(3) $\frac{P}{m} - h$
(4) $\frac{P}{mh}$

$\frac{P = mgh}{mh \quad mh}$
$\frac{P}{mh} = g$

4. If $bx - 2 = K$, then x equals

(1) $\frac{K}{b} + 2$
(2) $\frac{K-2}{b}$
(3) $\frac{2-K}{b}$
(4) $\frac{K+2}{b}$

$bx - 2 = K$
$+2 \quad +2$
$\frac{bx}{b} = \frac{K+2}{b}$
$x = \frac{K+2}{b}$

5. If $x = 2a - b^2$, then a equals

1) $\frac{x - b^2}{2}$
2) $\frac{x + b^2}{2}$
3) $\frac{b^2 - x}{2}$
4) $x + b^2$

$x = 2a - b^2$
$+b^2 \quad +b^2$
$\frac{x + b^2}{2} = \frac{2a}{2}$
$\frac{x + b^2}{2} = a$

6. If $2m + 2p = 16$, p equals

1) $8 - m$
2) $16 - m$
3) $16 + 2m$
4) $9m$

$2m+2p=16$
$-2m \quad -2m$
$\frac{2p}{2} = \frac{16-2m}{2}$
$p = 8-m$

7. If $3ax + b = c$, then x equals

1) $c - b + 3a$
2) $c + b - 3a$
3) $\frac{c-b}{3a}$
4) $\frac{b-c}{3a}$

$3ax+b=c$
$-b \quad -b$
$\frac{3ax}{3a} = \frac{c-b}{3a}$
$x = \frac{c-b}{3a}$

8. If the formula for the perimeter of a rectangle is $P = 2l + 2w$, then w can be expressed as

1) $w = \frac{2l-P}{2}$
2) $w = \frac{P-2l}{2}$
3) $w = \frac{P-l}{2}$
4) $w = \frac{P-2w}{2l}$

$P = 2l + 2w$
$-2l \quad -2l$
$\frac{P-2l}{2} = \frac{2w}{2}$
$\frac{P-2l}{2} = w$

9. The formula for electrical power, P, is $P = I^2R$, where I is current and R is resistance. The formula for I in terms of P and R is

1) $I = \left(\frac{P}{R}\right)^2$
2) $I = \sqrt{\frac{P}{R}}$
3) $I = (P - R)^2$
4) $I = \sqrt{P - R}$

$\frac{P}{R} = \frac{I^2R}{R}$
$\sqrt{\frac{P}{R}} = \sqrt{I^2}$
$\sqrt{\frac{P}{R}} = I$

10. The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of r, in terms of h and V, is

1) $r = \sqrt{\frac{V}{\pi h}}$
2) $r = \sqrt{V\pi h}$
3) $r = 2V\pi h$
4) $r = \frac{V}{2\pi}$

$\frac{V}{\pi h} = \frac{\pi r^2 h}{\pi h}$
$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$
$\sqrt{\frac{V}{\pi h}} = r$

11. The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for n, the number of sides of the polygon, in terms of S.

$S = 180n - 360$
$+360 \quad +360$
$S + 360 = 180n$
$\frac{S+360}{180} = \frac{180n}{180}$
$\frac{S+360}{180} = n$

12. Using the formula for the volume of a cone, $V=\frac{1}{3}\pi r^2 h$, express r in terms of V, h, and π.

$\frac{3V}{\pi h}=\frac{\pi r^2 h}{\pi h} \rightarrow \frac{3V}{\pi h}=r^2$

$\sqrt{\frac{3V}{\pi h}}=r$

13. Solve for x in terms of t and y:

$t\left(\frac{x+y}{t}\right)=6t$

$x+y=6t$

$-y \quad -y$

$x=6t-y$

14. Solve for x in terms of t and y:

$xy\left(\frac{t+1}{xy}\right)=6xy$

$\frac{t+1}{6y}=\frac{6xy}{6y}$

$\frac{t+1}{6y}=x$

15. Solve for t in terms of x and y:

$t\left(\frac{x}{t}\right)=(y+1)t$

$\frac{x}{y+1}=\frac{t(y+1)}{y+1}$

$\frac{x}{y+1}=t$

16. Solve for c in terms of a, b, d, and k:

$k\left(\frac{a+b+2c^2}{k}\right)=(d+1)k$

$a+b+2c^2=kd+k$

$-a-b \quad -a-b$

$\frac{2c^2}{2}=\frac{kd+k-a-b}{2}$

$c^2=\frac{kd+k-a-b}{2}$

$c=\sqrt{\frac{kd+k-a-b}{2}}$

17. Solve for x in terms of m and y:

$\left(\frac{y}{x+1}\right)(x+1) = (m)(x+1)$

$y = m(x+1)$

$y = mx + m$

$-m \quad -m$

$\frac{y-m}{m} = \frac{mx}{m}$

$\frac{y-m}{m} = x$

18. Solve for k in terms of x and t:

$\left(\frac{x}{t+k^2}\right)(t+k^2) = (2)(t+k^2)$

$x = 2(t+k^2)$

$x = 2t + 2k^2$

$-2t \quad -2t$

$\frac{x-2t}{2} = \frac{2k^2}{2}$

$\frac{x-2t}{2} = k^2$

$\frac{x-2t}{2} = k$

19. Solve for k in terms of x, y, z, and d:

$\left(\frac{x+2y-z}{k-1}\right)(d)(k-1)$

$x + 2y - z = d(k-1)$

$x + 2y - z = dk - d$

$+d \quad +d$

$\frac{x+2y-z+d}{d} = \frac{dk}{d}$

$\frac{x+2y-z+d}{d} = k$

20. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2

$2\left(A = \frac{1}{2}h(b_1 + b_2)\right)2$

$2A = h(b_1 + b_2)$

$2A = hb_1 + hb_2$

$-hb_2 \quad -hb_2$

$\frac{2A - hb_2}{h} = \frac{hb_1}{h}$

$\frac{2A - hb_2}{h} = b_1$

21. The formula for converting degrees Fahrenheit (F) to degrees Kelvin (K) is:

$9\left(K = \frac{5}{9}(F + 459.67)\right)9$

Solve for F, in terms of K.

$9K = 5(F + 459.67)$

$9K = 5F + 2298.35$

$-2298.35 \quad -2298.35$

$9K - 2298.35 = 5F$

$\frac{9K - 2298.35}{5} = \frac{5F}{5}$

$\frac{9K - 2298.35}{5} = F$

Literal Equations Solving for an Expression
Use rules of equation solving to manipulate the original expression into the desired expression.

1. If $4(x+y)=8$, what is the value of $x+y$?

$\frac{4(x+y)}{4}=\frac{8}{4}$

$x+y=2$

2. If $4x+2y=10$, what is the value of $2x+y$?

$\frac{4x+2y}{2}=\frac{10}{2}$

$2x+y=5$

3. If $3xy+3z-4=8$, what is the value of $xy+z$?

get xy and z alone on one side

$+4 \quad +4$

$\frac{3xy+3z=12}{3}$

$xy+z=4$

4. If $12x^2+4y-7=9$, what is the value of $3x^2+y$?

$+7 \quad +7$

$\frac{12x^2+4y=16}{4}$

$3x^2+y=4$

5. If $-12x+4z+2=14$, what is the value of $3x-z$?

$-2 \quad -2$

$\frac{-12x+4z=12}{-4}$

$3x-z=-3$

6. If $2x-4y+8=16$, what is the value of $-x+2y$?

$-8 \quad -8$

$\frac{2x-4y=8}{-2 \quad -2}$ → $-x+2y=-4$

7. If $8x-10y+7=11$, what is the value of $4x-5y$?

$-7 \quad -7$

$\frac{8x-10y=4}{2 \quad 2}$ → $4x-5y=2$

8. If $6x-4y+2x=12$, what is the value of $6x-3y$?

$\frac{8x-4y=12}{4 \quad 4}$ → $3(2x-y)=(3)3$

$6x-3y=9$

divide it down first

9. If $10x+6y-7=14$, what is the value of $-5x-3y$?

$+7 \quad +7$

$\frac{10x+6y=16}{-2 \quad -2}$ → $-5x-3y=-8$

10. The value of $\frac{m}{n}=5$, what is the value of $\frac{n}{m}$?

$\frac{m}{n}=\frac{5}{1}$

$\frac{n}{m}=\frac{1}{5}$

When asked for reciprocal, find reciprocal on both sides

11. If $\frac{j}{k}=\frac{1}{2}$, what is the value of $\frac{k}{j}-5$?

$\frac{j}{k}=\frac{1}{2} \rightarrow \frac{k}{j}=2$

$-5 \quad -5$

$\frac{k}{j}-5=-3$

12. If $\frac{x}{y}=-4$, what is the value of $\frac{8y}{x}$?

$8\left(\frac{y}{x}\right)=\left(-\frac{1}{4}\right)8 \qquad \frac{8y}{x}=-\frac{8}{4}$

$\frac{8y}{x}=-2$

13. If $\frac{x}{y}=3$, what is the value of $\frac{2y}{x}$?

$2\left(\frac{y}{x}\right)=\left(\frac{1}{3}\right)2 \qquad \frac{2y}{x}=\frac{2}{3}$

14. If $\frac{x}{y}=\frac{1}{3}$, what is the value of $\frac{3y}{x}+2$.

$3\left(\frac{y}{x}\right)=\left(\frac{1}{3}\right)3 \rightarrow \frac{3y}{x}=9$

$+2 \quad +2$

$\frac{3y}{x}+2=11$

15. If $2\frac{p}{q}=8$, what is the value of $-3\frac{q}{p}$.

$\frac{2p}{2}=\frac{8}{2}$

$\frac{p}{q}=4$

$-3\left(\frac{q}{p}\right)=\left(\frac{1}{4}\right)-3 \rightarrow -3\frac{q}{p}=-\frac{3}{4}$

Manipulating Equations

Replace the appropriate variable with the given manipulation in parenthesis
Get rid of the parenthesis
Manipulate the new formula so that the original formula is intact.

1. The formula below can be used to find the perimeter, P, of a rectangle given its length, l, and width, w.

$P = 2l + 2w$

If the length is increased by 5, how is the perimeter affected?

$l+5$

Perimeter is increased by 10

$P=2(l+5)+2w$

$P=2l+10+2w$

$P=2l+2w+10$

2. The formula below can be used to find the force, F, given the mass, m, and acceleration, a.

$F = ma$

If the mass is multiplied by 6, how will the force be affected?

$6m$

Force is multiplied by 6

$F=6ma$

3. The formula $y = 3x + 10$ represents the cost of a car service, y, traveling x miles. How would increasing the number of miles by 5 affect the cost of the trip?

$x+5$

$y=3(x+5)+10$

$y=3x+15+10$

$y=3x+10+15$

The cost increases by $15

4. The formula below gives the cost of a birthday party, c, for n guests. How will increasing the number of guests by 8 affect the cost of the birthday party?

$c = 20 + 2(n-10)$ $\quad n+8$

The cost increases by $16.

$C=20+2(n+8-10)$

$C=20+2n+16-2(10)$

$C=20+2n-20+16$

$C=20+2(n-10)+16$

5. The area of a circle, A, can be found using its radius, r, using the formula given below.

$A = \pi r^2$

If the radius is doubled, how will the area be affected?

$A = \pi(2r)^2$

$A = \pi(4r^2)$

$A = \boxed{4}\pi r^2$

the area is multiplied by 4.

6. The volume of a sphere, V, can be found using its radius, using the formula $V = \frac{4}{3}\pi r^3$.

If the radius is multiplied by 2, how will the volume be affected?

$2r$

$V = \frac{4}{3}\pi(2r)^3$

$V = \frac{4}{3}\pi(8r^3)$

$V = \boxed{8}\left(\frac{4}{3}\pi r^3\right)$

the volume is multiplied by 8.

7. The formula $r = \sqrt{\frac{A}{\pi}}$ can be used to find the radius of a circle, r, given its area, A.

If the area is multiplied by 9, how would the radius be affected?

$9A$

$r = \sqrt{\frac{9A}{\pi}}$

$r = \sqrt{9}\cdot\sqrt{\frac{A}{\pi}}$

$r = \boxed{3}\sqrt{\frac{A}{\pi}}$

the radius is multiplied by 3

8. The formula $C = \frac{5}{9}(F - 32)$ can be used to find the Celsius temperature, C, given its Fahrenheit temperature, F.

If the Fahrenheit temperature is increased by 18, how would the Celsius temperature be affected?

$F+18$

$C = \frac{5}{9}(F + 18 - 32)$

$C = \frac{5}{9}F + 10 - \frac{5}{9}(32)$

$C = \frac{5}{9}(F - 32)\boxed{+10}$

the Celcius temperature increases by 10.

9. The lateral area of a cone, L, is given by the equation $L = \pi rl$ where r is the radius and l is the slant height. If the radius is multiplied by 4 and the slant height is divided by 2, how is the lateral area affected?

$4r$ $\frac{1}{2}l$

$L = \pi rl$

$L = \pi(4r)(\frac{1}{2}l)$

$L = \boxed{2}\pi rl$

The lateral area is multiplied by 2.

10. The formula below can be used to find the power, P, given the current, I, and resistance, R.

$P = I^2R$

If the current is doubled and the resistance is cut in half, how will the power be affected?

$2I$ $\frac{1}{2}R$

$P = (2I)^2(\frac{1}{2}R)$

$P = (4I^2)(\frac{1}{2}R)$

$P = \boxed{2}I^2R$

The power is multiplied by 2.

11. The volume of a cylinder, V, can be found using its radius, r, and height, h, using the formula $V = \pi r^2 h$.

If the radius is multiplied by 4 and the height is divided by 4, how will the volume be affected?

$4r$ $\frac{1}{4}h$

$V = \pi(4r)^2(\frac{1}{4})h$

$V = \pi(16r^2)(\frac{1}{4}h)$

$V = \boxed{4}\pi r^2 h$

The volume is multiplied by 4.

12. The volume of a cone, V, can be found using its radius, r, and height, h, using the formula $V = \frac{1}{3}\pi r^2 h$.

If the radius is divided by 3 and the height is multiplied by 3, how will the volume be affected?

$\frac{1}{3}r$ $3h$

$V = \frac{1}{3}\pi(\frac{1}{3}r)^2(3h)$

$V = \frac{1}{3}\pi(\frac{1}{9}r^2)(3h)$

$V = \boxed{\frac{1}{3}}(\frac{1}{3}\pi r^2 h)$

The Volume is divided by 3.

13. The area of a circle, A, is given by the equation $A = \pi r^2$ where r is the radius.
Circle X has a radius of *2r* and Circle Y has a radius of *3r*. What is the ratio of the area of Circle X to the area of Circle Y?

X
$A = \pi(2r)^2$
$A = \pi(4r^2)$
$A = 4\pi r^2$

Y
$A = \pi(3r)^2$
$A = \pi(9r^2)$
$A = 9\pi r^2$

$\frac{4\pi r^2}{9\pi r^2}$

$\boxed{\frac{4}{9}}$

14. The volume of a sphere, V, can be found using its radius, using the formula $V = \frac{4}{3}\pi r^3$.

Sphere A has a radius of *2r* and Sphere B has a radius of *4r*. What is the ratio of the volume of Sphere B to the volume of Sphere A?

A
$V = \frac{4}{3}\pi(2r)^3$
$V = \frac{4}{3}\pi(8r^3)$
$V = 8(\frac{4}{3}\pi r^3)$

B
$V = \frac{4}{3}\pi(4r)^3$
$V = \frac{4}{3}\pi(64r^3)$
$V = 64(\frac{4}{3}\pi r^3)$

$\frac{64(\frac{4}{3}\pi r^3)}{8(\frac{4}{3}\pi r^3)}$

$\boxed{\frac{8}{1}}$

15. The formula $P = I^2R$ can be used to find the power, *P*, given the current, *I*, and resistance, *R*. If event A has a current of *3I* and event B has a current of $\frac{1}{2}I$, what is the ratio of the power of event A to event B?

A
$P = (3I)^2R$
$P = (9I^2)R$
$P = 9I^2R$

B
$P = (\frac{1}{2}I)^2R$
$P = (\frac{1}{4}I^2)R$
$P = \frac{1}{4}I^2R$

$\frac{9I^2R}{\frac{1}{4}I^2R}$

$\frac{9}{1} \cdot \frac{4}{1} = \boxed{\frac{36}{1}}$

16. The formula below can be used to find the surface area of a cube, *SA*, given the side length, *s*.
$SA = 6s^2$
If square A has side length *4s* and square B has side length $2s$, what is the ratio of the surface area of square B to square A?

A
$SA = 6(4s)^2$
$SA = 6(16s^2)$
$SA = 16(6s^2)$

B
$SA = 6(2s)^2$
$SA = 6(4s^2)$
$SA = 4(6s^2)$

$\frac{4(6s^2)}{16(6s^2)}$

$\frac{4}{16} = \boxed{\frac{1}{4}}$

Unit Analysis
Start with piece of information you are given.
Multiply by each comparison as a fraction until you end up with the desired unit.
***To get units to cancel, the same unit must be on top and bottom.**

1. A recipe calls for 6 ounces of flour to make 2 batches of cookies. How many batches of cookies can be made with 18 ounces of flour?

$18\text{oz} \cdot \frac{2 \text{ batches}}{6 \text{ oz}} = \frac{18(2)}{6} = 6 \text{ batches}$

2. Jessica gets paid $86 for working a 4 hour shift. How many hours would Jessica have to work in order to make $258?

$\$258 \cdot \frac{4 \text{ hours}}{\$86} = \frac{258(4)}{86} = 12 \text{ hours}$

3. Water flows out of a pool at a rate of 6.5 gallons per minute. How many gallons of water will flow out of the pool after 12 minutes?

$12 \text{ min} \cdot \frac{6.5 \text{ gal}}{1 \text{ min}} = \frac{12(6.5)}{1} = 78 \text{ gallons}$

4. Horsepower and watts are units of power. They are directly proportional such that 5 horsepower is equal to 3730 watts. How much power, to the nearest tenth of a horsepower, is equal to 12,000 watts?

$12{,}000 \text{ watts} \cdot \frac{5 \text{ hp}}{3730 \text{ watts}} = \frac{12{,}000(5)}{3730} = 16.1 \text{ hp}$

5. In a shipment of 950 calculators, it is found that 23 of them have defects. At this rate, to the nearest calculator, how many defective calculators will there be in a shipment of 50,000 calculators?

$50{,}000 \text{ calc} \cdot \frac{23 \text{ def}}{950 \text{ calc}} = \frac{50{,}000(23)}{950} = 1210.526316$

1211 calcs

6. The standard college cross country race is 10 kilometers. How many miles if this race rounded to the nearest hundredth of a mile? (1 kilometer = 0.6214 miles).

$10\text{ km} \cdot \frac{.6214\text{ miles}}{1\text{ km}} = 10(.6214) = 6.21\text{ miles}$

7. 60 CDs stacked on top of each other forms a cylinder that is 4.5 inches tall. How many CDs would need to be stacked on top of each other to create a column that is 7 inches tall? Round your answer to the nearest CD.

$7\text{ in} \cdot \frac{60\text{ CDs}}{4.5\text{ in}} = \frac{7(60)}{4.5} = 93.\overline{3} = 93\text{ CDs}$

8. Jaquan walks 120 meters in 42.1 seconds. If he walks at this same rate, how many meters will he walk in 4 minutes rounded to the nearest meter? (60 seconds = 1 minute)

$4\text{ min} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{120\text{ met}}{42.1\text{ sec}} = \frac{4(60)(120)}{42.1} = 684.0855\ldots$

684 meters

9. A certain item needs 12mm of fabric to be created. What is the maximum number of complete items that can be created with 4 meters of tape? (1 meter = 1000 mm)

$4\text{ met} \cdot \frac{1000\text{ mm}}{1\text{ met}} \cdot \frac{1\text{ item}}{12\text{ mm}} = \frac{4(1000)}{12} = 333.\overline{3}\text{ items}$

333 items

10. An energy shot comes in a 4 fluid ounce countainter. How many full containers can be filled from 2.5 liters of the energy drink?

1 fluid ounce = 29.574 milliliters

1000 milliliters = 1 liter

$2.5\text{ lit} \cdot \frac{1000\text{ ml}}{1\text{ lit}} \cdot \frac{1\text{ oz}}{29.574\text{ ml}} \cdot \frac{1\text{ container}}{4\text{ oz}}$

$\frac{2.5(1000)}{29.574(4)} = 21.1334\ldots$

21 containers

11. Marlena runs 400 meters in 61.4 seconds. At this rate, how many meters, to the nearest meter, will Marlena run in 3.5 minutes? (60 seconds = 1 minute)

$3.5 \text{ min} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{400 \text{ met}}{61.4 \text{ sec}} = \frac{3.5(60)(400)}{61.4} = 1368.078176$

1368 meters

12. The current price of silver is $17.69 per ounce. How many pounds of silver, to the nearest pound, are there in an $80,000 deposit of silver? (16 ounces = 1 pound)

$\$80{,}000 \cdot \frac{1 \text{ oz}}{\$17.69} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{80{,}000}{17.69(16)} = 282.6455625$

283 Pounds

13. To make a certain bakery cookie, the maker needs 2.1 ounces of chocolate. How many pounds of chocolate are needed to make 32 cookies? (1 pound = 16 ounces)

$32 \text{ cookies} \cdot \frac{2.1 \text{ oz}}{1 \text{ cookies}} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{32(2.1)}{16} = 4.2$ Pounds

14. A new unit, Schlansky, is created such that 77 Schlanskys is equivalent to 50 inches. 120 Schlanskys is equivalent to how many feet, to the nearest unit? (12 inches = 1 foot)

$120 \text{ Schlanskys} \cdot \frac{50 \text{ inches}}{77 \text{ Schlanskys}} \cdot \frac{1 \text{ ft}}{12 \text{ inches}} = \frac{120(50)}{77(12)} = 6.49...$

6 ft

15. One ounce of corbomite can provide power for people in 4 homes. If homes have an average of 5.5 people in them, how many people could be powered by 3.2 ounces of corbomite? Round your answer to the nearest person.

$3.2 \text{ oz} \cdot \frac{4 \text{ homes}}{1 \text{ oz}} \cdot \frac{5.5 \text{ ppl}}{1 \text{ home}} = 3.2(4)(5.5) = 70.4$

70 ppl

Percents (Basic)
Decimal(whole) = part
***Convert percent to a decimal by dividing by 100.**
*The percent of something not happening is 100 – the percent it will happen. For example, if something has an 80% of happening, it has a 20% chance of not happening (100 – 80).

Create an equation to represent each and solve

1. Find 80% of 90

$.8(90)=x$
$72=x$

2. Find 38% of 120

$.38(120)=x$
$45.6=x$

3. Find 70% of 122

$.7(122)=x$
$85.4=x$

4. Find 31% of 49

$.31(49)=x$
$15.19=x$

5. 60% of what number is 30?

$\frac{.6x}{.6}=\frac{30}{.6}$
$x=50$

6. 24% of what number is 126?

$\frac{.24x}{.24}=\frac{126}{.24}$
$x=525$

7. 75% of what number is 210?

$\frac{.75x}{.75}=\frac{210}{.75}$
$x=280$

8. 30% of what number is 120?

$\frac{.3x}{.3}=\frac{120}{.3}$
$x=400$

9. Find 12% of 68

$.12(68)=x$
$8.16=x$

10. 45% of what number is 54?

$\frac{.45x}{.45}=\frac{54}{.45}$
$x=120$

11. David conducted a study and found that 32% of students like to sing. If a class has 250 students, how many would be expected to like to sing?

$.32(250) = X$

$80 = X$

12. A building has dimensions 30 feet long, 40 feet wide, and 80 feet tall. A model is to be made of this building where all of the dimensions are 15% of the original. How tall will the model be?

$.15(80) = X$

$12 = X$

55% Profit

13. 45% of a baker's revenue goes to cost and the rest of his profit. How much profit does this baker make if his revenue is $250?

$.55(250) = X$

$\$137.50 = X$

20% did not study

14. 80% of the students in a class studied for their math test. If there are 32 students in the class, how many did not study for their math test?

$.2(32) = X$

$6.4 = X$

40% did not

15. 60% of a class did their homework. If there are 20 students in the class, how many students did not do their homework?

$.4(20) = X$

$8 = X$

Percents (Increasing or Decreasing by a Percent)

If you're increasing or decreasing by a percent, $1 \pm decimal$.
For example: Increases by 12% each year, multiply by $1+.12=1.12$
Decreases by 20% each year, multiply by $1-.20=.80$

Create an expression that represents the following:

1. A number is increased by 25%

$1+.25$
$1.25x$

2. A number is decreased by 10%

$1-.1$
$.9x$

3. A number is decreased by 22%

$1-.22$
$.78x$

4. A number is increased by 37%

$1+.37$
$1.37x$

5. A number is increased by 12.5%

$1+.125$
$1.125x$

6. A number is decreased by 17.2%

$1-.172$
$.828x$

7. Byron purchases groceries that total $52. How much will his bill be after 7% sales tax is added?

$1+.07$
1.07

$1.07(52)=x$
$\$55.64=x$

8. A $500 television goes on sale for 20% off. How much will the cost of the television be without tax?

$1-.2=.8$

$.8(800)=x$
$\$400=x$

9. The population of a county increased by 30%. If the initial population was 4500, what did the population increase to?

$1+.3$
1.3

$1.3(4500)=x$
$5850=x$

1+.12
1.12

10. The population of Smallville has increased by 12% since 2010. If the population was 15,000 in 2010, what is the population now after the 12% increase?

$1.12(15{,}000) = x$

$16{,}800 = x$

1-.15=.85

11. The amount of Jessica's student loan debt has decreased by 15%. If she initially owed $34,000, how much does she owe now after the 15% decrease?

$.85(34{,}000) = x$

$\$28{,}900 = x$

12. Jamie paid $32.40 for a sweater after 8% sales tax. What is the cost of the sweater before tax?

1+.08
1.08

$\frac{1.08x}{1.08} = \frac{32.40}{1.08}$

$x = \$30$

13. Jackie paid $22.50 for a scarf during a 25% off sale. What was the original price of the scarf?

1-.25=.75

$\frac{.75x}{.75} = \frac{22.50}{.75}$

$x = \$30$

14. After an increase of 30%, the value of a stock is $1950. What was the initial investment on the stock?

1+.3
1.3

$\frac{1.3x}{1.3} = \frac{1950}{1.3}$

$x = \$1500$

15. An item is on sale for $51 during a 40% off sale. What was the original price of the item?

1-.4
.6

$\frac{.6x}{.6} = \frac{51}{.6}$

$x = \$85$

Percents (Compound Percents)
If you are applying multiple percents/fractions, multiply them by each other.

Create a mathematical expression for the following

1. What is 25% of 50% of a number?

$.25(.5)(x)$
$.125x$

2. What is 80% of 20% of a number?

$.8(.2)x$
$.16x$

3. What is 62% of 80% of a number?

$.62(.8)(x)$
$.496x$

4. What is 10% of 5% of a number?

$.1(.05)(x)$
$.005x$

5. A number is increased by 20% and then increased an additional 10%.

$1+.2=1.2$ $1+.1=1.1$
$1.2(1.1)(x)=1.32x$

6. A number is increased by 25% and then decreased by 15%.

$1+.25=1.25$ $1-.15=.85$
$1.25(.85)(x)=1.0625x$

7. A number is decreased by 18% and then increased by 10%.

$1-.18=.82$ $1+.1=1.1$
$.82(1.1)(x)=.902x$

8. A number is decreased by 40% and then increased by 8%.

$1-.4=.6$ $1+.08=1.08$
$.6(1.08)x=.648x$

9. A jacket has an original price of $80. It is on sale for 30% off and Jackson has a 20% off coupon. Without sales tax, how much will Jackson have to pay for the jacket?

$1-.3=.7$ $1-.2=.8$
$.7(.8)(80)=\$44.80$

1−.10=.9

10. Chloe wants to buy a car that has an original price of $8000. The dealership is offering 10% off and her mom will pay for 25% of the final cost. How much will Chloe have to pay without tax?

1−.25=.75

.9(.75)(8000) = $5400

1+.08=1.08

11. The population of a city was 52,000 in 2010. The population increased by 8% by 2011 and then increased 11% by 2012. What was the population in 2012?

1+.11
1.11

1.08(1.11)(52,000)
62337.6

1−.32=.68 → 1+.05=1.05

12. Jessica buys a coat through a website for 32% off but there is a 5% shipping fee. If the original price of the coat was $129, what is the cost of her order to the nearest cent without tax?

.68(1.05)(129) = $92.10

1−.2=.8

13. David went to the store to buy a jacket that has a price tag of $75. He has a 20% off coupon and needs to pay 8% sales tax. How much will he pay for the jacket?

1+.08 = 1.08

1.08(.8)(75) = $64.80

1+.62=1.62 1−.4=.6

14. Phil spent $5,500 buying a stock. It increased 62% and then decreased 40%. What is the current value of Phil's stock?

1.62(.6)(5500) = $5346

1−.25=.75

15. Natalia went shopping with her mom. There was a pair of boots with an original price of $150. Natalia's mom had a 25% off coupon and said Natalia would only have to pay for 40% of the final cost. How much did Natalia have to pay for the boots?

.4

.75(.4)(150)
$45

Percents (Percent of change)

$$\frac{\textit{amount of change}}{\textit{original}} = \frac{\textit{percent of change}}{100}$$

What is the percent of change from:

1. 20 to 25?

$25-20=5$ $\quad \frac{5}{20}=\frac{x}{100}$ $\quad \frac{20x}{20}=\frac{500}{20}$ $\quad x=25\%$ increase

2. 25 to 20?

$25-20=5$ $\quad \frac{5}{25}=\frac{x}{100}$ $\quad \frac{25x}{25}=\frac{500}{25}$ $\quad x=20\%$ decrease

3. 50 to 60?

$60-50=10$ $\quad \frac{10}{50}=\frac{x}{100}$ $\quad \frac{50x}{50}=\frac{1000}{50}$ $\quad x=20\%$ increase

4. 80 to 70?

$80-70=10$ $\quad \frac{10}{80}=\frac{x}{100}$ $\quad \frac{80x}{80}=\frac{1000}{80}$ $\quad x=12.5\%$ decrease

5. 60 to 72?

$72-60=12$ $\quad \frac{12}{60}=\frac{x}{100}$ $\quad \frac{60x}{60}=\frac{1200}{60}$ $\quad x=20\%$ increase

6. 50 to 30?

$50-30=20$ $\quad \frac{20}{50}=\frac{x}{100}$ $\quad \frac{50x}{50}=\frac{2000}{50}$ $\quad x=40\%$ decrease

7. 12 to 10?

$12-10=2$ $\quad \frac{2}{12}=\frac{x}{100}$ $\quad \frac{12x}{12}=\frac{200}{12}$ $\quad x=16.6$ decrease

8. 80 to 96?

$96-80=16$ $\quad \frac{16}{80}=\frac{x}{100}$ $\quad \frac{80x}{80}=\frac{1600}{80}$ $\quad x=20\%$ increase

9. A dress with an original price of $125 went on clearance for $68.75. What is the percent of decrease of the dress?

$125-68.75=56.25$ $\quad \frac{56.25}{125}=\frac{x}{100}$ $\quad \frac{125x}{125}=\frac{5625}{125}$ $\quad x=45\%$

10. The population of a town went from 23,000 to 24,380 over the course of a year. What was the percent of increase of the population?

$24380 - 23000 = 1380$

$\frac{1380}{23000} = \frac{x}{100}$

$23000x = 138000$

$\frac{23000x}{23000} = \frac{138000}{23000}$

$x = 6\%$

11. The price of a stock changed from \$2.56 per share to \$4.12 per share. What is the percent of change rounded to the nearest tenth of a percent?

$4.12 - 2.56 = 1.56$

$\frac{1.56}{2.56} = \frac{x}{100}$

$2.56x = 156$

$\frac{2.56x}{2.56} = \frac{156}{2.56}$

$x = 60.9\%$

12. In 2011, the Schlansky Zoo had 5 elephants. In 2012, the Schlansky Zoo had 8 elephants. What is the percent of increase of the population of elephants in the Schlansky Zoo from 2011 to 2012?

$8 - 5 = 3$

$\frac{3}{5} = \frac{x}{100}$

$5x = 300$

$\frac{5x}{5} = \frac{300}{5}$

$x = 60\%$

13. The price of gas in 2019 was \$2.29 per gallon. The price of gas in 2021 was \$3.23 per gallon. What was the percent of increase of the cost of a gallon of gas from 2019 to 2021?

$3.23 - 2.29 = .94$

$\frac{.94}{2.29} = \frac{x}{100}$

$2.29x = 94$

$\frac{2.29x}{2.29} = \frac{94}{2.29}$

$x = 41.\overline{0}\%$

14. Before SAT Prep, Mr. Schlansky averaged 6 students per day in tutorial. During SAT Prep, Mr. Schlansky averaged 11 students per day in tutorial. What was the percent of change in the average number of students attending Mr. Schlansky's tutorial?

$11 - 6 = 5$

$\frac{5}{6} = \frac{x}{100}$

$6x = 500$

$\frac{6x}{6} = \frac{500}{6}$

$x = 83.\overline{3}\%$

Evaluating Functions

Algebraically: Substitute the value in parenthesis in for x.

Graphically: Find the y value for the x value that is in parenthesis.

1. If $f(x)=6x-3$, $g(x)=-x^2+x$, $h(x)=2\sqrt{x^2+2}$ find:

a) f(3)

$f(3)=6(3)-3$

$f(3)=15$

b) f(0)

$f(0)=6(0)-3$

$f(0)=-3$

c) $f(x+5)$

$f(x+5)=6(x+5)-3$

$f(x+5)=6x+30-3$

$f(x+5)=6x+27$

d) g(0)

$g(0)=-(0)^2+0$

$g(0)=0$

e) g(4.2)

$g(4.2)=-(4.2)^2+4.2$

$g(4.2)=-13.44$

f) $g(x-4)$

$g(x-4)=-(x-4)^2+x-4$

$=-(x-4)(x-4)+x-4$

$=-(x^2-8x+16)+x-4$

$=-x^2+8x-16+x-4$

$=-x^2+9x-20$

g) h(1)

$h(1)=2\sqrt{(1)^2+2}$

$h(1)=2\sqrt{3}$

h) h(7)

$h(7)=2\sqrt{(7)^2+2}$

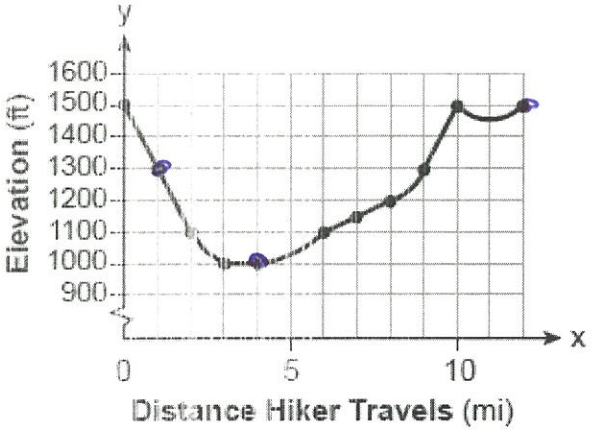
$h(7)=2\sqrt{51}$

i) $h(-2x)$

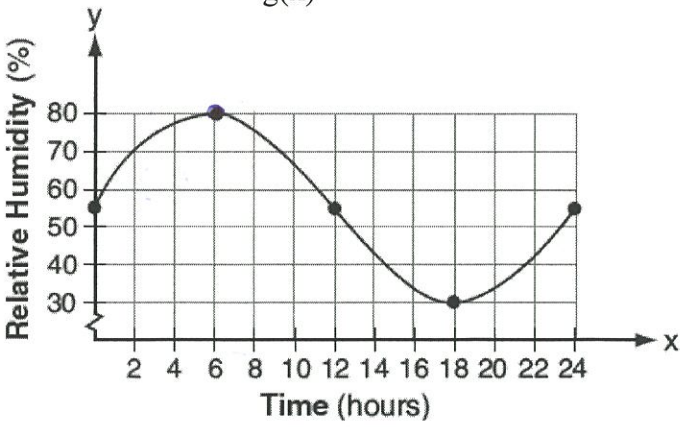
$h(-2x)=2\sqrt{(-2x)^2+2}$

$h(-2x)=2\sqrt{4x^2+2}$

2. f(x)



g(x)



a) Evaluate f(4)

$f(4)=1000$

b) Evaluate f(12)

$f(12)=1500$

c) Evaluate f(1)

$f(1)=1300$

d) Evaluate g(6)

$g(6)=80$

e) Evaluate g(18)

$g(18)=30$

f) Evaluate g(24)

$g(24)=55$

Operations with Polynomials
Adding: Combine like terms
Subtracting: Distribute the negative and then combine like terms (Keep Change Change)
***Subtracting from: from comes first!**
Multiplying: Box Method (Multiply to determine what's in the boxes, add to combine like terms)
Dividing: Divide every term in the numerator by the denominator
*Use Multiple Choice Strategy if Multiple Choice

1. If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals

1) $-5x^2 - 11x + 13$
2) $5x^2 + 11x - 13$
3) $-5x^2 - x + 1$
4) $5x^2 - x + 1$

$(3x^2+5x-6)-(-2x^2-6x+7)$
$3x^2+5x-6+2x^2+6x-7$
$5x^2+11x-13$

2. What is the result when $4x^2 - 17x + 36$ is subtracted from $2x^2 - 5x + 25$?

1) $6x^2 - 22x + 61$
2) $2x^2 - 12x + 11$
3) $-2x^2 - 22x + 61$
4) $-2x^2 + 12x - 11$

$(2x^2-5x+25)-(4x^2-17x+36)$
$2x^2-5x+25-4x^2+17x-36$
$-2x^2+12x-11$

3. Which expression is equivalent to $2(3g - 4) - (8g + 3)$?

1) $-2g - 1$
2) $-2g - 5$
3) $-2g - 7$
4) $-2g - 11$

$6g-8-8g-3$
$-2g-11$

4. What is the product of $2x + 3$ and $4x^2 - 5x + 6$?

1) $8x^3 - 2x^2 + 3x + 18$
2) $8x^3 - 2x^2 - 3x + 18$
3) $8x^3 + 2x^2 - 3x + 18$
4) $8x^3 + 2x^2 + 3x + 18$

$(2x+3)(4x^2-5x+6)$

	$4x^2$	$-5x$	$+6$
$2x$	$8x^3$	$-10x^2$	$+12x$
$+3$	$+12x^2$	$-15x$	$+18$

$8x^3+2x^2-3x+18$

5. Multiply $(2x^2 + 3x - 2)(x - 2)$

	$2x^2$	$+3x$	-2
x	$2x^3$	$+3x^2$	$-2x$
-2	$-4x^2$	$-6x$	$+4$

$2x^3-x^2-8x+4$

6. The result when $6x^2 - 13x + 12$ is subtracted from $-3x^2 + 6x + 7$ is a polynomial in the form $ax^2 + bx + c$. What is the value of $a+b+c$?

$(-3x^2+6x+7) - (6x^2-13x+12)$

$-3x^2+6x+7-6x^2+13x-12$

$-9x^2+19x-5$

ax^2+bx+c

$a=-9$

$b=19$

$c=-5$

$a+b+c$

$-9+19+-5$

(5)

7. The product of $2x^2 + 7x - 10$ and $x + 5$ is expressed in its standard form of $ax^3 + bx^2 + cx + d$. What is $a+b-d$?

$(2x^2+7x-10)(x+5)$

	$2x^2$	$+7x$	-10
x	$2x^3$	$+7x^2$	$-10x$
$+5$	$+10x^2$	$+35x$	-50

$2x^3+17x^2+25x-50$

ax^3+bx^2+cx+d

$a=2$ $c=25$

$b=17$ $d=-50$

$a+b-d$

$2+17+(+50)$

(69)

8. When express in simplest form, $(3x^2 + 4x - 8) - (-2x^2 + 4x + 2)$ can be written as $ax^2 + bx + c$. What is the value of $2b + \frac{1}{2}c$?

$3x^2+4x-8+2x^2-4x-2$

$5x^2+0x-10$

ax^2+bx+c

$a=5$

$b=0$

$c=-10$

$2b+\frac{1}{2}c$

$2(0)+\frac{1}{2}(-10)$

(−5)

9. The expression $5x + 4x^2(2x + 7) - 6x^2 - 9x$ is a polynomial which can be written in the form $ax^3 + bx^2 + cx + d$. What is the value of $2a - cd$?

$5x+8x^3+28x^2-6x^2-9x$

$8x^3+22x^2-4x+0$

ax^3+bx^2+cx+d

$a=8$

$b=22$

$c=-4$

$d=0$

$2a-cd$

$2(8)-(-4)(0)$

(16)

10. The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is written in its standard form $ax^2 + bx + c$. What is the value of $c - 2b$?

$3x^2-3-x^2+7x-10$

$2x^2+7x-13$

ax^2+bx+c

$a=2$

$b=7$

$c=-13$

$c-2b$

$-13-2(7)$

(−27)

Square Binomial Theorem

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

You need to recognize and apply this rule.

When given $a^2 + 2ab + b^2$, take the square root of the first term and the last term to determine a and b.

Express the following as trinomials

1. $(m+7)^2$

$m^2+14m+49$

2. $(y-4)^2$

$y^2-8y+16$

3. $(x-9)^2$

$x^2-18x+81$

4. $(z+2)^2$

z^2+4z+4

5. $(x+y)^2$

$x^2+2xy+y^2$

6. $(m-n)^2$

$m^2-2mn+n^2$

7. $(x-2y)^2$

$x^2-4xy+4y^2$

8. $(2m+n)^2$

$4m^2+4mn+n^2$

9. $(4y-5)^2$

$16y^2-40y+25$

10. $(3x+y)^2$

$9x^2+6xy+y^2$

Express the following as binomials squared

11. $\sqrt{x^2} \boxed{+} 6x + \sqrt{9}$

$(x+3)^2$

12. $\sqrt{y^2} \boxed{-} 8y + \sqrt{16}$

$(y-4)^2$

13. $\sqrt{m^2} \boxed{+} 2mn + \sqrt{n^2}$

$(m+n)^2$

14. $\sqrt{k^2} \boxed{-} 2kj + \sqrt{j^2}$

$(k-j)^2$

15. $\sqrt{x^2} \boxed{-} 2cx + \sqrt{c^2}$

$(x-c)^2$

16. $\sqrt{n^2} \boxed{+} 2nq + \sqrt{q^2}$

$(n+q)^2$

17. $\sqrt{y^2} \boxed{-} 2yx + \sqrt{x^2}$

$(y-x)^2$

18. $\sqrt{x^4} \boxed{-} 2mx^2 + \sqrt{m^2}$

$(x^2-m)^2$

19. $\sqrt{4a^2} \boxed{-} 4ab + \sqrt{b^2}$

$(2a-b)^2$

20. $\sqrt{m^2} \boxed{+} 10mn + \sqrt{25n^2}$

$(m+5n)^2$

21. $\sqrt{9x^2} \boxed{+} 12xy + \sqrt{4y^2}$

$(3x+2y)^2$

22. $\sqrt{a^6} \boxed{+} 8a^3b + \sqrt{16b^2}$

$(a^3+4b)^2$

Identities

1) Put both sides in standard form using operations with polynomials.
2) Set the coefficients of the left hand side equal to the corresponding coefficients on the right hand side.
3) Solve for each variable.

1. Solve for a: $x^2+6x+8=(x+2)(x+a)$

$x^2+6x+8 = x^2+2x+ax+2a$

$x^2+6x+8 = x^2+(2+a)x+2a$

$6=2+a \quad 8=2a$

$-2 \quad -2 \quad \frac{8}{2}=\frac{2a}{2}$

$4=a \quad 4=a$

	x	$+2$
x	x^2	$+2x$
$+a$	$+ax$	$+2a$

$x^2+2x+ax+2a$

2. Solve for h: $(x-6)^2-h=x^2-12x+40$

$x^2-12x+36-h = x^2-12x+40$

$36-h=40 \quad -36 \quad -36$

$\frac{-h}{-1}=\frac{4}{-1}$

$h=-4$

	x	-6
x	x^2	$-6x$
-6	$-6x$	$+36$

$x^2-12x+36$

3. Solve for m and p: $(x-m)(px+3)=2x^2-9x-18$

$px^2+3x-mpx-3m=2x^2-9x-18$

$px^2+(3-mp)x-3m=2x^2-9x-18$

$p=2$

$3-mp=-9$

$3-m(2)=-9$

$3-2m=-9$

$-3 \quad -3$

$\frac{-2m}{-2}=\frac{-12}{-2}$

$m=6$

$\frac{-3m}{-3}=\frac{-18}{-3}$

$m=6$

	x	$-m$
px	px^2	$-mpx$
$+3$	$+3x$	$-3m$

$px^2+3x-mpx-3m$

4. Solve for a and b: $(x^2+a)(x+b)=x^3+x^2+3x+3$

$x^3+bx^2+ax+ab = x^3+1x^2+3x+3$

$b=1 \quad a=3$

$ab=3$

$(3)(1)=3$

	x^2	$+a$
x	x^3	$+ax$
$+b$	$+bx^2$	$+ab$

$x^3+bx^2+ax+ab$

5. Solve for h and k: $3x^3 - 8x^2 + 13 = (3x^2 + hx - 4)(x - 2) + k$

$3x^3 - 8x^2 + 13 = 3x^3 + (h-6)x^2 + (-2h-4)x + 8 + k$

$-8 = h - 6$ $\quad +6 \quad +6$

$-2 = h$

$13 = 8 + k$

$-8 \quad -8$

$5 = k$

×	$3x^2$	$+hx$	-4
x	$3x^3$	$+hx^2$	$-4x$
-2	$-6x^2$	$-2hx$	$+8$

$3x^3 + (h-6)x^2 + (-2h-4)x + 8$

6. Algebraically determine the values of h and k to correctly complete the identity stated below.

$2x^3 - 10x^2 + 11x - 7 = (x-4)(2x^2 + hx + 3) + k$

$2x^3 - 10x^2 + 11x - 7 = 2x^3 + (h-8)x^2 + (-4h+3)x - 12 + k$

$-10 = h - 8$ $\quad +8 \quad +8$

$-2 = h$

$-7 = -12 + k$

$+12 \quad +12$

$5 = k$

×	$2x^2$	$+hx$	$+3$
x	$2x^3$	$+hx^2$	$+3x$
-4	$-8x^2$	$-4hx$	-12

$2x^3 + (h-8)x^2 + (-4h+3)x - 12$

7. Algebraically determine the values of h and k to correctly complete the identity stated below.

$x^3 - 8x^2 + 5x + 53 = (x-5)^2(x+h) + k$

$= (x-5)(x-5)(x+h) + k$

$= (x^2 - 10x + 25)(x+h) + k$

$x^3 - 8x^2 + 5x + 53 = x^3 + (h-10)x^2 + (25-10h)x + 25h + k$

$-8 = h - 10$ $\quad +10 \quad +10$

$2 = h$

$53 = 25h + k$

$53 = 25(2) + k$

$53 = 50 + k$

$-50 \quad -50$

$3 = k$

×	x	-5
x	x^2	$-5x$
-5	$-5x$	$+25$

$x^2 - 10x + 25$

×	x^2	$-10x$	$+25$
x	x^3	$-10x^2$	$+25x$
$+h$	$+hx^2$	$-10hx$	$+25h$

$x^3 + (h-10)x^2 + (25-10h)x + 25h$

8. Solve for a and b: $(x+a)(x^2 - 3x + b) = x^3 - x^2 - 5x + 2$

$x^3 + (a-3)x^2 + (-3a+b)x + ab = x^3 - x^2 - 5x + 2$

$a - 3 = -1$ $\quad +3 \quad +3$

$a = 2$

$-3a + b = -5$

$-3(2) + b = -5$

$-6 + b = -5$

$+6 \quad +6$

$b = 1$

×	x^2	$-3x$	$+b$
x	x^3	$-3x^2$	$+bx$
$+a$	$+ax^2$	$-3ax$	$+ab$

$x^3 + (a-3)x^2 + (-3a+b)x + ab$

Adding and Subtracting Fractions

1) Find the Least Common Denominator
   - -Integers: Find the least common multiple (the smallest # every # divides into)
   - -Variables: List all factors in all denominators and put them together
2) Multiply top and bottom of each fraction by what is missing
3) Combine numerators and keep denominators

1. $\frac{2x}{2(3)}+\frac{y}{(2)3}$

$\frac{2x}{6}+\frac{3y}{6}$

$\boxed{\frac{2x+3y}{6}}$

2. $\frac{3}{3}\left(\frac{3x}{10}\right)-\left(\frac{x}{15}\right)\frac{2}{2}$

$\frac{9x}{30}-\frac{2x}{30}=\boxed{\frac{7x}{30}}$

3. $\frac{2}{2}\left(\frac{2}{x}\right)+\left(\frac{x}{2}\right)\frac{x}{x}$ F1: 2, F2: x, LCD: 2x

$\frac{4}{2x}+\frac{x^2}{2x}$

$\boxed{\frac{4+x^2}{2x}}$

4. $\frac{5}{5}\left(\frac{3}{x}\right)+\left(\frac{2}{5}\right)\frac{x}{x}$ F1: 5, F2: x, LCD: 5x

$\frac{15}{5x}+\frac{2x}{5x}$

$\boxed{\frac{15+2x}{5x}}$

5. $\frac{2}{2}\left(\frac{a}{x}\right)+\frac{b}{2x}$ F1: 2, F2: x, LCD: 2x

$\frac{2a}{2x}+\frac{b}{2x}$

$\boxed{\frac{2a+b}{2x}}$

6. $\frac{3}{3}\left(\frac{3}{7n}\right)+\left(\frac{7}{3n}\right)\frac{7}{7}$ F1: 21, F2: n, LCD: 21n

$\frac{9}{21n}+\frac{49}{21n}$

$\boxed{\frac{58}{21n}}$

7. $\frac{x}{x}\left(\frac{1}{x+1}\right)+\left(\frac{1}{x}\right)\frac{x+1}{x+1}$ F1: x, F2: x+1, LCD: x(x+1)

$\frac{x}{x(x+1)}+\frac{x+1}{x(x+1)}$

$\boxed{\frac{2x+1}{x(x+1)}}$

8. $\frac{x+3}{x+3}\left(\frac{1}{x}\right)+\left(\frac{1}{x+3}\right)\frac{x}{x}$ F1: x, F2: x+3, LCD: x(x+3)

$\frac{x+3}{x(x+3)}+\frac{x}{x(x+3)}$

$\boxed{\frac{2x+3}{x(x+3)}}$

9. $\frac{2}{x-4}+\frac{3}{x+4}$ (x+4)(x-4)

F1: x-4
F2: x+4
LCD: (x-4)(x+4)

$\frac{2(x+4)}{(x-4)(x+4)}+\frac{3(x-4)}{(x-4)(x+4)}$

$\frac{2(x+4)+3(x-4)}{(x-4)(x+4)}$ → $\frac{2x+8+3x-12}{(x-4)(x+4)}$

$\frac{5x-4}{(x-4)(x+4)}$

10. $\frac{9}{c+8}+\frac{2}{c}$ c(c+8)

F1: c
F2: c+8
LCD: c(c+8)

$\frac{9c}{c(c+8)}+\frac{2(c+8)}{c(c+8)}$

$\frac{9c-2(c+8)}{c(c+8)}$ → $\frac{9c-2c-16}{c(c+8)}$

$\frac{7c-16}{c(c+8)}$

11. $\frac{1}{2x+3}+\frac{4}{1}$ (2x+3)

$\frac{1}{2x+3}+\frac{4(2x+3)}{2x+3}$

$\frac{1+4(2x+3)}{2x+3}$ → $\frac{1+8x+12}{2x+3}$

$\frac{8x+13}{2x+3}$

12. $\frac{4}{x+5}-\frac{2}{1}$ (x+5)

$\frac{4}{x+5}-\frac{2(x+5)}{x+5}$

$\frac{4-2(x+5)}{x+5}$ → $\frac{4-2x-10}{x+5}$

$\frac{-2x-6}{x+5}$

13. $\frac{y-2}{y+3}+\frac{y}{1}$ (y+3)

$\frac{y-2}{y+3}+\frac{y(y+3)}{y+3}$

$\frac{y-2+y(y+3)}{y+3}$ → $\frac{y-2+y^2+3y}{y+3}$

$\frac{y^2+4y-2}{y+3}$

14. $\frac{4x}{x+2}+\frac{x-5}{x+2}$ x+2

$\frac{4x(x+2)}{x+2}+\frac{x-5}{x+2}$

$\frac{4x(x+2)+x-5}{x+2}$ → $\frac{4x^2+8x+x-5}{x+2}$

$\frac{4x^2+9x-5}{x+2}$

15. $\frac{x-1}{(x+3)^2}+\frac{2}{x+3}$ (x+3)

$\frac{x-1}{(x+3)^2}+\frac{2(x+3)}{(x+3)^2}$

$\frac{x-1+2x+6}{(x+3)^2}$

$\frac{3x+5}{(x+3)^2}$

16. $\frac{3x+5}{(x-2)^2}-\frac{3}{x-2}$ (x-2)

$\frac{3x+5}{(x-2)^2}-\frac{3(x-2)}{(x-2)^2}$

$\frac{3x+5-3(x-2)}{(x-2)^2}$

$\frac{3x+5-3x+6}{(x-2)^2}$ → $\frac{11}{(x-2)^2}$

Operations with Complex Numbers

Replace i^2 with -1!!!!!!

Adding: Combine like terms

Subtracting: Distribute the negative and then combine like terms (Keep Change Change)

Multiplying: Box Method (Multiply to determine what's in the boxes, add to combine like terms)

Dividing: Multiply by the conjugate of the denominator (switch middle sign).

*Use Multiple Choice Strategy if possible

1. $(-2+9i)+(6+8i)$

$-2+9i+6+8i$

$4+17i$

2. $(-10+2i)+(7+6i)$

$-10+2i+7+6i$

$-3+8i$

3. $(5-2i)-(2-3i)$

$5-2i-2+3i$

$3+i$

4. $(-2+2i)-(8-i)$

$-2+2i-8+i$

$-10+3i$

5. $(7-2i)\bullet(8+3i)$

	7	$-2i$
8	56	$-16i$
$+3i$	$21i$	$-6i^2$

$56+5i-6i^2$

$56+5i-6(-1)$

$56+5i+6$

$62+5i$

6. $(6-i)\bullet(8-5i)$ $48-38i+5i^2$

	6	$-i$
8	48	$-8i$
$-5i$	$-30i$	$+5i^2$

$48-38i+5(-1)$

$48-38i-5$

$43-38i$

7. The expression $(3-7i)^2$ is equivalent to

1) $-40+0i$
2) $40-42i$
3) $58+0i$
4) $58-42i$

	3	$-7i$
3	9	$-21i$
$-7i$	$-21i$	$+49i^2$

$9-42i+49i^2$

$9-42i+49(-1)$

$9-42i-49$

$-40-42i$

8. Given i is the imaginary unit, $(2-yi)^2$ in simplest form is

1) $y^2-4yi+4$
2) $-y^2-4yi+4$
3) $-y^2+4$
4) y^2+4

	2	$-yi$
2	4	$-2yi$
$-yi$	$-2yi$	$+y^2i^2$

$4-4yi+y^2i^2$

$4-4yi+y^2(-1)$

$4-4yi-y^2$

9. If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$2xi^2 - 8xi$

$2x(-1) - 8xi$

$-2x - 8xi$

10. $\frac{3}{(2+5i)} \cdot \frac{(2-5i)}{(2-5i)}$

$\frac{6-15i}{4-25i^2}$

$\frac{6-15i}{4-25(-1)}$

$\frac{6-15i}{4+25}$

$\frac{6-15i}{33}$

$\frac{6}{33} - \frac{15i}{33}$

11. $\frac{7}{(5-i)} \cdot \frac{(5+i)}{(5+i)}$

$\frac{35+7i}{25-i^2}$

$\frac{35+7i}{25-(-1)}$

$\frac{35+7i}{25+1}$

$\frac{35+7i}{26}$

$\frac{35}{26} + \frac{7i}{26}$

12. $\frac{2}{(6+2i)} \cdot \frac{(6-2i)}{(6-2i)}$

$\frac{12-4i}{36-4i^2}$

$\frac{12-4i}{36-4(-1)}$

$\frac{12-4i}{36+4}$

$\frac{12-4i}{40}$

$\frac{3}{10} - \frac{1}{10}i$

13. $\frac{9}{(-9-5i)} \cdot \frac{(-9+5i)}{(-9+5i)}$

$\frac{-81+45i}{81-25i^2}$

$\frac{-81+45i}{81-25(-1)}$

$\frac{-81+45i}{81+25}$

$\frac{-81+45i}{106}$

$\frac{-81}{106} + \frac{45i}{106}$

14. $\frac{(7-2i)(3+4i)}{(3-4i)(3+4i)}$

	7	$-2i$
3	21	$-6i$
$+4i$	$+28i$	$-8i^2$

$21 + 22i - 8i^2$

$21 + 22i - 8(-1)$

$21 + 22i + 8$

$29 + 22i$

$\frac{29+22i}{9-16i^2}$

$\frac{29+22i}{9-16(-1)}$

$\frac{29+22i}{9+16}$

$\frac{29+22i}{25}$

$\frac{29}{25} + \frac{22}{25}i$

15. $\frac{(2+7i)(1+3i)}{(1-3i)(1+3i)}$

	2	$+7i$
1	2	$7i$
$+3i$	$+6i$	$+21i^2$

$2 + 13i + 21i^2$

$2 + 13i + 21(-1)$

$2 + 13i - 21$

$-19 + 13i$

$\frac{-19+13i}{1-9i^2}$

$\frac{-19+13i}{1-9(-1)}$

$\frac{-19+13i}{10}$

$\frac{-19}{10} + \frac{13i}{10}$

16. $\frac{(8-3i)(5+i)}{(5-i)(5+i)}$

	8	$-3i$
5	40	$-15i$
$+i$	$+8i$	$-3i^2$

$40 - 7i - 3i^2$

$40 - 7i - 3(-1)$

$40 - 7i + 3$

$43 - 7i$

$\frac{43-7i}{25-i^2}$

$\frac{43-7i}{25-(-1)}$

$\frac{43-7i}{25+1}$

$\frac{43-7i}{26}$

$\frac{43}{26} - \frac{7i}{26}$

17. $\frac{(-2-5i)(7-2i)}{(7+2i)(7-2i)}$

	-2	$-5i$
7	-14	$-35i$
$-2i$	$+4i$	$+10i^2$

$-14 - 31i + 10i^2$

$-14 - 31i + 10(-1)$

$-14 - 31i - 10$

$-24 - 31i$

$\frac{-24-31i}{49-4i^2}$

$\frac{-24-31i}{49-4(-1)}$

$\frac{-24-31i}{49+4}$

$\frac{-24-31i}{53}$

$\frac{-24}{53} - \frac{31i}{53}$

Multiply first and last with the conjugate

Multiply middle with the conjugate

Dividing Polynomials: (Synthetic Division)
Negative the value of what you are dividing by and put it outside
Bring the first number down
Multiply, Add, Multiply, Add, etc.
Decrease the first terms exponent by 1, the last number is the remainder. The remainder goes over the divisor.
(Put 0 as a placeholder if necessary)

Divide each of the following polynomials

1. $\frac{x^2+3x-4}{x+4}$

$-4 \mid 1 \;\; 3 \;\; -4$
$\downarrow \;\; -4 \;\; 4$
$1 \;\; -1 \;\; 0$

$x-1$

2. $\frac{x^2+7x+5}{x+1}$

$-1 \mid 1 \;\; 7 \;\; 5$
$\downarrow \;\; -1 \;\; -6$
$1 \;\; 6 \;\; -1$

$x+6-\frac{1}{x+1}$

3. $\frac{x^2-10x-21}{x+2}$

$-2 \mid 1 \;\; -10 \;\; -21$
$\downarrow \;\; -2 \;\; 24$
$1 \;\; -12 \;\; 3$

$x-12+\frac{3}{x+2}$

4. $\frac{-x^2-8x+33}{x+10}$

$-10 \mid -1 \;\; -8 \;\; 33$
$\downarrow \;\; 10 \;\; -20$
$-1 \;\; 2 \;\; 13$

$-x+2+\frac{13}{x+10}$

$5x^4+17x^3+10x^2+0x-5$

5. $\frac{5x^4+17x^3+10x^2-5}{x+3}$

$-3 \mid 5 \;\; 17 \;\; 10 \;\; 0 \;\; -5$
$\downarrow \;\; -15 \;\; -6 \;\; -12 \;\; 36$
$5 \;\; 2 \;\; 4 \;\; -12 \;\; 31$

$5x^3+2x^2+4x-12+\frac{31}{x+3}$

$2x^4-3x^3+6x^2+0x-5$

6. $\frac{2x^4-3x^3+6x^2-5}{x+3}$

$-3 \mid 2 \;\; -3 \;\; 6 \;\; 0 \;\; -5$
$\downarrow \;\; -6 \;\; 27 \;\; -99 \;\; 297$
$2 \;\; -9 \;\; 33 \;\; -99 \;\; 292$

$2x^3-9x^2+33x-99+\frac{292}{x+3}$

7. $\frac{2x^3-x-2}{x-4}$

$2x^3+0x^2-x-2$
$x-4$

$4 \mid 2 \;\; 0 \;\; -1 \;\; -2$
$\downarrow \;\; 8 \;\; 32 \;\; 124$
$2 \;\; 8 \;\; 31 \;\; 122$

$2x^2+8x+31+\frac{122}{x-4}$

8. $\frac{2x^3-3x^2+2x+5}{x-5}$

$5 \mid 2 \;\; -3 \;\; 2 \;\; 5$
$\downarrow \;\; 10 \;\; 35 \;\; 185$
$2 \;\; 7 \;\; 37 \;\; 190$

$2x^2+7x+37+\frac{190}{x-5}$

9. $\frac{x^3+5x^2-1}{x+2}$

$$\begin{array}{r|cccc} -2 & 1 & 5 & 0 & -1 \\ & & -2 & -6 & 12 \\ \hline & 1 & 3 & -6 & 11 \end{array}$$

$x^2+3x-6+\frac{11}{x+2}$

10. $\frac{x^4-2x^2-7x+12}{x+6}$

$$\begin{array}{r|ccccc} -6 & 1 & 0 & -2 & -7 & 12 \\ & & -6 & 36 & -204 & 1266 \\ \hline & 1 & -6 & 34 & -211 & 1278 \end{array}$$

$x^3-6x^2+34x-211+\frac{1278}{x+6}$

11. $\frac{2x^3+5x^2-31x-84}{x+3}$

$$\begin{array}{r|cccc} -3 & 2 & 5 & -31 & -84 \\ & & -6 & 3 & 84 \\ \hline & 2 & -1 & -28 & 0 \end{array}$$

$2x^2-x-28$

12. $\frac{4x^3+12x^2-5}{x+5}$

$$\begin{array}{r|cccc} -5 & 4 & 12 & 0 & -5 \\ & & -20 & 40 & -200 \\ \hline & 4 & -8 & 40 & -205 \end{array}$$

$4x^2-8x+40-\frac{205}{x+5}$

13. $\frac{6x^3-5x+3}{x-3}$

$$\begin{array}{r|cccc} 3 & 6 & 0 & -5 & 3 \\ & & 18 & 54 & 147 \\ \hline & 6 & 18 & 49 & 150 \end{array}$$

$6x^2+18x+49+\frac{150}{x-3}$

14. $\frac{5x^3-60}{x-2}$

$$\begin{array}{r|cccc} 2 & 5 & 0 & 0 & -60 \\ & & 10 & 20 & 40 \\ \hline & 5 & 10 & 20 & -20 \end{array}$$

$5x^2+10x+20-\frac{20}{x-2}$

15. $\frac{x^2+x-4}{x-3}$

$$\begin{array}{r|ccc} 3 & 1 & 1 & -4 \\ & & 3 & 12 \\ \hline & 1 & 4 & 8 \end{array}$$

$x+4+\frac{8}{x-3}$

16. $\frac{-3x^2+10x-6}{x+1}$

$$\begin{array}{r|ccc} -1 & -3 & 10 & -6 \\ & & 3 & -13 \\ \hline & -3 & 13 & -19 \end{array}$$

$-3x+13-\frac{19}{x+1}$

Dividing Polynomials: (Long Division)
Divide (First term divided by first term)
Multiply (Top term by outside term)
Subtract (Keep/Change/Change or Distribute the Negative)
Bring Down (The next term)
(Put 0 as a placeholder if necessary)
The remainder goes over the divisor

Divide each of the following polynomials

1. $\frac{4x-8}{2x-3}$

$2 - \frac{2}{2x-3}$

$2x-3 \overline{)\, 4x-8}$

$-(+4x+6)$

-2

2. $\frac{12x+8}{6x+1}$

$2 + \frac{6}{6x+1}$

$6x+1 \overline{)\, 12x+8}$

$-(+12x+2)$

6

3. $\frac{3x-4}{3x+2}$

$1 - \frac{6}{3x+2}$

$3x+2 \overline{)\, 3x-4}$

$-(+3x+2)$

-6

4. $\frac{10x+5}{2x+2}$

$\frac{5}{2} - \frac{5}{2x+2}$

$2x+2 \overline{)\, 10x+5}$

$-(+10x+10)$

-5

5. $\frac{6x-7}{3x-1}$

$2 - \frac{5}{3x-1}$

$3x-1 \overline{)\, 6x-7}$

$-(+6x+2)$

-5

6. $\frac{8x-4}{2x-6}$

$4 + \frac{20}{2x-6}$

$2x-6 \overline{)\, 8x-4}$

$-(+8x+24)$

20

7. $\frac{7x-1}{7x+2}$

$1 - \frac{3}{7x+2}$

$7x+2 \overline{)\, 7x-1}$

$-(+7x+2)$

-3

8. $\frac{15x+3}{5x+2}$

$3 - \frac{3}{5x+2}$

$5x+2 \overline{)\, 15x+3}$

$-(+15x+6)$

-3

9. $\frac{6x^2+10x-7}{3x-1}$

$2x+4-\frac{3}{3x-1}$

$3x-1 \overline{)6x^2+10x-7}$

$+-6x^2+2x$

$12x-7$

$+-12x+4$

-3

10. $\frac{10x^2-34x-19}{5x+3}$

$2x-8+\frac{5}{5x+3}$

$5x+3 \overline{)10x^2-34x-19}$

$+-10x^2+6x$

$-40x-19$

$++40x+24$

5

11. $\frac{2x^2+3x-27}{2x-5}$

$x+4-\frac{7}{2x-5}$

$2x-5 \overline{)2x^2+3x-27}$

$+-2x^2+5x$

$8x-27$

$+-8x+20$

-7

12. $\frac{9x^2-2}{3x+1}$

$3x-1-\frac{1}{3x+1}$

$3x+1 \overline{)9x^2+0x-2}$

$+-9x^2+3x$

$-3x-2$

$++3x+1$

-1

13. $\frac{6x^3+x^2-15x+5}{3x-4}$

$2x^2+3x+1+\frac{9}{3x-4}$

$3x-4 \overline{)6x^3+x^2-15x+5}$

$+-6x^3+8x^2$

$9x^2-15x$

$+-9x^2+12x$

$3x+5$

$+-3x+4$

9

14. $\frac{8x^3-8x^2+12x-12}{2x-1}$

$4x^2-2x+5-\frac{7}{2x-1}$

$2x-1 \overline{)8x^3-8x^2+12x-12}$

$+-8x^3+4x^2$

$-4x^2+12x$

$++4x^2+2x$

$10x-12$

$+-10x+5$

-7

Points on a Graph

To determine if a point is on a graph, substitute x and y into the equation.

1. Which point lies on the line whose equation is $2x - 3y = 9$?

(1) (-1,-3) (2) (-1,3) (3) (0,3) (4) (0,-3)

$2(-1)-3(-3)=9$ $2(-1)-3(3)=9$ $2(0)-3(3)=9$ $2(0)-3(-3)=9$

$0+9=9$, $9=9$ ✓

2. Which point lies on the line whose equation is $y + 3x = 6$?

1) (3,6) 2) (2,0) 3) (0,2) 4) (4,12)

$6+3(3)=6$ $0+3(2)=6$ $2+3(0)=6$ $12+3(4)=6$

$0+6=6$, $6=6$ ✓

3. Which point lies on the line whose equation is $3y - x = 2$?

1) (3,-1) 2) (2,-1) 3) (7,3) 4) (-1,2)

$3(-1)-3=2$ $3(-1)-2=2$ $3(3)-7=2$ $3(2)+1=2$

$9-7=2$, $2=2$ ✓

4. Which point lies on the line whose equation is $4x + 5y - 3 = 0$?

1) (4,5) 2) (2,-3) 3) (-6,4) 4) (2,-1)

$4(4)+5(5)-3=0$ $4(2)+5(-3)-3=0$ $4(-6)+5(4)-3=0$ $4(2)+5(-1)-3=0$

$8-5-3=0$, $0=0$ ✓

5. The point $(x,3)$ is on the graph whose equation is $y = 2x + 5$. What is the value of x?

$3 = 2x + 5$

$-5 \quad -5$

$-2 = 2x$

$\frac{-2}{2} = \frac{2x}{2}$

$-1 = x$

6. The point $(2,k)$ is on the graph whose equation is $2x + 3y = 6$. What is the value of k?

$2(2)+3(k)=6$

$4+3k=6$

$-4 \quad -4$

$\frac{3k}{3} = \frac{2}{3}$

$k = \frac{2}{3}$

7. The point $(-4,b)$ is on the graph whose equation is $y + x^2 = 6$. What is the value of b?

$b+(-4)^2=6$

$b+16=6$

$-16 \quad -16$

$b = -10$

8. The point $(a,-2)$ is on the graph whose equation is $2y + x^2 = 12$. What is the positive value of a?

$2(-2)+a^2=12$

$-4+a^2=12$

$+4 \quad +4$

$\sqrt{a^2} = \sqrt{16}$

$a = 4$

Systems of Inequalities

Substitute each coordinate in for x and y into both equations.
A solution must work in both inequalities.

1. Which ordered pair is in the solution set of the following system of linear inequalities?

$y < 2x + 2$
$y \geq -x - 1$

1) (0, 3)
2) (2, 0)
3) (−1, 0)
4) (−1, −4)

1) $3 < 2(0) + 2$ ✗
$3 \geq -0 - 1$ ✓

2) $0 < 2(2) + 2$ ✓
$0 \geq -2 - 1$ ✓

3) $0 < 2(-1) + 2$ ✗
$0 \geq +1 - 1$ ✓

4) $-4 < 2(-1) + 2$ ✓
$-4 \geq 1 - 1$ ✗

2. Which ordered pair is in the solution set of the following system of inequalities?

$y < \frac{1}{2}x + 4$
$y \geq -x + 1$

1) (−5, 3)
2) (0, 4)
3) (3, −5)
4) (4, 0)

1) $3 < \frac{1}{2}(-5) + 4$ ✗
$3 \geq 5 + 1$ ✗

2) $4 < \frac{1}{2}(0) + 4$ ✗
$4 \geq 0 + 1$ ✗

3) $-5 < \frac{1}{2}(3) + 4$ ✓
$-5 \geq -3 + 1$ ✗

4) $0 < \frac{1}{2}(4) + 4$ ✓
$0 \geq -4 + 1$ ✓

3. Which ordered pair is *not* in the solution set of $y > -\frac{1}{2}x + 5$ and $y \leq 3x - 2$?

1) (5,3)
2) (4,3)
3) (3,4)
4) (4,4)

1) $3 > -\frac{1}{2}(5) + 5$ ✓
$3 \leq 3(5) - 2$ ✓

2) $3 > -\frac{1}{2}(4) + 5$ ✗
$3 \leq 3(4) - 2$ ✓

3) $4 > -\frac{1}{2}(3) + 5$ ✓
$4 \leq 3(3) - 2$ ✓

4) $4 > -\frac{1}{2}(4) + 5$ ✓
$4 \leq 3(4) - 2$ ✓

4. Which ordered pair is in the solution set of the following system:

$y + x > 2$
$y \leq 3x - 2$

1) (1, −2)
2) (1, 6)
3) (4, 10)
4) (−3, 2)

1) $-2 + 1 > 2$ ✗
$-2 \leq 3(1) - 2$ ✓

2) $6 + 1 > 2$ ✓
$6 \leq 3(1) - 2$ ✗

3) $10 + 4 > 2$ ✓
$10 \leq 3(4) - 2$ ✓

4) $2 + -3 > 2$ ✗
$2 \leq 3(-3) - 2$ ✗

5. Which ordered pair is in the solution set of the following system:

$2y > 3x + 4$
$y + 5 \geq -2x$

1) (3, 2)
2) (−2, −1)
3) (−3, 8)
4) (−8, 2)

1) $2(2) > 3(3) + 4$ ✗
$2 + 5 \geq -2(3)$ ✓

2) $2(-1) > 3(-2) + 4$ ✗
$-1 + 5 \geq -2(-2)$ ✓

3) $2(8) > 3(-3) + 4$ ✓
$8 + 5 \geq -2(-3)$ ✓

4) $2(2) > 3(-8) + 4$ ✓
$2 + 5 \geq -2(-8)$ ✗

5. Which ordered pair is in the solution set of the following system: $2y > 3x+4$, $y+5 \geq -2x$

1) $2(2) > 3(3)+4$
1) (3,2) $2+5 \geq -2(3)$
2) (−2,−1) $4 > 10$ ✗, $7 \geq -6$ ✓

3) (−3,8)
4) (−8,2)

3) $2(8) > 3(-3)+4$
$8+5 \geq -2(-3)$
$16 > -5$ ✓
$13 \geq 6$ ✓

2) $2(-1) > 3(-2)+4$
$-1+5 \geq -2(-2)$
$-2 > -2$ ✗
$4 \geq 4$ ✓

4) $2(2) > 3(-8)+4$
$2+5 \geq -2(-8)$
$4 > -20$ ✓
$7 \geq 16$ ✗

6. Which ordered pair is in the solution set of the following system: $2y \geq 3x-16$, $y+2x > -5$

1) (5,−8)
2) (2,6)

1) $2(-8) \geq 3(5)-16$
$-8+2(5) > -5$
$-16 \geq -1$ ✗
$2 > -5$ ✓

2) $2(6) \geq 3(2)-16$
$6+2(2) > -5$
$12 \geq -10$ ✓
$10 > -5$ ✓

3) (−3,1)
4) (−8,0)

3) $2(1) \geq 3(-3)-16$
$1+2(-3) > -5$
$2 \geq -27$ ✓
$-5 > -5$ ✗

4) $2(0) \geq 3(-8)-16$
$0+2(-8) > -5$
$0 \geq -40$ ✓
$-16 > -5$ ✗

7. Which ordered pair is *not* in the solution set of the following system: $2y+3x \leq 14$, $4x-y < 2$

1) (−4,1)
2) (2,6)

1) $2(1)+3(-4) \leq 14$
$4(-4)-1 < 2$
$-12 \leq 14$ ✓
$-17 < 2$ ✓

2) $2(6)+3(2) \leq 14$
$4(2)-6 < 2$
$18 \leq 14$ ✗
$2 < 2$ ✗

3) (−4,−1)
4) (1,4)

3) $2(-1)+3(-4) \leq 14$
$4(-4)-(-1) < 2$
$-14 \leq 14$ ✓
$-15 < 2$ ✓

4) $2(1)+3(1) \leq 14$
$4(1)-4 < 2$
$14 \leq 14$ ✓
$0 < 2$ ✓

8. Which ordered pair is in the solution set of the following system: $3x+4y > 20$, $x < 3y-18$

1) (0,6)
2) (4,10)

1) $3(0)+4(6) > 20$
$0 < 3(6)-18$
$24 > 20$ ✓
$0 < 0$ ✗

2) $3(4)+4(10) > 20$
$4 < 3(10)-18$
$52 > 20$ ✓
$4 < 12$ ✓

3) (−5,5)
4) (10,8)

3) $3(-5)+4(5) > 20$
$-5 < 3(5)-18$
$5 > 20$ ✗
$-5 < -3$ ✓

4) $3(10)+4(8) > 20$
$10 < 3(8)-18$
$62 > 20$ ✓
$10 < 6$ ✗

Factoring:

Greatest Common Factor: GCF( )

Difference of Two Squares (2 Terms): $(\sqrt{\ }+\sqrt{\ })(\sqrt{\ }-\sqrt{\ })$

Trinomials (3 Terms): (x)(x)

1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number

Grouping: (4 Terms or More)

1) Group the first two terms together and the last two terms together
2) Factor out the GCF in each group

*You should be left with the same factor. If signs are reversed, factor out a negative

3) Combine coefficients and keep like term.

***Factor further if necessary**

Factor each expression

1. $\frac{4x}{4}+\frac{8}{4}$ GCF
$4(x+2)$

2. $\frac{12x}{6}+\frac{18}{6}$ GCF
$6(2x+3)$

3. $\frac{x^2}{x}-\frac{7x}{x}$ GCF
$x(x-7)$

4. $\frac{2x^2}{2x}-\frac{4xy}{2x}$ GCF
$2x(x-2y)$

5. $\frac{5x^2y}{5x}-\frac{20x}{5x}$ GCF
$5x(xy-4)$

6. $\frac{2x^3}{2x}+\frac{6x^2}{2x}-\frac{10ax}{2x}$ GCF
$2x(x^2+3x-5a)$

7. $\sqrt{x^2}-\sqrt{64}$ DOTS
$(x+8)(x-8)$

8. $\sqrt{y^2}-\sqrt{36}$ DOTS
$(y+6)(y-6)$

9. $\sqrt{x^2}-\sqrt{a^2}$ DOTS
$(x+a)(x-a)$

10. $\sqrt{k^2}-\sqrt{j^2}$ DOTS
$(k+j)(k-j)$

11. $\sqrt{x^2}\sqrt{6}$ DOTS (Kinda)

$(x+\sqrt{6})(x-\sqrt{6})$

12. x^2-10 DOTS (Kinda)

$(x+\sqrt{10})(x-\sqrt{10})$

13. $\sqrt{y^2}\sqrt{x}$ DOTS (Kinda)

$(y+\sqrt{x})(y-\sqrt{x})$

14. $\sqrt{x^2}\sqrt{ab}$ DOTS (Kinda)

$(x+\sqrt{ab})(x-\sqrt{ab})$

15. $\sqrt{9x^2}\sqrt{6y^4}$ DOTS

$(3x+4y^2)(3x-4y^2)$

16. $\sqrt{4t^2}\sqrt{25}$ DOTS

$(2t+5)(2t-5)$

17. $x^2+4x-12$ Trinomial

$(x+6)(x-2)$

18. y^2+3y+2

$(y+2)(y+1)$

19. $m^2-8m+15$

$(m-5)(m-3)$

20. $x^2-8x-20$

$(x-10)(x+2)$

21. $y^2+5y-14$

$(y+7)(y-2)$

22. $x^2+|x-12$

$(x+4)(x-3)$

23. $x^2-3x-10$

$(x-5)(x+2)$

24. $x^2-7x+12$

$(x-4)(x-3)$

25. $x^2-9x-36$

$(x-12)(x+3)$

26. $y^2-21y+110$

$(y-11)(y-10)$

27. $x^4 - 8x^2 - 9$ Trinomial

DOTS $(x^2 - 9)(x^2 + 1)$

$(x+3)(x-3)(x^2+1)$

28. $x^4 + x^2 - 2$ Trinomial

$(x^2+2)(x^2-1)$ DOTS

$(x^2+2)(x+1)(x-1)$

29. $\frac{2x^2}{2} - \frac{50}{2}$ GCF

$2(x^2 - 25)$ DOTS

$2(x+5)(x-5)$

30. $\frac{2x^2}{2} - \frac{8x}{2} - \frac{10}{2}$ GCF

$2(x^2 - 4x - 5)$ Trinomial

$2(x-5)(x+1)$

31. $\frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$ GCF

$3(x^2 + 3x - 4)$ Trinomial

$3(x+4)(x-1)$

32. $\frac{6x^2}{6} - \frac{54}{6}$ GCF

$6(x^2 - 9)$ DOTS

$6(x+3)(x-3)$

33. $\frac{2x^2}{2} + \frac{14x}{2} + \frac{24}{2}$ GCF

$2(x^2 + 7x + 12)$ Trinomial

$2(x+4)(x+3)$

34. $\frac{5x^2}{5} - \frac{500}{5}$ GCF

$5(x^2 - 100)$ DOTS

$5(x+10)(x-10)$

35. $\frac{ax^2}{a} - \frac{2ax}{a} - \frac{8a}{a}$ GCF

$a(x^2 - 2x - 8)$ Trinomial

$a(x-4)(x+2)$

36. $\frac{yx^2}{y} - \frac{64y}{y}$ GCF

$y(x^2 - 64)$ DOTS

$y(x+8)(x-8)$

37. $\frac{x^5}{x}+\frac{2x^3}{x}-\frac{8x}{x}$ GCF

$x(x^4+2x^2-8)$ Trinomial

$x(x^2+4)(x^2-2)$

38. $\sqrt{x^4}-\sqrt{81}$ DOTS

$(x^2+9)(x^2-9)$ DOTS

$(x^2+9)(x+3)(x-3)$

39. $(x^3-5x^2)+(2x-10)$ Grouping

$x^2(x-5)+2(x-5)$

$(x^2+2)(x-5)$

40. $(x^3+3x^2)+(4x+12)$ Grouping

$x^2(x+3)+4(x+3)$

$(x^2+4)(x+3)$

41. $(x^3+12x^2)-(2x-24)$ Grouping

$x^2(x+12)-2(x+12)$

$(x^2-2)(x+12)$

42. $(x^3+6x^2)-(3x-18)$ Grouping

$x^2(x+6)-3(x+6)$

$(x^2-3)(x+6)$

43. $(x^3+3x^2)-(9x-27)$ Grouping

$x^2(x+3)-9(x+3)$

DOTS $(x^2-9)(x+3)$

$(x+3)(x-3)(x+3)$

44. $(x^3+10x^2)-(9x-90)$ Grouping

$x^2(x+10)-9(x+10)$

DOTS $(x^2-9)(x+10)$

$(x+3)(x-3)(x+10)$

Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor
3) Set each factor equal to zero

If given a binomial squared, take the square root of both sides (the last two steps of the completing the square method)

1. $y^2-5y+6=0$

Trinomial $(y-6)(y+1)=0$

$y-6=0 \quad | \quad y+1=0$

$+6\ +6 \quad | \quad -1\ -1$

$y=6 \quad | \quad y=-1$

2. $x^2+4x=0$

GCF $x(x+4)=0$

$x=0 \quad | \quad x+4=0$

$-4\ -4$

$x=-4$

3. $a^2-8a=20$

$-20\ -20$

Trinomial $a^2-8a-20=0$

$(a-10)(a+2)=0$

$a-10=0 \quad | \quad a+2=0$

$+10\ +10 \quad | \quad -2\ -2$

$a=10 \quad | \quad a=-2$

4. $x^2=25$

$-25\ -25$

DOTS $x^2-25=0$

$(x+5)(x-5)=0$

$x+5=0 \quad | \quad x-5=0$

$-5\ -5 \quad | \quad +5\ +5$

$x=-5 \quad | \quad x=5$

5. $2x^2+5x=0$

GCF $x(2x+5)=0$

$x=0 \quad | \quad 2x+5=0$

$-5\ -5$

$\frac{2x}{2}=\frac{-5}{2}$

$x=-\frac{5}{2}$

6. $x^2+5x=14$

$-14\ -14$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x+7=0 \quad | \quad x-2=0$

$-7\ -7 \quad | \quad +2\ +2$

$x=-7 \quad | \quad x=2$

7. $n^2=3n+18$

$-3n-18\ -3n-18$

Trinomial $n^2-3n-18=0$

$(n-6)(n+3)=0$

$n-6=0 \quad | \quad n+3=0$

$+6\ +6 \quad | \quad -3\ -3$

$n=6 \quad | \quad n=-3$

8. $4x^2=64$

$-64\ -64$

$4x^2-64=0$ → divide away integer GCF

$\frac{4x^2-64}{4}=\frac{0}{4}$

DOTS $x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0 \quad | \quad x-4=0$

$-4\ -4 \quad | \quad +4\ +4$

$x=-4 \quad | \quad x=4$

9. $3x^2+3x-6=0$ → Divide away integer GCF

$\frac{3}{3}\quad\frac{3}{3}$

Trinomial $x^2+x-2=0$

$(x+2)(x-1)=0$

$x+2=0$ | $x-1=0$

$-2\ -2$ | $+1\ +1$

$x=-2$ | $x=1$

10. $(x^3+10x^2-9x-90)=0$ → Grouping

$\frac{x^2}{x^2}\quad\frac{x^2}{x^2}\quad\frac{-9}{-9}\quad\frac{-9}{-9}$

$x^2(x+10)-9(x+10)=0$

DOTS $(x^2-9)(x+10)=0$

$(x+3)(x-3)(x+10)=0$

$x+3=0$ | $x-3=0$ | $x+10=0$

$-3\ -3$ | $+3\ +3$ | $-10\ -10$

$x=-3$ | $x=3$ | $x=-10$

11. $x^2-5=-1$

$+1\ +1$

DOTS $x^2-4=0$

$(x+2)(x-2)=0$

$x+2=0$ | $x-2=0$

$-2\ -2$ | $+2\ +2$

$x=-2$ | $x=2$

12. $2x^2-12x=32$ → divide away integer GCF

$-32\ -32$

$2x^2-12x-32=0$

$\frac{2}{2}\quad\frac{2}{2}\quad\frac{2}{2}\quad\frac{0}{2}$

Trinomial $x^2-6x-16=0$

$(x-8)(x+2)=0$

$x-8=0$ | $x+2=0$

$+8\ +8$ | $-2\ -2$

$x=8$ | $x=-2$

13. $x^3+5x^2-4x=20$

$-20\ -20$

Grouping $(x^3+5x^2-4x-20)=0$

$\frac{x^2}{x^2}\quad\frac{x^2}{x^2}\quad\frac{-4}{-4}\quad\frac{-4}{-4}$

$x^2(x+5)-4(x+5)=0$

$(x^2-4)(x+5)=0$

DOTS $(x+2)(x-2)(x+5)=0$

$x+2=0$ | $x-2=0$ | $x+5=0$

$-2\ -2$ | $+2\ +2$ | $-5\ -5$

$x=-2$ | $x=2$ | $x=-5$

14. $x^4-5x^2=-4$

$+4\ +4$

Trinomial $x^4-5x^2+4=0$

DOTS $(x^2-4)(x^2-1)=0$

$(x+2)(x-2)(x+1)(x-1)$

$x+2=0$ | $x-2=0$ | $x+1=0$ | $x-1=0$

$-2\ -2$ | $+2\ +2$ | $-1\ -1$ | $+1\ +1$

$x=-2$ | $x=2$ | $x=-1$ | $x=1$

15. $2x^4-6x^3=20x^2$

$-20x^2\ -20x^2$

GCF $2x^4-6x^3-20x^2=0$

$\frac{2x^2}{2x^2}\quad\frac{-6x^3}{2x^2}\quad\frac{-20x^2}{2x^2}$

$2x^2(x^2-3x-10)=0$ → Trinomial

$2x^2(x-5)(x+2)=0$

$2x^2=0$ | $x-5=0$ | $x+2=0$

$\frac{2}{2}\quad\frac{0}{2}$ | $+5\ +5$ | $-2\ -2$

$\sqrt{x^2}=0$ | $x=5$ | $x=-2$

$x=0$

16. $x^3=x$

$-x\ -x$

GCF $x^3-x=0$

$x(x^2-1)=0$ → DOTS

$x(x+1)(x-1)=0$

$x=0$ | $x+1=0$ | $x-1=0$

$-1\ -1$ | $+1\ +1$

$x=-1$ | $x=1$

17. $10x^2=x^4+9$

$-10x^2\ -10x^2$

$0=x^4-10x^2+9$ → Trinomial

$0=(x^2-9)(x^2-1)$ ← DOTS

$(x+3)(x-3)(x+1)(x-1)$

$x+3=0$ | $x-3=0$ | $x+1=0$ | $x-1=0$

$-3\ -3$ | $+3\ +3$ | $-1\ -1$ | $+1\ +1$

$x=-3$ | $x=3$ | $x=-1$ | $x=1$

18. $-36x=x^4-4x^3-9x^2$

$+36x\quad+36x$

$0=x^4-4x^3-9x^2+36x$ → GCF

$0=x(x^3-4x^2-9x+36)$ → Grouping

$\frac{x^2}{x^2}\quad\frac{x^2}{x^2}\quad\frac{-9}{-9}\quad\frac{36}{-9}$

$0=x(x^2(x-4)-9(x-4))$

$0=x(x^2-9)(x-4)$ → DOTS

$0=x(x+3)(x-3)(x-4)$

$x=0$ | $x+3=0$ | $x-3=0$ | $x-4=0$

$-3\ -3$ | $+3\ +3$ | $+4\ +4$

$x=-3$ | $x=3$ | $x=4$

In terms of x

When given something is less than, more than, twice, etc. of something else, call something x and express everything else in terms of x.

1. The tigers played 78 games. If they won 8 more games than they lost, how many games did they win?

W: X+8 35+8=43
L: X

X+X+8=78
2X+8=78
−8 −8
2X=70
2 2
X=35

2. Jackie and Lisa are both saving money. If Lisa has $12 less than twice as much saved as Jackie, and together they have saved $348, how much money has Lisa saved?

L: 2X−12 2(120)−12=228
J: X

2X−12+X=348
3X−12=348
+12 +12
3X=360
3 3
X=120

3. Danielle's age is 2 years less than Jessica's age. If the sum of their ages is 56, how old is Jessica?

D: X−2
J: X=29

X+X−2=56
2X−2=56
+2 +2
2X=58
2 2
X=29

4. Jamie is 5 years older than her sister Amy. If the sum of their ages is 19, how old is Jamie?

J: X+5 7+5=12
A: X

X+X+5=19
2X+5=19
−5 −5
2X=14
2 2
X=7

5. Jose has studied for three times as long as Miguel has for his SAT Exam. If together they have studied for 52 hours, how long as Miguel studied?

J: 3X
M: X=13

4X=52
4 4
X=13

6. Jack's age is 5 more than half Jill's age. If the sum of their ages is 29, determine the age of Jack and Jill.

$\frac{1}{2}(16)+5=13$

Jack: $\frac{1}{2}x+5$

Jill: x

16

$2\left(\frac{1}{2}x+5+x\right)=(29)2$

$x+10+2x=58$

$3x+10=58$

$-10\quad -10$

$\frac{3x}{3}=\frac{48}{3}$

$x=16$

7. The population of Town B is 10% higher than Town A. If there are a total of 105,000 people in both towns, what is the population of Town B?

A: x

B: $1.1x$

$1.1(50{,}000)$

55,000

$1x+1.1x=105{,}000$

$\frac{2.1x}{2.1}=\frac{105{,}000}{2.1}$

$x=50{,}000$

8. Ben has 20% more DVDs than Jake has. If they have a total of 66 DVDs, how many DVDs does Jake have?

$1+.2=1.2$

B: $1.2x$

J: $x=30$

$1.2x+1x=66$

$\frac{2.2x}{2.2}=\frac{66}{2.2}$

$x=30$

9. Three times as many robins as cardinals visited a bird feeder. If a total of 20 robins and cardinals visited the feeder, how many were robins?

R: $3x$

$3(5)=15$

C: x

$3x+x=20$

$\frac{4x}{4}=\frac{20}{4}$

$x=5$

10. At Genesee High School, the sophomore class has 60 more students than the freshman class. The junior class has 50 fewer students than twice the students in the freshman class. The senior class is three times as large as the freshman class. If there are a total of 1,424 students at Genesee High School, how many students are in the freshman class?

F: $x=202$

So: $x+60$

J: $2x-50$

Se: $3x$

$x+x+60+2x-50+3x=1424$

$7x+10=1424$

$-10\quad -10$

$\frac{7x}{7}=\frac{1414}{7}$

$x=202$

11. The length of a rectangle is 3 inches more than its width. The area of the rectangle is 40 square inches. What is the length, in inches, of the rectangle?

$l: x+3 \mid 5+3=8$

$w: x$

$A: 40$

$A = lw$

$40 = x(x+3)$

$40 = x^2+3x$

$-40 \quad -40$

$0 = x^2+3x-40$

$0 = (x+8)(x-5)$

$x+8=0 \quad x-5=0$

$-8 \quad -8 \quad +5 \quad +5$

$x=-8 \quad x=5$

12. A rectangle has an area of 24 square units. The width is 5 units less than the length. What is the length, in units, of the rectangle?

$l: x=8$

$w: x-5$

$A: 24$

$A = lw$

$24 = x(x-5)$

$24 = x^2-5x$

$-24 \quad -24$

$0 = x^2-5x-24$

$0 = (x-8)(x+3)$

$x-8=0 \quad x+3=0$

$+8 \quad +8 \quad -3 \quad -3$

$x=8 \quad x=-3$

13. A rectangular park is three blocks longer than it is wide. The area of the park is 40 square blocks. If w represents the width, write an equation in terms of w for the area of the park. Find the length and the width of the park.

$l: w+3 \quad 5+3=8$

$w: w=5$

$A: 40$

$A = lw$

$40 = w(w+3)$

$40 = w^2+3w$

$-40 \quad -40$

$0 = w^2+3w-40$

$(w+8)(w-5)$

$w+8=0 \quad w-5=0$

$-8 \quad -8 \quad +5 \quad +5$

$w=-8 \quad w=5$

14. Jack is building a rectangular dog pen that he wishes to enclose. The width of the pen is 25% more than the length. If the area of the dog pen is 45 square yards, what is the width of the dog pen?

$1.25 = 1.25$

$l: x$

$w: 1.25x$

$A: 45$

$1.25(6) \quad (7.5)$

$A = lw$

$45 = x(1.25x)$

$45 = 1.25x^2$

$\frac{45}{1.25} = \frac{1.25x^2}{1.25}$

$36 = x^2$

$\pm 6 = x$

$6 = x$

15. Tamara has two sisters. One of the sisters is 7 years older than Tamara. The other sister is 3 years younger than Tamara. The product of Tamara's sisters' ages is 24. How old is Tamara?

$T: x=5$

$S1: x+7$

$S2: x-3$

	x	$+7$
x	x^2	$+7x$
-3	$-3x$	-21

$x^2+4x-21$

$(x+7)(x-3) = 24$

$x^2+4x-21 = 24$

$-24 \quad -24$

$x^2+4x-45 = 0$

$(x+9)(x-5) = 0$

$x+9=0 \quad x-5=0$

$-9 \quad -9 \quad +5 \quad +5$

$x=-9 \quad x=5$

Modeling Linear Functions

Per/Eachx + one time fee

Per or each goes in front of x, the one time fee or one time starting amount goes at the end. x represents what the amount you're paying per is for. For example, if something costs $5 per hour, x is hours.

The slope is per/each.

The y-intercept is your one time fee or one time starting amount.

Same: Set the two equations equal to each other

1. The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

1) The commercial costs $0 to produce and $110 per airing up to $900.
2) The commercial costs $110 to produce and $900 each time it is aired.
3) The commercial costs $900 to produce and $110 each time it is aired.
4) The commercial costs $1010 to produce and can air an unlimited number of times.

2. A cell phone company charges $60.00 a month for up to 1 gigabyte of data. The cost of additional data is $0.05 per megabyte. If d represents the number of additional megabytes used and c represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?

1) $c = 60 - 0.05d$
2) $c = 60.05d$
3) $c = 60d - 0.05$
4) $c = 60 + 0.05d$

3. A plumber has a set fee for a house call and charges by the hour for repairs. The total cost of her services can be modeled by $c(t) = 125t + 95$. Which statements about this function are true?

I. A house call fee costs $95.
II. The plumber charges $125 per hour.
III. The number of hours the job takes is represented by t.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

4. A company that manufactures radios first pays a start-up cost, and then spends a certain amount of money to manufacture each radio. If the cost of manufacturing r radios is given by the function $c(r) = 5.25r + 125$, then the value 5.25 best represents

1) the start-up cost
2) the profit earned from the sale of one radio
3) the amount spent to manufacture each radio
4) the average number of radios manufactured

5. A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function?

1) y is the total cost, x is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.
2) y is the total cost, x is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
3) x is the total cost, y is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
4) x is the total cost, y is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

6. Each day, a local dog shelter spends an average of \$2.40 on food per dog. The manager estimates the shelter's daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$. Which statements regarding the function $E(x)$ are correct?

I. x represents the number of dogs at the shelter per day
II. x represents the number of volunteers at the shelter per day
III. 30 represents the shelter's total expenses per day.
IV. 30 represents the shelter's nonfood expenses per day.

1) I and III
2) I and IV
3) II and III
4) II and IV

7. Last weekend, Emma sold lemonade at a yard sale. The function $P(c) = .50c - 9.96$ represented the profit, $P(c)$, Emma earned selling c cups of lemonade. Sales were strong, so she raised the price for this weekend by 25 cents per cup. Which function represents her profit for this weekend?

1) $P(c) = .25c - 9.96$
2) $P(c) = .50c - 9.71$
3) $P(c) = .50c - 10.21$
4) $P(c) = .75c - 9.96$

$.50 + .25 = .75$

8. The amount Mike gets paid weekly can be represented by the expression $2.50a + 290$, where a is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?

1) $2.50a$, the amount he is guaranteed to be paid each week
2) $2.50a$, the amount he earns when he sells a accessories
3) 290, the amount he is guaranteed to be paid each week
4) 290, the amount he earns when he sells a accessories

9. A car leaves Albany, NY, and travels west toward Buffalo, NY. The equation $D = 280 - 59t$ can be used to represent the distance, D, from Buffalo after t hours. In this equation, the 59 represents the

1) car's distance from Albany
2) speed of the car
3) distance between Buffalo and Albany
4) number of hours driving

miles per hour

10. A high school club is researching a tour package offered by the Island Kayak Company. The company charges $35 per person and $245 for the tour guide. Which function represents the total cost, $C(x)$, of this kayak tour package for x club members?

1) $C(x) = 35x$
2) $C(x) = 35x + 245$
3) $C(x) = 35(x + 245)$
4) $C(x) = 35 + (x + 245)$

35x+245

11. A gardener is planting two types of trees:

Type A is 36 inches tall and grows at a rate of 15 inches per year.
Type B is 48 inches tall and grows at a rate of 10 inches per year.

Algebraically determine exactly how many years it will take for these trees to be the same height.

$A(x) = 15x + 36$
$B(x) = 10x + 48$

$15x + 36 = 10x + 48$
$-10x \quad -10x$
$5x + 36 = 48$
$-36 \quad -36$
$\frac{5x}{5} = \frac{12}{5}$
$x = 2.4$

equal

12. A local business was looking to hire a landscaper to work on their property. They narrowed their choices to two companies. Flourish Landscaping Company charges a flat rate of $120 per hour. Green Thumb Landscapers charges $70 per hour plus a $1600 equipment fee. Write a system of equations representing how much each company charges. Determine and state the number of hours that must be worked for the cost of each company to be the same. If it is estimated to take at least 35 hours to complete the job, which company will be less expensive? Justify your answer.

$F(x) = 120x$
$G(x) = 70x + 1600$

$120x = 70x + 1600$
$-70x \quad -70x$
$\frac{50x}{50} = \frac{1600}{50}$
$x = 32$

$F(35) = 120(35)$
$F(35) = 4200$
$G(35) = 70(35) + 1600$
$G(35) = 4050$

Green thumb will be less expensive.

13. Ian is borrowing $1000 from his parents to buy a notebook computer. He plans to pay them back at the rate of $60 per month. Ken is borrowing $600 from his parents to purchase a snowboard. He plans to pay his parents back at the rate of $20 per month. Write an equation that can be used to determine after how many months the boys will owe the same amount. ⇒ equal
Determine algebraically and state in how many months the two boys will owe the same amount.
State the amount they will owe at this time.

$I(x) = 1000 - 60x$

$K(x) = 600 - 20x$

$1000 - 60x = 600 - 20x$

$+60x \quad +60x$

$1000 = 600 + 40x$

$-600 \quad -600$

$\frac{400}{40} = \frac{40x}{40}$

$10 = x$

$1000 - 60(10) = 400$

They will owe $400.

14. Next weekend Marnie wants to attend either carnival *A* or carnival *B*. Carnival *A* charges $6 for admission and an additional $1.50 per ride. Carnival *B* charges $2.50 for admission and an additional $2 per ride.

a) In function notation, write $A(x)$ to represent the total cost of attending carnival *A* and going on x rides. In function notation, write $B(x)$ to represent the total cost of attending carnival *B* and going on x rides.

b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. equal

c) Marnie wants to go on five rides. Determine which carnival would have the lower total cost. Justify your answer.

$A(x) = 1.50x + 6$

$B(x) = 2x + 2.50$

$1.50x + 6 = 2x + 2.50$

$-1.50x \quad -1.50x$

$6 = .50x + 2.50$

$-2.50 \quad -2.50$

$\frac{3.50}{.5} = \frac{.5x}{.5}$

$7 = x$

$A(5) = 1.50(5) + 6 = 13.5$

$B(5) = 2(5) + 2.5 = 12.5$

Carnival B will have a lower cost.

15. Central High School had five members on their swim team in 2010. Over the next several years, the team increased by an average of 10 members per year. The same school had 35 members in their chorus in 2010. The chorus saw an increase of 5 members per year. In what year will the number of students on the swim team be the same as the number of students in chorus?

$s(x) = 10x + 5$

$c(x) = 5x + 35$

$10x + 5 = 5x + 35$

$-5x \quad -5x$

$5x + 5 = 35$

$-5 \quad -5$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

2010 + 6 = 2016

Modeling Exponential Functions

Exponential/ Interest/Depreciation Problems: $A = P(1 \pm r)^t$, where A is the current amount, P is the initial amount (y-intercept), r is the rate as a decimal (divide by 100), and t is time.
Given an exponential function: What is in front of the parenthesis is the INITIAL amount, what is inside the parenthesis is 1 + the rate or 1 – the rate.

Example: $A = 500(1.2)^t$: 500 is initial amount, rate is .2 or 20% growth (1 + .2)

$A = 500(0.8)^t$: 500 is initial amount, rate is .2 or 20% decay (1 - .2)

1. Anne invested $1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) $A = 1000(1 - 0.013)^2$
2) $A = 1000(1 + 0.013)^2$
3) $A = 1000(1 - 1.3)^2$
4) $A = 1000(1 + 1.3)^2$

A=A
P=1000
r=.013
t=2
$A=1000(1+.013)^2$

2. Dylan invested $600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

A=A
P=600
r=.016
t=2
$A=600(1+.016)^2$
$A=600(1.016)^2$
$A=619.35$

3. A certain car depreciates at a rate of 15% each year. If the car was initially worth $8125, what is the value of the car, rounded to the nearest cent, 11 years later?

A=A
P=8125
r=.15
t=11
$A=8125(1-.15)^{11}$
$A=8125(.85)^{11}$
$A=1359.66$

4. Cassandra bought an antique dresser for $500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?

A=A
P=500
r=.06
t=3
$A=500(1+.06)^3$
$A=500(1.06)^3$
$A=596$

5. Kathy plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. What is the value of the car after 3 years rounded to the nearest cent?

$A = A$
$P = 21{,}000$
$r = .14$
$t = 3$

$A = 21{,}000(1 - .14)^3$
$A = 21{,}000(.86)^3$
$A = 13{,}357.18$

6. Sheba opened a retirement account with $36,500. Her account grew at a rate of 7% per year compounded annually. She made no deposits or withdrawals on the account. At the end of 20 years, what was the account worth, to the *nearest dollar*?

$A = A$
$P = 36{,}500$
$r = .07$
$t = 20$

$A = 36{,}500(1 + .07)^{20}$
$A = 36{,}500(1.07)^{20}$
$A = 141243$

7. The value of a truck bought new for $28,000 decreases 9.5% each year. Write an exponential function to represent this function and predict the value of the truck to the nearest cent after 10 years.

$A = A$
$P = 28{,}000$
$r = .095$
$t = 10$

$A = 28{,}000(1 - .095)^{10}$
$A = 28{,}000(.905)^{10}$
$A = 10{,}319.15$

8. Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. What is the value of Kirsten's account after 5 years?

$A = A$
$P = 1000$
$r = .03$
$t = 5$

$A = 1000(1 + .03)^5$
$A = 1000(1.03)^5$
$A = 1159.27$

9. Marilyn collects old dolls. She purchases a doll for $450. Research shows this doll's value will increase by 2.5% each year. Write an equation that determines the value, V, of the doll t years after purchase. Assuming the doll's rate of appreciation remains the same, will the doll's value be doubled in 20 years? Justify your reasoning.

$A = V$
$P = 450$
$r = .025$
$t = t$

$V = 450(1 + .025)^t$
$V = 450(1.025)^t$

$V = 450(1.025)^{20}$
$V = 737.38$

No, $737.38 < 2(450)$

10. The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly rate of appreciation of the comic book is

1) 17%
2) 1.7%
3) 1.017%
4) 0.017%

$1.017 = 1 + .017$

$.017(100) = 1.7\%$

11. Milton has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where x is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?

1) It decreases 78% per year.
2) It decreases 22% per year.
3) It increases 78% per year.
4) It increases 22% per year.

$.78 = 1 - .22$

$.22(100) = 22\%$

12. The equation $A = 1300(1.02)^t$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?

1) 0.02% decay
2) 0.02% growth
3) 2% decay
4) 2% growth

$1.02 = 1 + .02$

$.02(100) = 2\%$

13. A car's depreciated value can be represented by the function $v(t) = 25500(.83)^t$. What was the initial value of the car and what is the depreciation rate?

initial value: 25,500

depreciation rate: 17%

$.83 = 1 - .17$

$.17(100) = 17\%$

14. The value, $v(t)$, of a car depreciates according to the function $v(t) = P(.85)^t$, where P is the purchase price of the car and t is the time, in years, since the car was purchased. State the percent that the value of the car *decreases* by each year. Justify your answer.

$.85 = 1 - .15$

$.15(100) = 15\%$

15. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?

1) 0.98, the percent of money in the account initially
2) 0.98, the percent of money in the account after x years
3) 5000, the amount of money in the account initially
4) 5000, the amount of money in the account after x years

16. The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where x represents the number of years since being discovered. What is the percent of change each year?

$.95 = 1 - .05$

$.05(100) = 5\%$

17. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Explain what 20 and 1.014 represent in the context of the problem.

20 represents the initial population of rabbits in the lab

$1.014 = 1 + .014$

$.014(100) = 1.4\%$

1.014 represents a 1.04% daily increase in the population of rabbits.

18. The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

300 represents the initial amount of the substance.

$.5 = 1 - .5$

$.5(100)$

50%

.5 represents a 50% decrease of mg of the substance each year.

Irregular Time (Half Life, Double Time, Or a given percent every x unit of time)

$A = P(1 \pm r)^{\frac{t}{h}}$ where h is the amount of time the rate is applied. For example, if the rate increases by 15% every 5 years, r = .15 and h = 5.

1. The population of a town increases by 10% every 3.2 years. If the current population is 27,000, what will be the population of the town 5 years from now? Round to the nearest person.

P r h t

A = A
P = 27,000
r = .1
t = 5
h = 3.2

$A = P(1+r)^{\frac{t}{h}}$

$A = 27{,}000(1+.1)^{\frac{5}{3.2}}$

A = 31,336

2. A stock has been increasing by 8% every 5 months. What will be the value of the stock after 12 months?

with $8,000 P r h t

A = A
P = 8000
r = .08
t = 12
h = 5

$A = P(1+r)^{\frac{t}{h}}$

$A = 8000(1+.08)^{\frac{12}{5}}$

A = $9622.92

3. Phil is trying to get himself back into shape and wants to ease his way back into distance running. He will start by running 2 miles each day but every 4 days, he will increase his distance by 60%. How many miles will Phil be running after 10 days rounded to the *nearest mile*?

P r h t

A = A
P = 2
r = .6
t = 10
h = 4

$A = P(1+r)^{\frac{t}{h}}$

$A = 2(1+.6)^{\frac{10}{4}}$

A = 6 miles

4. The number of subscribers to a YouTube channel decreases by 8% every 6 months. If the channel currently has 2340 subscribers, how many subscribers will there be 14 months from now?

P r h t

A = A
P = 2340
r = .08
t = 14
h = 6

$A = P(1+r)^{\frac{t}{h}}$

$A = 2340(1-.08)^{\frac{14}{6}}$

A = 1926..

5. Jabba went to the movies on Friday night and bought a large popcorn. Every 20 minutes, Jabba eats 40% of the remaining amount of popcorn in his bucket. If there were 967 pieces of popcorn initially in Jabba's bucket, how many pieces of popcorn, to the *nearest piece of popcorn*, will be left 90 minutes into the movie?

A = A
P = 967
r = .4
t = 90
h = 20

$A = P(1 \pm r)^{\frac{t}{h}}$

$A = 967(1 - .4)^{\frac{90}{20}}$

$A = 97$

6. The amount of views of a YouTube video increases by 70% every 5 days. If it currently has 1120 views, how many full views will the video have 12 days from now?

A = A
P = 1120
r = .7
t = 12
h = 5

$A = P(1 \pm r)^{\frac{t}{h}}$

$A = 1120(1 + .7)^{\frac{12}{5}}$

$A = 4002$

7. A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) $300(.30)^{\frac{14}{365}}$ 2) $300(1.30)^{\frac{14}{365}}$ 3) $300(.30)^{\frac{365}{14}}$ 4) $300(1.30)^{\frac{365}{14}}$

A = A
P = 300
r = .3
t = 365
h = 14

$A = P(1 \pm r)^{\frac{t}{h}}$

$A = 300(1 + .3)^{\frac{365}{14}}$

8. Jay borrowed \$50,000 from Aaron and they came to an agreement regarding how the interest will be paid. Every week, the loan will accumulate 2% interest. If Jay repays the loan after 21 days, how much money will he have to repay Aaron rounded to the *nearest cent*?

~~2 weeks~~ 7 days

A = A
P = 50,000
r = .02
t = 21
h = 7

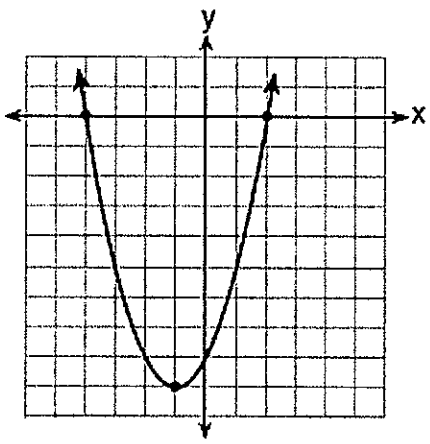
$A = P(1 \pm r)^{\frac{t}{h}}$

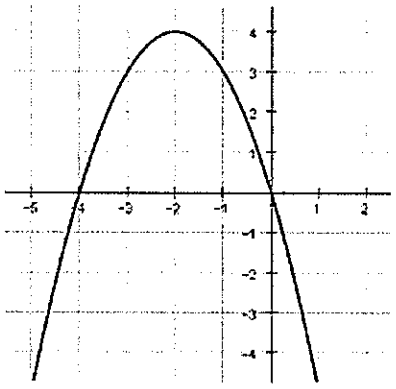
$A = 50{,}000(1 + .02)^{\frac{21}{7}}$

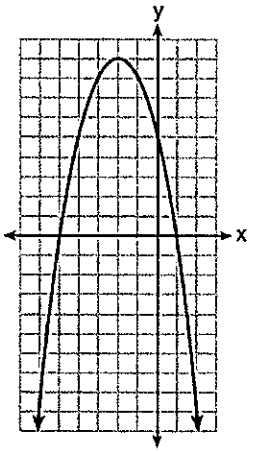
$A = \$53{,}060.40$

Finding the Vertex Of a Parabola Given the Zeros

The x coordinate of the vertex is equidistant between the zeros/roots/x-intercepts

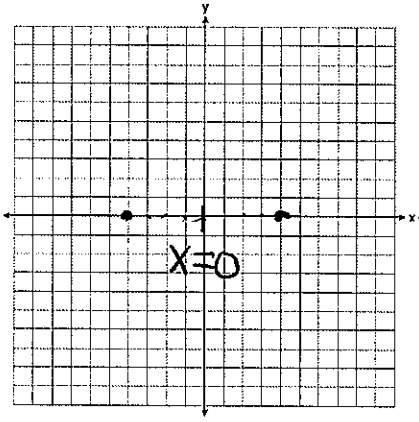




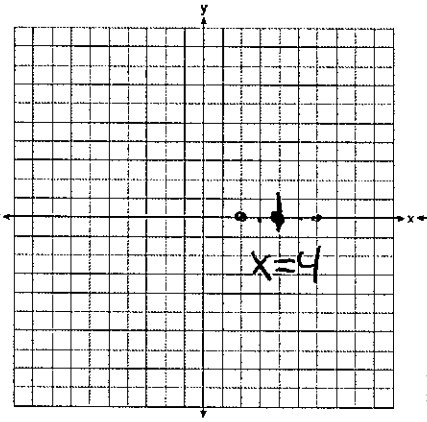


Find the x coordinate of the vertex if the roots are:

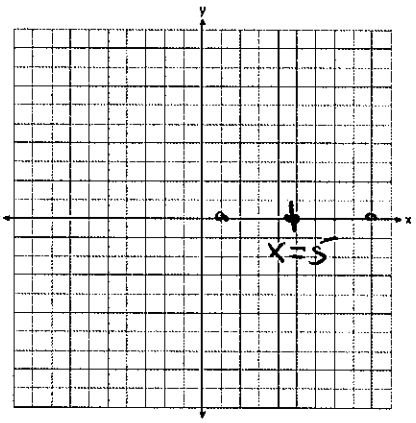
1. -4 and 4



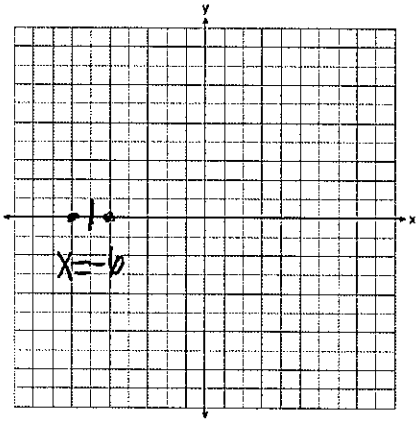
2. 2 and 6



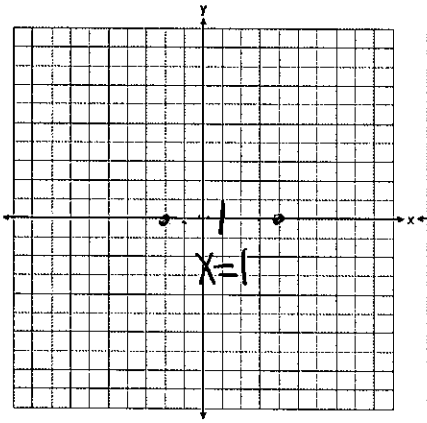
3. 1 and 9



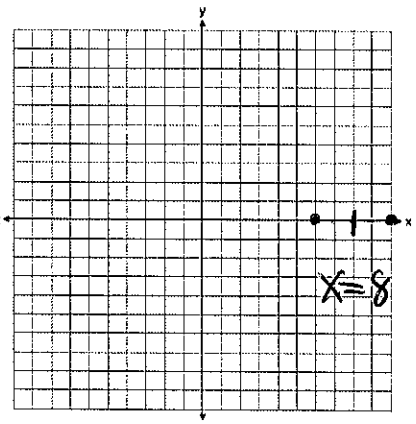
4. -7 and -5



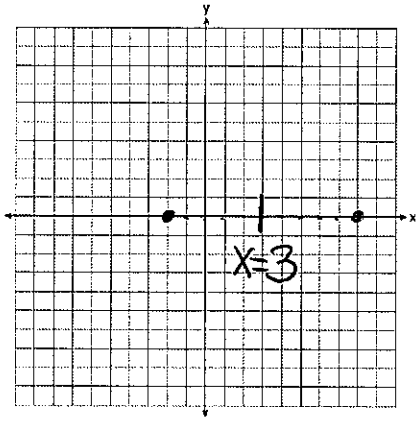
5. -2 and 4



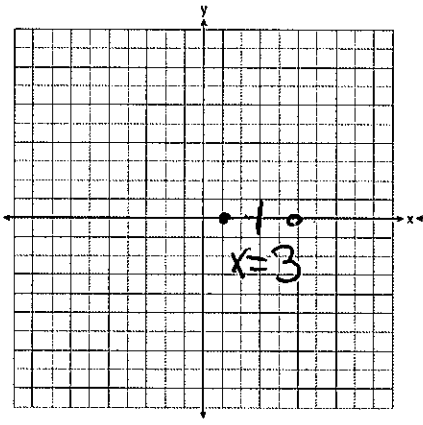
6. 6 and 10



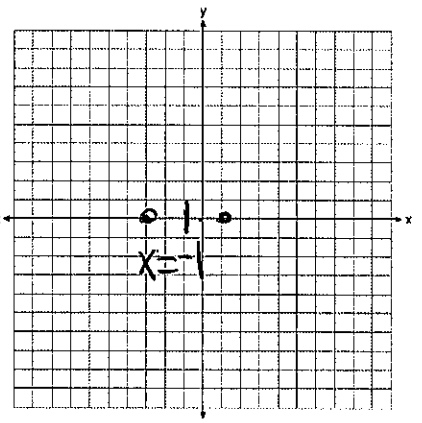
7. -2 and 8



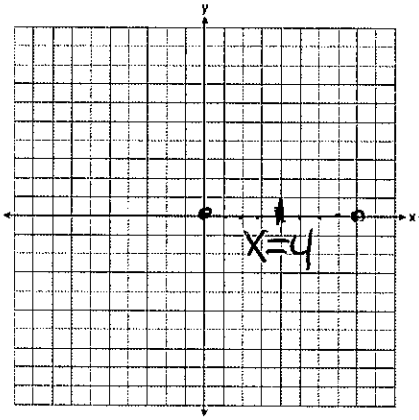
8. 1 and 5



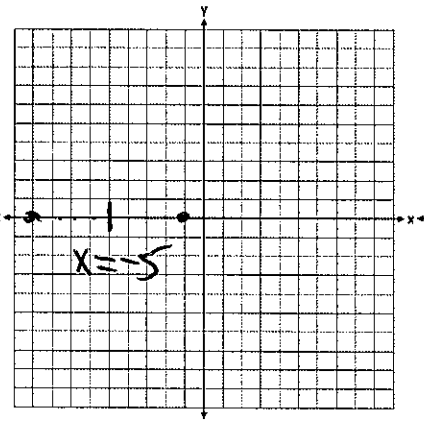
9. -3 and 1



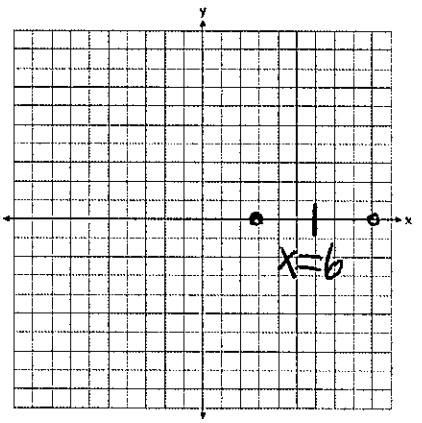
10. 0 and 8



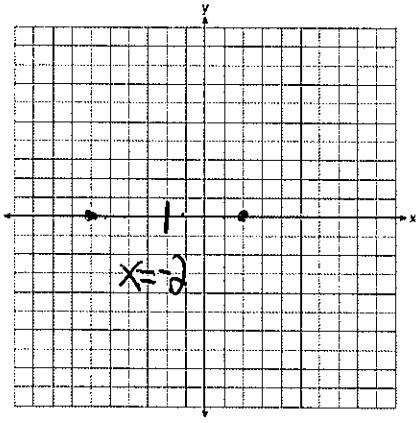
11. -9 and -1



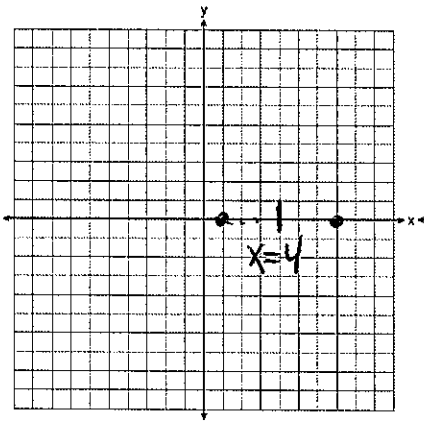
12. 3 and 9



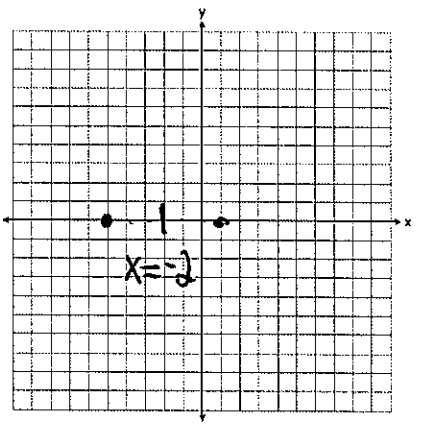
13. -6 and 2



14. 1 and 7



15. -5 and 1



Vertex Form (Graphing)

Vertex Form of a Parabola: $f(x)=a(x-v)^2+t$ where (v,t) is the vertex

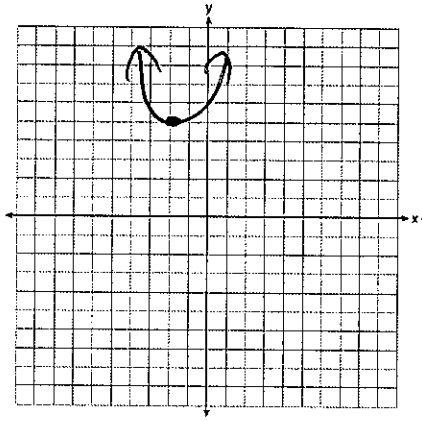
Pull the vertex from the equation by negating the x and not negating the y. Then determine if it opens up or down by checking the sign of the a value.

The a value affects how narrow/wide the parabola is and if it opens up or down.

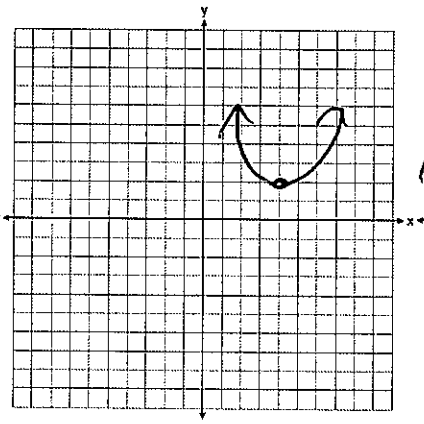
As $|a|$ increases, the parabola gets narrower. As $|a|$ decreases, the parabola gets wider

If a is positive, the parabola opens up If a is negative, the parabola opens down.

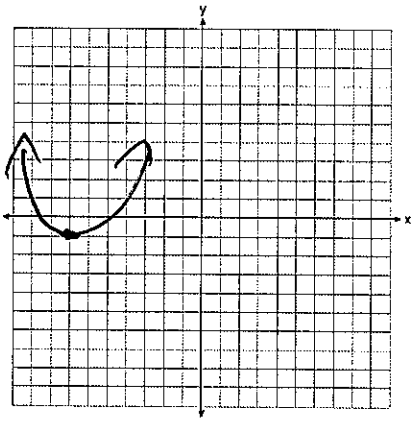
1. $y=(x+2)^2+5$ (-2, 5)



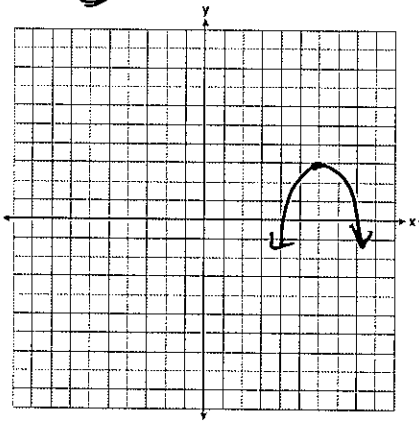
2. $y=(x-4)^2+2$ (4, 2)



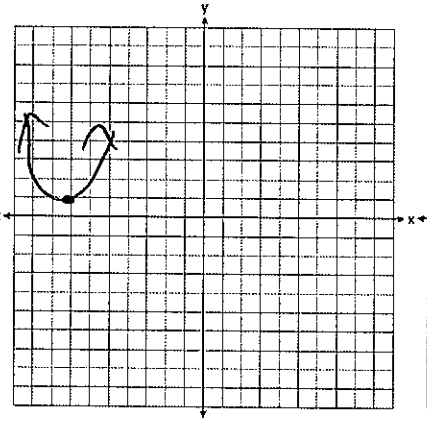
3. $y=(x+7)^2-1$ (-7, -1)



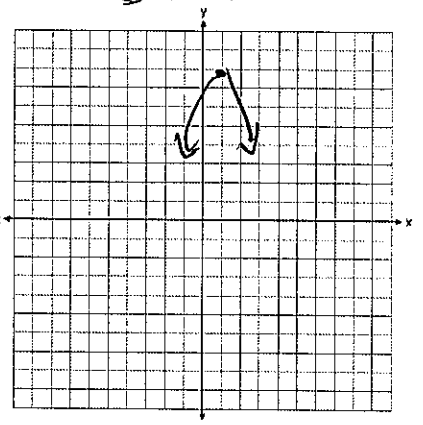
4. $y=-(x-6)^2+3$ (6, 3)



5. $y=2(x+7)^2+1$ (-7, 1)

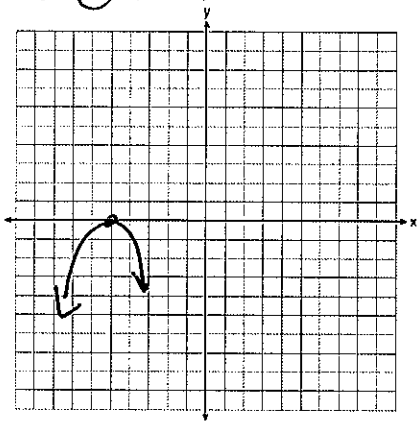


6. $y=-3(x-1)^2+8$ (1, 8)

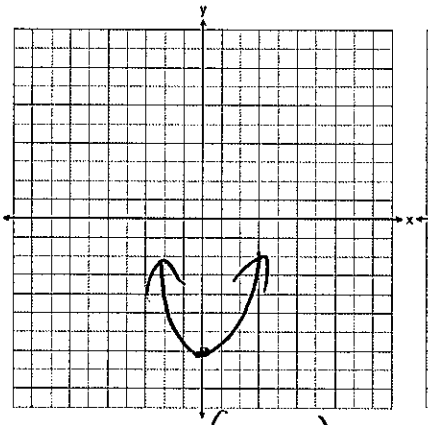


(-5,0) (0,-7) (-5,-5)

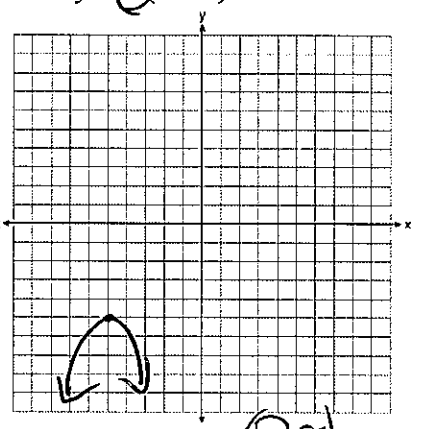
7. $y = -2(x+5)^2$



8. $y = x^2 - 7$

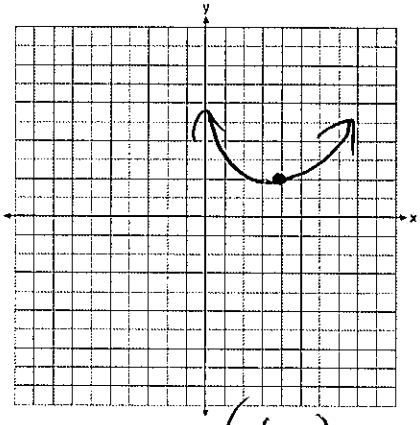


9. $y = -(x+5)^2 - 5$

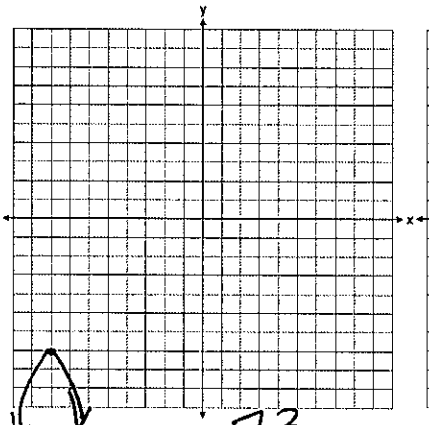


(-8,-7) (3,8)

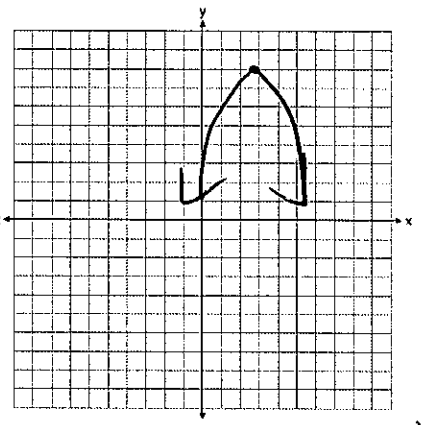
10. $y = \frac{1}{2}(x-4)^2 + 2$



11. $y = -4(x+8)^2 - 7$

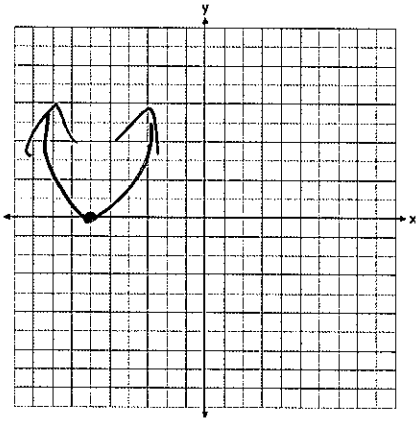


12. $y = -\frac{2}{3}(x-3)^2 + 8$

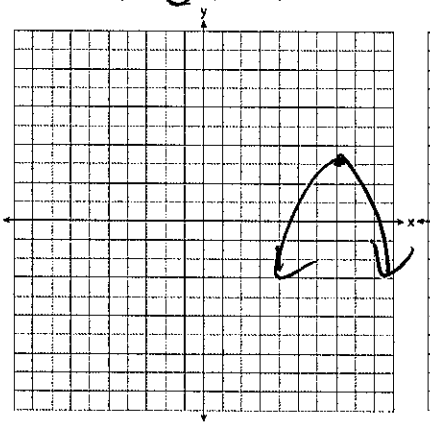


(-6,0) 7,3 (-1,-9)

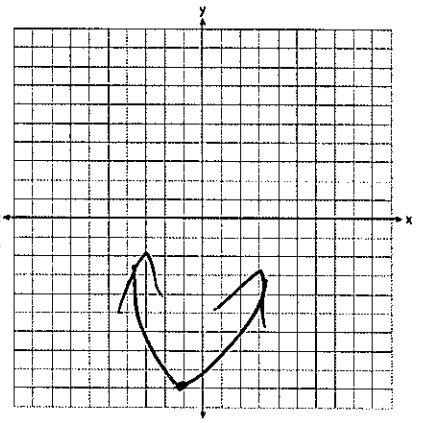
13. $y = 3(x+6)^2$



14. $y = -2(x-7)^2 + 3$



15. $y = (x+1)^2 - 9$



Vertex Form (Writing the Equation Given the Graph)

Vertex Form of a Parabola: $f(x)=a(x-v)^2+t$ where (v,t) is the vertex

To write the equation of a parabola in vertex form:

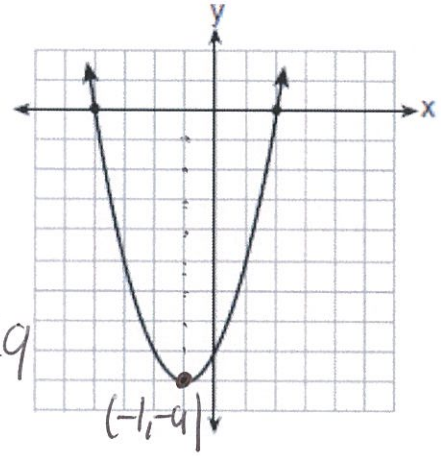
-Identify the vertex from the graph.

-Negate the x value and don't negate the y value when substituting it into $f(x)=a(x-v)^2+t$.

If the parabola opens up, a is positive. If the parabola opens down, a is negative. Do not worry about the integer value of the a value in this lesson.

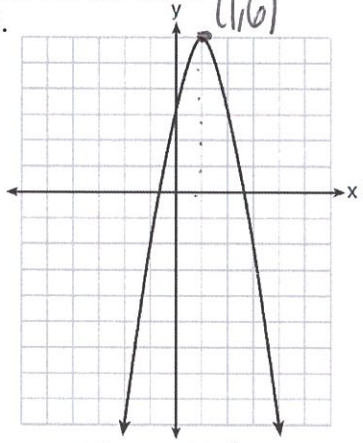
Write the equation of each parabola in vertex form

1.



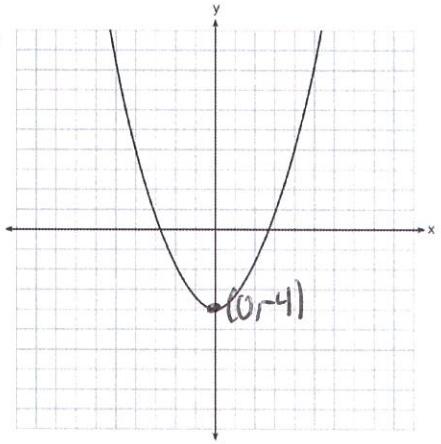
$y=(x+1)^2-4$

2.



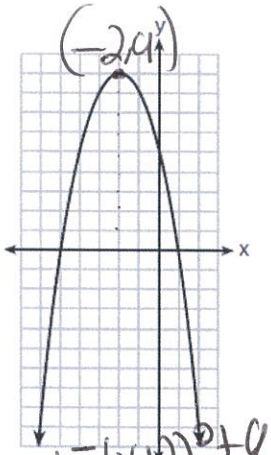
$y=(x-1)^2+6$

3.



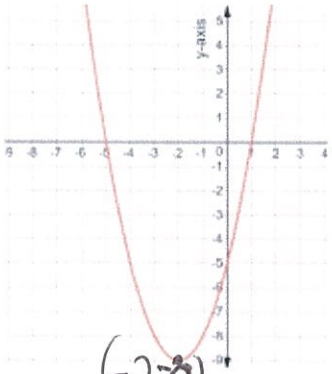
$y=x^2-4$

4.



$y=(x+2)^2+4$

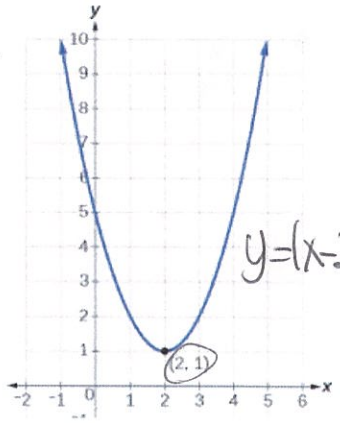
5.



$(-2,-4)$

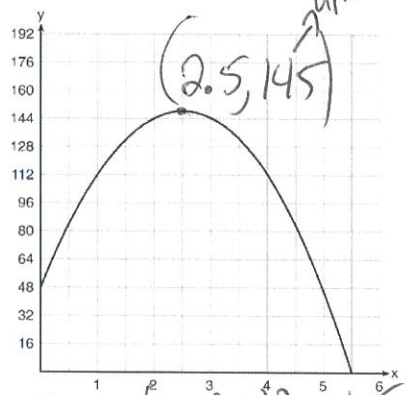
$y=(x+2)^2-4$

6.



$y=(x-2)^2+1$

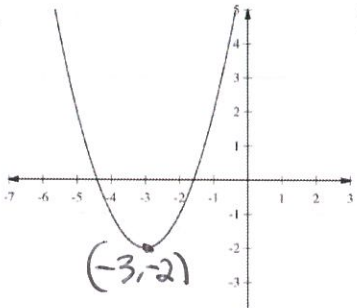
7.



approximate

$y=-(x-2.5)^2+145$

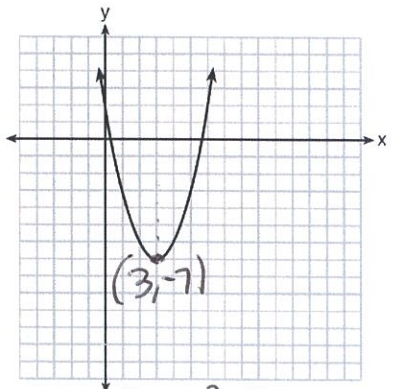
8.



$(-3,-2)$

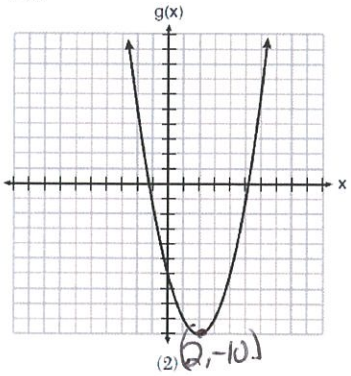
$y=(x+3)^2-2$

9.



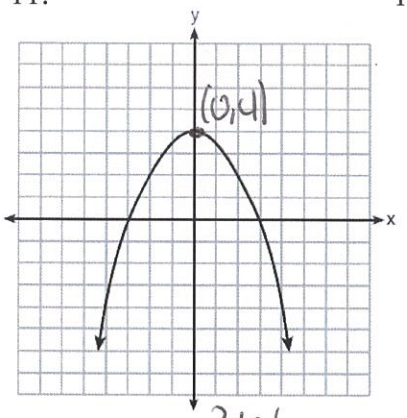
$y=(x-3)^2-7$

10.



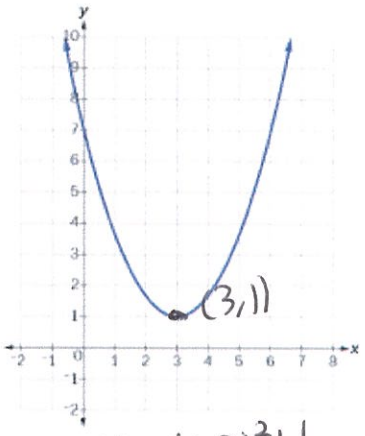
$y=(x-2)^2-10$

11.



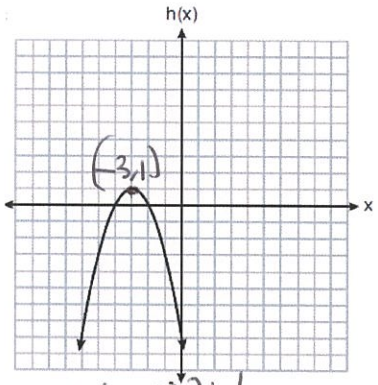
$y=-x^2+4$

12.



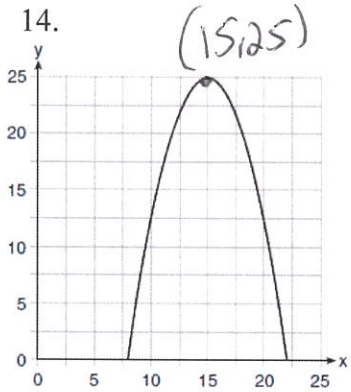
$y=(x-3)^2+1$

13.



$y=(x+3)^2+1$

14.

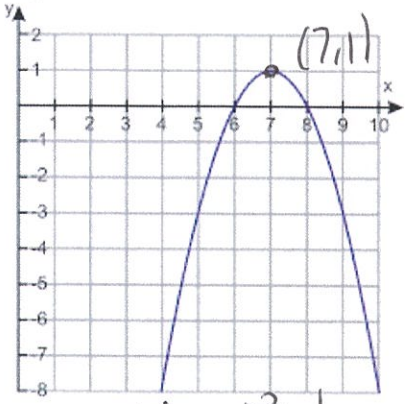


$y=-(x-15)^2+25$

15.

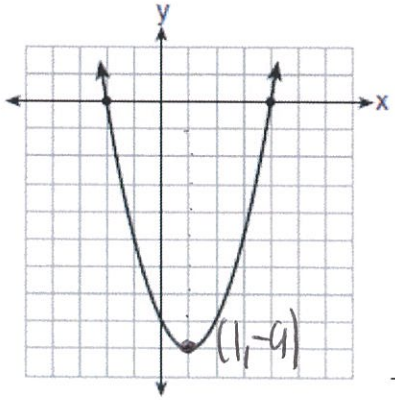
$y=-(x+6)^2+120$

16.



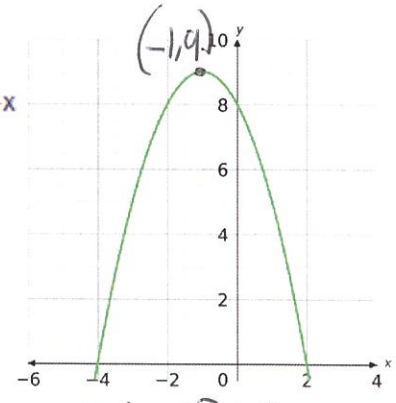
$y=-(x-7)^2+1$

17.



$y=(x-1)^2-9$

18.



$y=-(x+1)^2+9$

Factors and Zeros (Writing the Equation of a Parabola/Polynomial Equation in Factored Form)

The zeros are the x intercepts.

If a is a zero, $x-a$ is a factor.

To write the equation of a polynomial equation in factored form:

$f(x)=a(x-b)(x-c)$ where *b, c,* are zeros.

-Pull the zeros from the graph

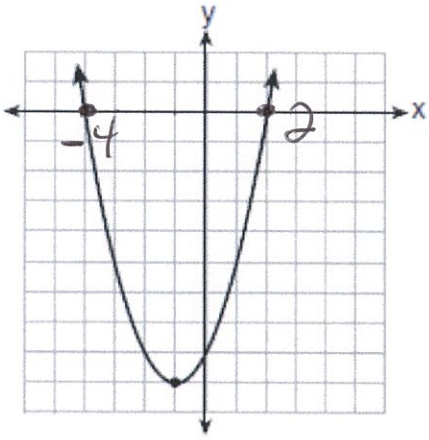
-List the corresponding factors

a is positive if the graph opens up to the right and negative if it opens down to the right. Do not worry about the numerical value of *a* in this lesson.

*Double roots bounce off the x axis, single roots pass through the x axis

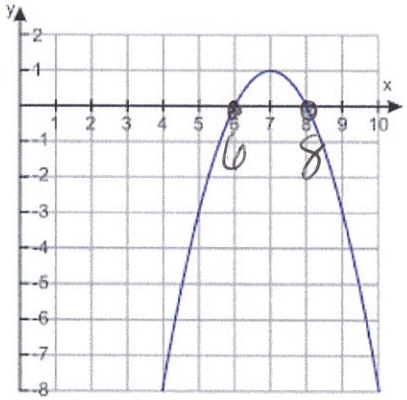
Write a possible polynomial equation in factored form

1.



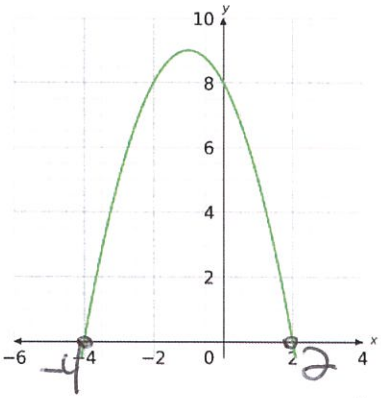
$y=(x+4)(x-2)$

2.



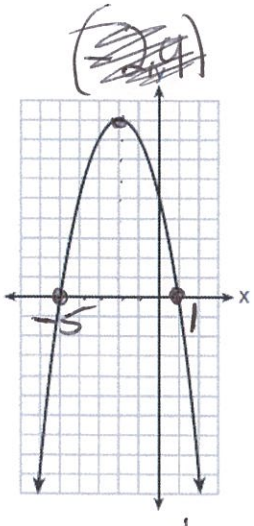
$y=-(x-6)(x-8)$

3.



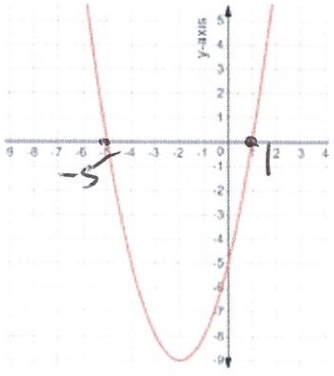
$y=-(x+4)(x-2)$

4.



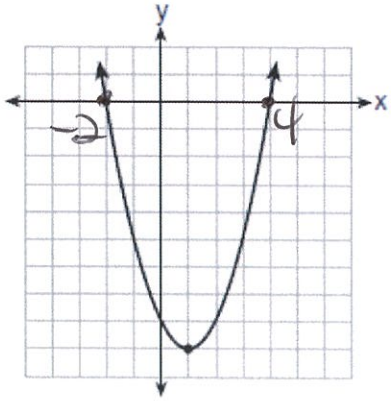
$y=-(x+5)(x-1)$

5.

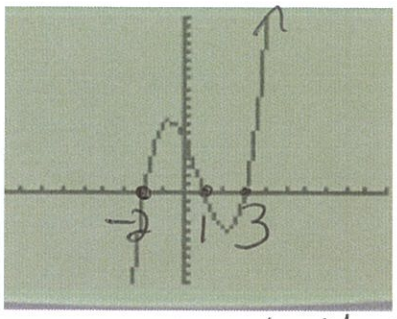


$y=(x+5)(x-1)$

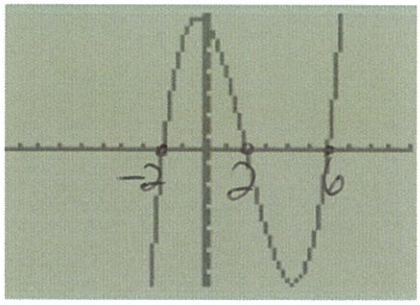
6.



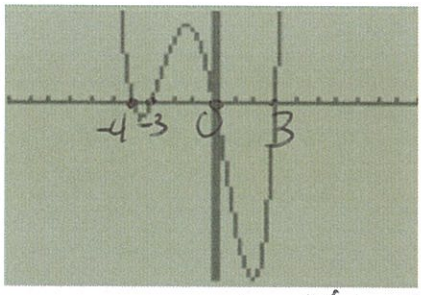
$y=(x+2)(x-4)$

7. 

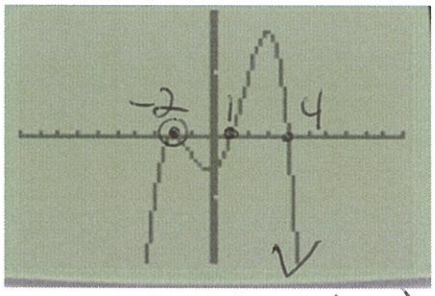
$y = (x+2)(x-1)(x-3)$

8. 

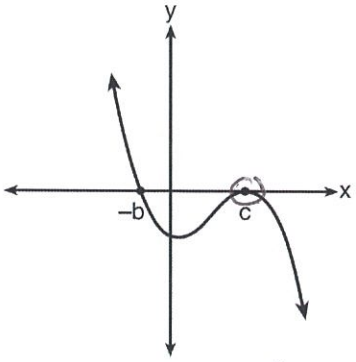
$y = (x+2)(x-2)(x-6)$

9. 

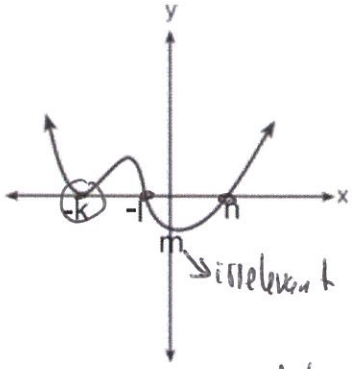
$y = x(x+4)(x+3)(x-3)$

10. 

$y = -(x+2)^2(x-1)(x-4)$

11. 

$y = -(x+b)(x-c)^2$

12. 

$y = (x+k)^2(x+l)(x-n)$

13. The graph below shows the polynomial $y = p(x)$.

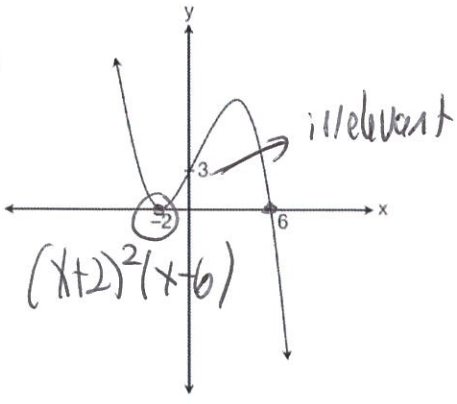
The factors of $p(x)$ are

(1) $(x+2)$, $(x-3)$, and $(x+6)$

(2) $(x-2)$, $(x+3)$, and $(x+6)$

(3) $(x-2)$, $(x-2)$, and $(x+6)$

(4) $(x+2)$, $(x+2)$, and $(x-6)$



$(x+2)^2(x+6)$

Writing Parabolas/Polynomial Equations with Constants

To write the equation of a parabola in vertex form:

-Identify the vertex from the graph.

-Negate the x value and don't negate the y value when substituting it into $f(x)=a(x-v)^2+t$.

To write the equation of a parabola/polynomial equation in factored form:

$f(x)=a(x-b)(x-c)$ where b, c, are zeros.

-Pull the zeros from the graph

-List the corresponding factors

*Double roots bounce off the x axis, single roots pass through the x axis

For either form, to find the *a* value:

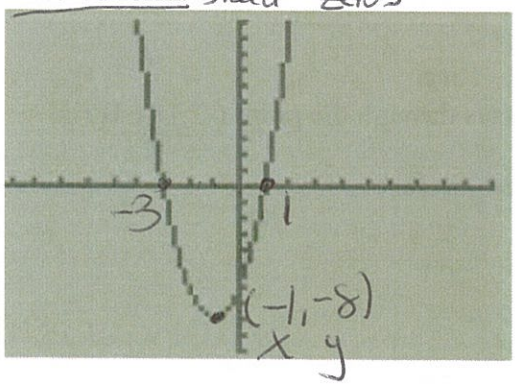
-Find a point on the graph that's not the zeros (if factored form) or vertex (if vertex form)

-Substitute the x coordinate for x and the y coordinate for y

-Solve for a

-Substitute a value back into your equation

1. The parabola shown below has $(-1,-8)$ as its vertex. Write the equation of the parabola in factored form. → need zeros



$y=a(x+3)(x-1)$

$-8=a(-1+3)(-1-1)$

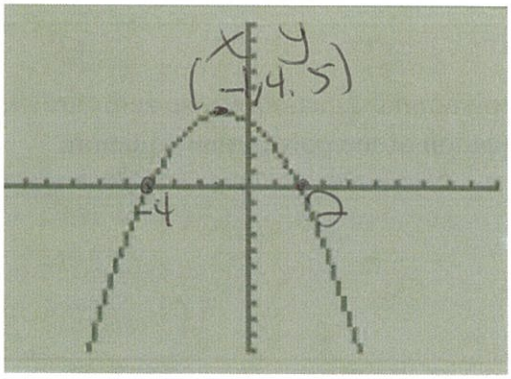
$-8=a(2)(-2)$

$\frac{-8}{-4}=\frac{-4a}{-4}$

$2=a$

$y=2(x+3)(x-1)$

2. The parabola shown below has $(-1,4.5)$ as its vertex. Write the equation of the parabola in factored form.



$y=a(x+4)(x-2)$

$4.5=a(-1+4)(-1-2)$

$4.5=a(3)(-3)$

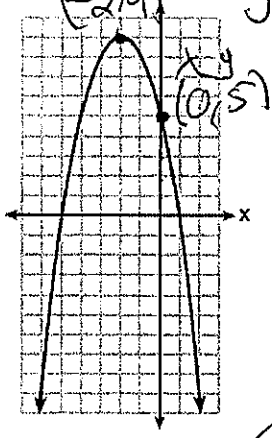
$\frac{4.5}{-9}=\frac{-9a}{-9}$

$-\frac{1}{2}=a$

$y=-\frac{1}{2}(x+4)(x-2)$

Write the equation of the parabolas below in vertex form.

3.



(-2,9)

$y = a(x+2)^2 + 9$

(0,5)

$5 = a(0+2)^2 + 9$

$5 = a(2)^2 + 9$

$5 = 4a + 9$

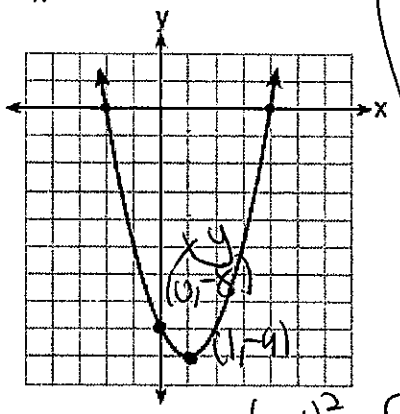
$-9 \quad -9$

$\frac{-4}{4} = \frac{4a}{4}$

$-1 = a$

$y = -1(x+2)^2 + 9$

4.



(0,-8)

(1,-9)

$-8 = a(0-1)^2 - 9$

$-8 = a(-1)^2 - 9$

$-8 = a - 9$

$+9 \quad +9$

$1 = a$

$y = a(x-1)^2 - 9$

$y = 1(x-1)^2 - 9$

5. A polynomial function whose zeros are 3, -1, and -2 passes through the point (4,60). Write the equation of the polynomial equation.

+y

$y = a(x-3)(x+1)(x+2)$

$60 = a(4-3)(4+1)(4+2)$

$60 = a(1)(5)(6)$

$\frac{60}{30} = \frac{30a}{30}$

$2 = a$

$y = 2(x-3)(x+1)(x+2)$

6. A polynomial function whose zeros are -4, 1, and -3 passes through the point (-2,18). Write the equation of the polynomial equation.

+y

$y = a(x+4)(x-1)(x+3)$

$18 = a(-2+4)(-2-1)(-2+3)$

$18 = a(2)(-3)(1)$

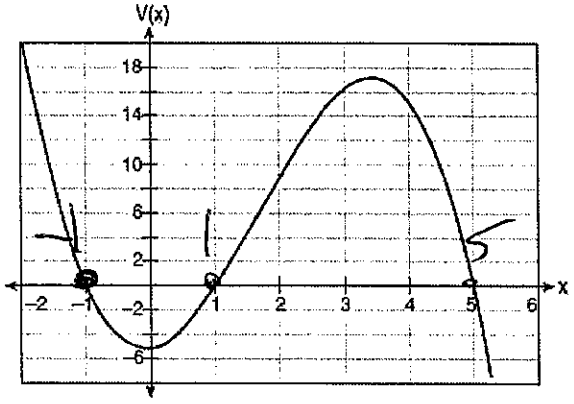
$\frac{18}{-6} = \frac{-6a}{-6}$

$-3 = a$

$y = -3(x+4)(x-1)(x+3)$

7. Write the equation of the polynomial graph below which passes through the point (0,-5).

+y



$y = a(x+1)(x-1)(x-5)$

$-5 = a(0+1)(0-1)(0-5)$

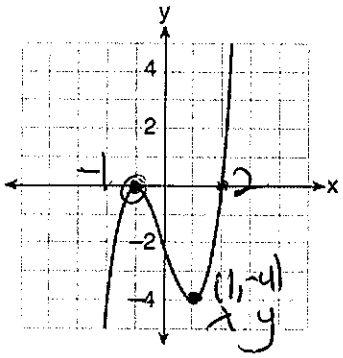
$-5 = a(1)(-1)(-5)$

$\frac{-5}{5} = \frac{5a}{5}$

$-1 = a$

$y = -1(x+1)(x-1)(x-5)$

8. Write the equation of the polynomial graph below.



$y = a(x+1)^2(x-2)$

$-4 = a(1+1)^2(1-2)$

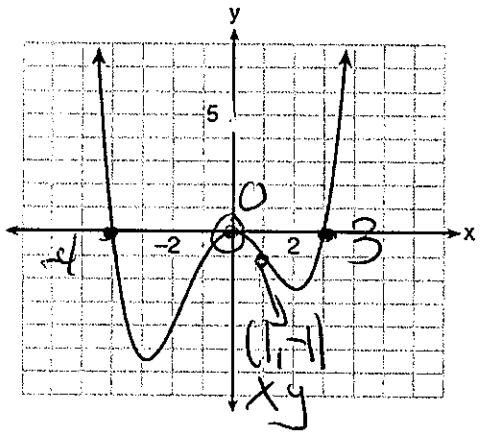
$-4 = a(2)^2(-1)$

$\frac{-4}{-4} = \frac{-4a}{-4}$

$1 = a$

$y = 1(x+1)^2(x-2)$

9. Write the equation of the polynomial graph below.



$y = ax^2(x+4)(x-3)$

$-1 = a(1)^2(1+4)(1-3)$

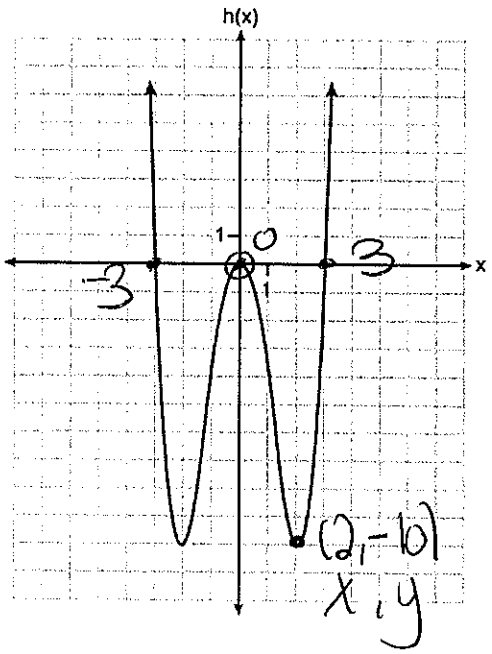
$-1 = a(1)(5)(-2)$

$\frac{-1}{-10} = \frac{-10a}{-10}$

$\frac{1}{10} = a$

$y = \frac{1}{10}x^2(x+4)(x-3)$

10. Write the equation of the polynomial graph below.



$y = ax^2(x+3)(x-3)$

$-10 = a(2)^2(2+3)(2-3)$

$-10 = a(4)(5)(-1)$

$\frac{-10}{-20} = \frac{-20a}{-20}$

$\frac{1}{2} = a$

$y = \frac{1}{2}x^2(x+3)(x-3)$

Factors and Zeros (Finding k)

The zeros are the x intercepts.

If $x-a$ is a factor, a is a zero and $p(a)=0$

To find k given a factor:

-Use the given factor to determine the corresponding zero/root.

-Replace x with *the* zero/root and y with 0.

-Solve for k

1. Consider the polynomial $p(x)=x^3+kx^2+x+6$. Find a value of k so that $x+1$ is a factor of P.

$0=(-1)^3+k(-1)^2+-1+6$

$0=-1+k-1+6$

$0=k+4$

$-4 \quad -4$

$-4=k$

-1 is a 0

$(-1,0)$

2. Consider the polynomial $p(x)=x^3+kx-30$. Find a value of k so that $x+3$ is a factor of P.

$0=(-3)^3+k(-3)-30$

$0=-27-3k-30$

$0=-3k-57$

$+57 \quad +57$

$\frac{57}{-3}=\frac{-3k}{-3}$

$-19=k$

-3 is a 0

$(-3,0)$

3. Given $p(x)=6x^3+31x^2+kx-12$, and $x+4$ is a factor. Find the value of k.

$0=6(-4)^3+31(-4)^2+k(-4)-12$

$0=-384+496-4k-12$

$0=-4k+100$

$+4k \quad +4k$

$\frac{4k}{4}=\frac{100}{4}$

$k=25$

-4 is a 0

$(-4,0)$

4. Given $z(x)=6x^3+bx^2-52x+15$, and $x+5$ is a factor. Find the value of b.

$0=6(-5)^3+b(-5)^2-52(-5)+15$

$0=-750+25b+260+15$

$0=25b-475$

$+475 \quad +475$

$\frac{475}{25}=\frac{25b}{25}$

$19=b$

-5 is a 0

$(-5,0)$

5. Given $p(x)=x^3+5x^2+kx-24$, and $x+3$ is a factor, algebraically determine the value of k.

$0=(-3)^3+5(-3)^2+k(-3)-24$ -3 is a 0

$0=-27+45-3k-24$ $(-3,0)$

$0=-3k-6$

$+6 \quad +6$

$6=-3k$

$\frac{6}{-3}=\frac{-3k}{-3}$

$-2=k$

6. Given $p(x)=x^3+kx^2-26x+5$, and $x-5$ is a factor, algebraically determine the value of k.

$0=(5)^3+k(5)^2-26(5)+5$ 5 is a 0

$0=125+25k-130+5$ $(5,0)$

$\frac{0}{25}=\frac{25k}{25}$

$0=k$

7. Given $p(x)=x^3+5x^2+kx-10$, and $x+2$ is a factor, algebraically determine the value of k.

$0=(-2)^3+5(-2)^2+k(-2)-10$ -2 is a 0

$0=-8+20-2k-10$ $(-2,0)$

$0=-2k+2$

$-2 \quad -2$

$-2=-2k$

$\frac{-2}{-2}=\frac{-2k}{-2}$

$1=k$

8. Given $p(x)=2x^3-x^2+kx+8$, and $x-2$ is a factor, algebraically determine the value of k.

$0=2(2)^3-(2)^2+k(2)+8$ 2 is a 0

$0=16-4+2k+8$ $(2,0)$

$0=2k+20$

$-20 \quad -20$

$-20=2k$

$\frac{-20}{2}=\frac{2k}{2}$

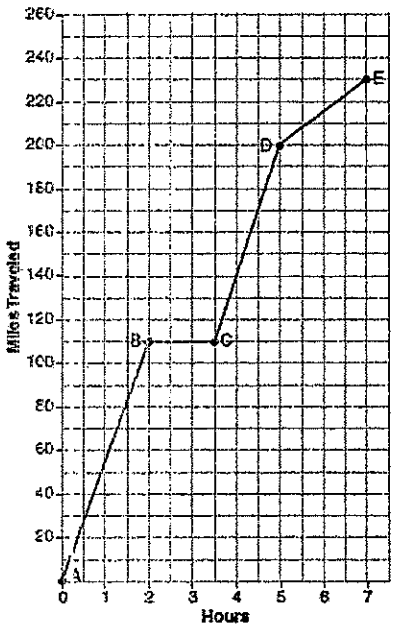
$-10=k$

Irregular Graphs

If the y axis is distance, 0 slope means there is no movement. The greater the slope, the faster the movement.

If the y axis is rate, 0 slope means the rate is staying the same. The greater the slope, the greater the rate is increasing.

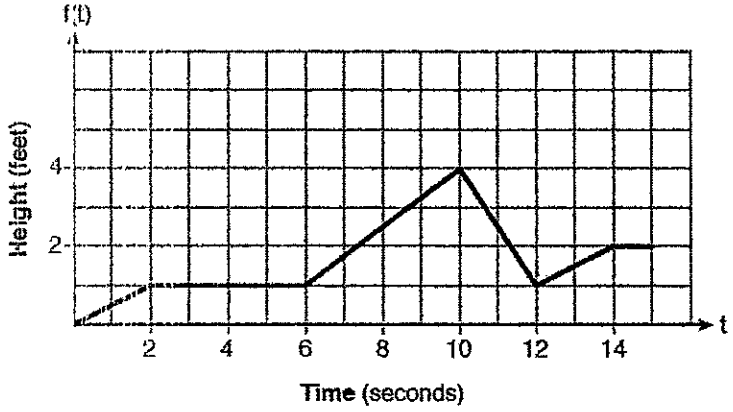
1. The graph below models Craig's trip to visit his friend in another state. In the course of his travels, he encountered both highway and city driving. Based on the graph, during which interval did Craig most likely drive in the city? Explain your reasoning. Explain what might have happened in the interval between *B* and *C*.



City: Between D and E because it was a slower rate.

Between B and C: He stopped for lunch because time passed but mileage did not increase.

2. The graph of $f(t)$ models the height, in feet, that a bee is flying above the ground with respect to the time it traveled in t seconds. State all time intervals when the bee's rate of change is zero feet per second. Explain your reasoning.

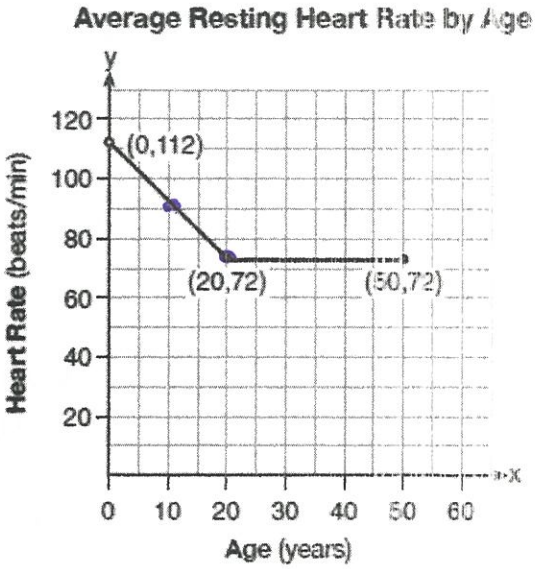


(2,4) and (14,15) because time passed but height did not increase/decrease.

3. A graph of average resting heart rates is shown below. The average resting heart rate for adults is 72 beats per minute, but doctors consider resting rates from 60-100 beats per minute within normal range.

Which statement about average resting heart rates is *not* supported by the graph?

1) A 10-year-old has the same average resting heart rate as a 20-year-old.
2) A 20-year-old has the same average resting heart rate as a 30-year-old.
3) A 40-year-old may have the same average resting heart rate for ten years.
4) The average resting heart rate for teenagers steadily decreases.

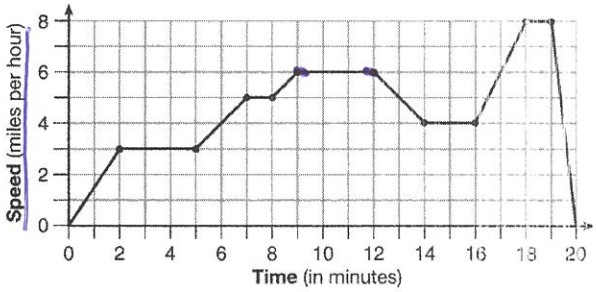


4. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

Which statement best describes what the jogger was doing during the 9 – 12 minute interval of her jog?

1) She was standing still.
2) She was increasing her speed.
3) She was decreasing her speed
4) She was jogging at a constant rate.

speed was constant

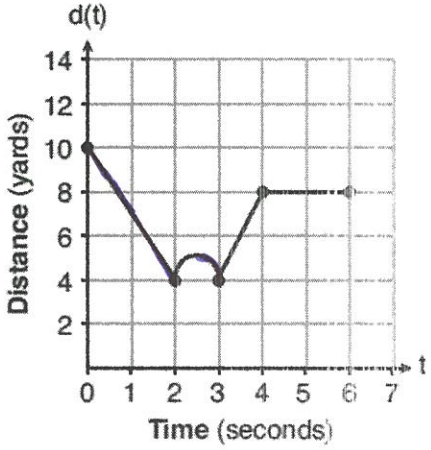


5. A child is playing outside. The graph below shows the child's distance, $d(t)$, in yards from home over a period of time, t, in seconds.

Which interval represents the child constantly moving closer to home?

1) $0 \le t \le 2$
2) $2 \le t \le 3$
3) $3 \le t \le 4$
4) $4 \le t \le 6$

(0,2)
(2.5,3)



Average rate of change: $\frac{f(b)-f(a)}{b-a}$

Use a table to organize your values. If given an equation, type it into $y=$. If given a graph, pull the values from the graph. $f(b)$ and $f(a)$ are y values. b and a are x values.
If asked which interval has the greatest rate of change, find the average rate of change for each interval.
"On average, between a and b, the function is increasing/decreasing x units per unit of time."

1. Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

1) 4.3
2) 4.5
3) 5.4
4) 6.0

$\frac{f(b)-f(a)}{b-a}$ $\frac{36.6-15}{4-0}=5.4$

2. A family is traveling from their home to a vacation resort hotel. The table below shows their distance from home as a function of time.
Determine the average rate of change between hour 2 and hour 7, including units.

Time (hrs)	0	2	5	7
Distance (mi)	0	140	375	480

$\frac{f(b)-f(a)}{b-a}$ $\frac{480-140}{7-2}=68$ miles per hour.

3. The table below shows the average diameter of a pupil in a person's eye as he or she grows older.
What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?

1) 2.4
2) 0.04
3) −2.4
4) −0.04

$\frac{f(b)-f(a)}{b-a}$ $\frac{2.3-4.7}{80-20}=-.04$

Age (years)	Average Pupil Diameter (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

4. An astronaut drops a rock off the edge of a cliff on the Moon. The distance, $d(t)$, in meters, the rock travels after t seconds can be modeled by the function $d(t) = 0.8t^2$. What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped?

1) 12
2) 20
3) 60
4) 80

x	y
5	20
10	80

$\frac{f(b)-f(a)}{b-a}$ $\frac{80-20}{10-5} = 12$

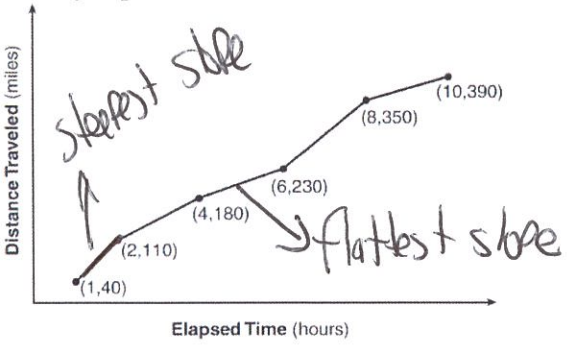
5. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Determine, to the *nearest tenth*, the average rate of change from day 50 to day 100.

x	y
50	40.08
100	80.32

$\frac{f(b)-f(a)}{b-a}$

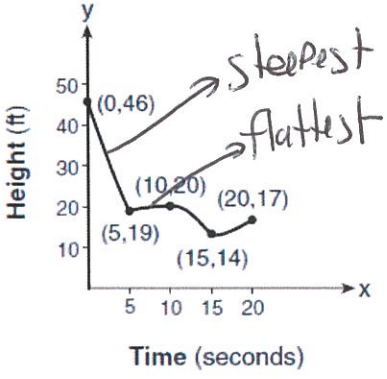
$\frac{80.32-40.08}{100-50} = .8$

6. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.
During which interval was their average speed the greatest? During which interval was their average speed the least?



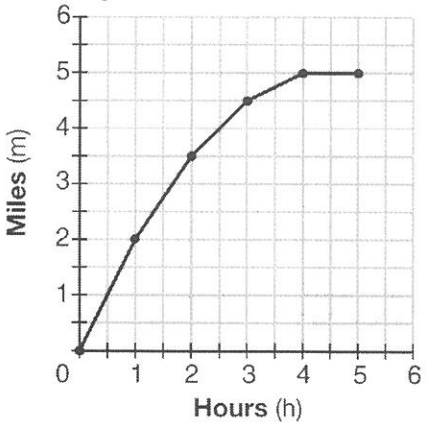
greatest: (1,2)
least: (4,6)

7. The graph below models the height of a remote-control helicopter over 20 seconds during flight.
Over which interval does the helicopter have the *slowest* average rate of change? Over which interval does the helicopter have the *fastest* average rate of change?



slowest: (5,10)
fastest: (0,5)

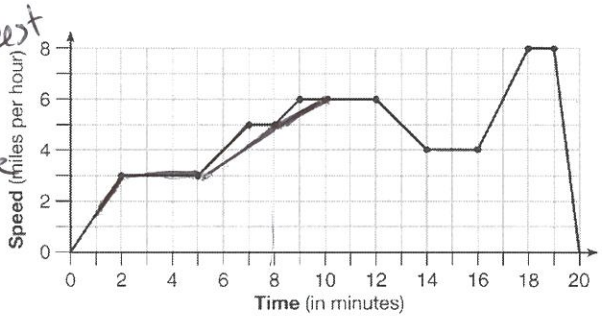
8. The graph below shows the distance in miles, m, hiked from a camp in h hours.
Which hourly interval had the greatest rate of change? Which hourly interval had the slowest rate of change?



(0,1) had the greatest rate of change because it has the steepest slope
(4,5) had the slowest rate of change because it has the flattest slope.

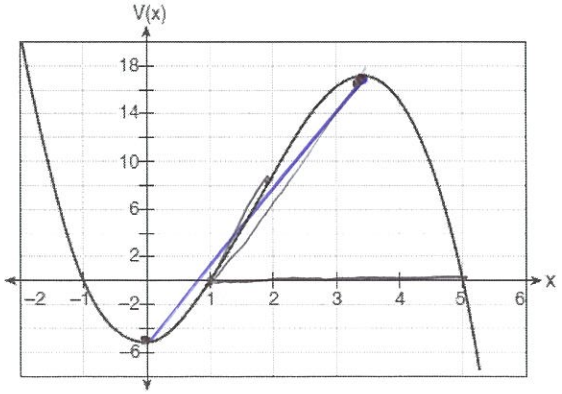
9. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.
During which interval was her average rate of change the fastest? During which interval was her average rate of change the slowest?

1) the first hour to the second hour — fastest, steepest slope
2) the second hour to the fourth hour — slowest, flattest slope
3) the sixth hour to the eighth hour
4) the eighth hour to the tenth hour



10. A cardboard box manufacturing company is building boxes with length represented by $x+1$, width by $5-x$, and height by $x-1$. The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?
1) [1, 2]
2) [1,3.5]
3) [1,5]
4) [0, 3.5]



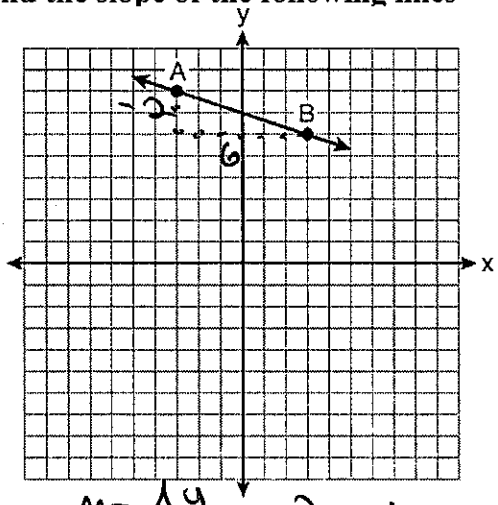
Slope of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

If given a graph, count the change in y on top and the change in x on the bottom.
If given points, subtract the "y"s on top subtract the "x"s on bottom.

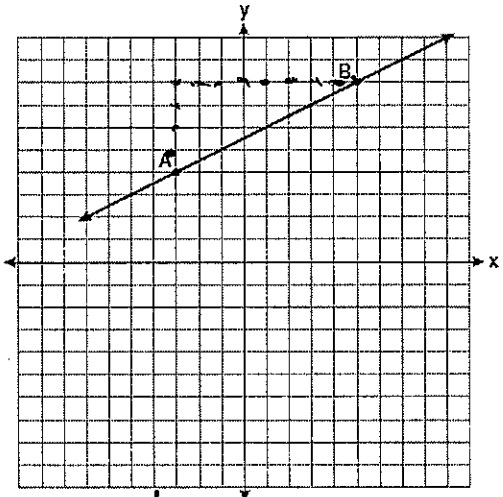
Find the slope of the following lines

1.



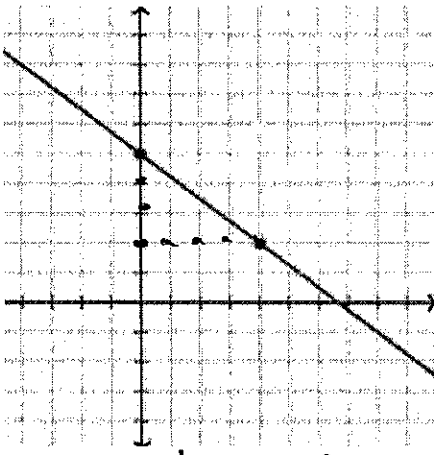
$m = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = \frac{-1}{3}$

2.



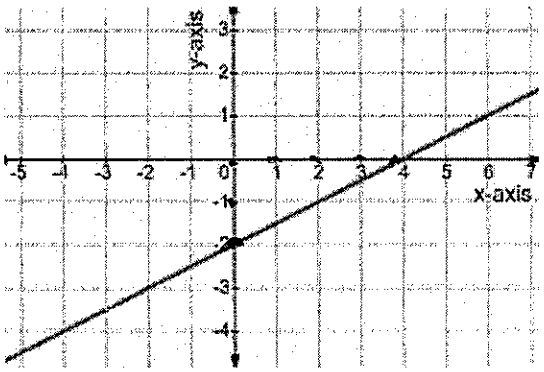
$m = \frac{\Delta y}{\Delta x} = \frac{4}{8} = \frac{1}{2}$

3.



$m = \frac{\Delta y}{\Delta x} = \frac{-3}{4}$

4.



$m = \frac{\Delta y}{\Delta x} = \frac{2}{4} = \frac{1}{2}$

$x_1\ y_1 \quad x_2\ y_2$

5. (3,10) and (-1,8)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-10}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

$x_1\ y_1 \quad x_2\ y_2$

6. (3,1) and (9,-1)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{9-3} = \frac{-2}{6} = \frac{-1}{3}$

$x_1\ y_1 \quad x_2\ y_2$

7. (-2,1) and (-4, -1)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-1}{-4+2} = \frac{-2}{-2} = 1$$

$x_1\ y_1 \quad x_2\ y_2$

8. (-3,1) and (5,2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-1}{5+3}$$

$$m = \frac{1}{8}$$

$x_1\ y_1 \quad x_2\ y_2$

9. (8,2) and (6,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-2}{6-8} \qquad m = \frac{2}{-2} \qquad m = -1$$

$x_1\ y_1 \quad x_2\ y_2$

10. (-6,1) and (-3,2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-1}{-3+6} \qquad m = \frac{1}{3}$$

$x_1\ y_1 \quad x_2\ y_2$

11. (a,5) and (b, 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-5}{b-a} \qquad m = \frac{-3}{b-a}$$

$x_1\ y_1 \quad x_2\ y_2$

12. (4,a) and (6, b)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{b-a}{6-4} \qquad m = \frac{b-a}{2}$$

$x_1\ y_1 \quad x_2\ y_2$

13. (1,2y) and (3, k)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{k-2y}{3-1} \qquad m = \frac{k-2y}{2}$$

$x_1\ y_1 \quad x_2\ y_2$

14. (2x,8) and (-1, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-8}{-1-2x} \qquad m = \frac{-4}{-1-2x}$$

$x_1\ y_1 \quad x_2\ y_2$

15. (0,3k) and (a, y)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y-3k}{a-0} \qquad m = \frac{y-3k}{a}$$

$x_1\ y_1 \quad x_2\ y_2$

16. (b,y) and (2a, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

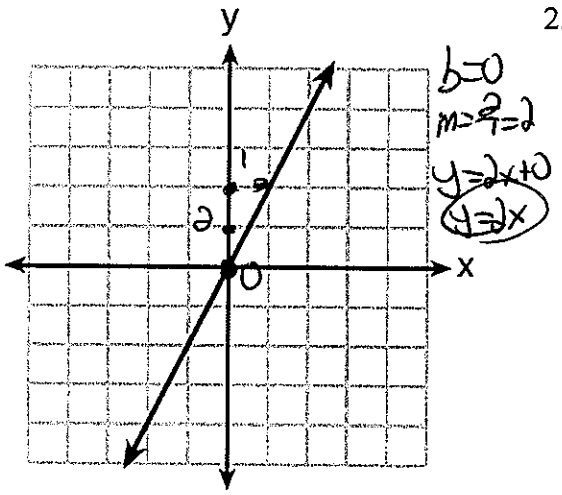
$$m = \frac{0-y}{2a-b} \qquad m = \frac{-y}{2a-b}$$

Equation of a line given a graph

Slope-Intercept Form: $y = mx + b$ where $m = slope$ and $b =$ y-intercept.

Write the equation of the following lines:

1.



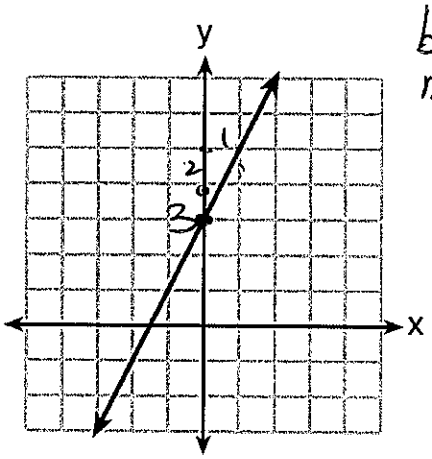
$b=0$

$m=\frac{2}{1}=2$

$y=2x+0$

$y=2x$

2.

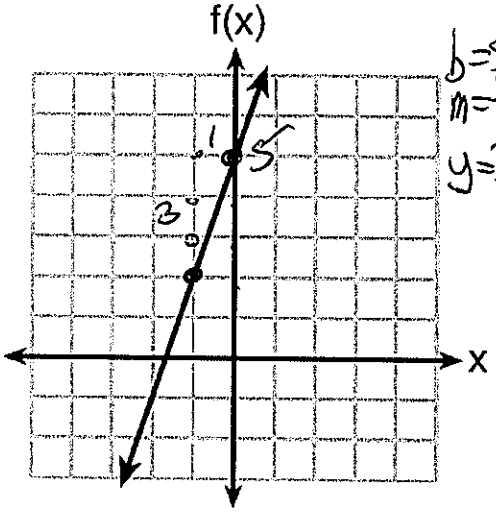


$b=3$

$m=\frac{2}{1}=2$

$y=2x+3$

3.

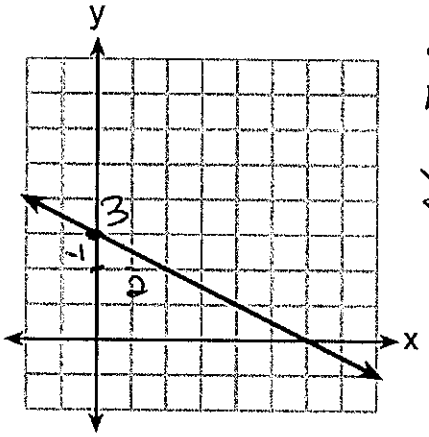


$b=5$

$m=\frac{3}{1}=3$

$y=3x+5$

4.

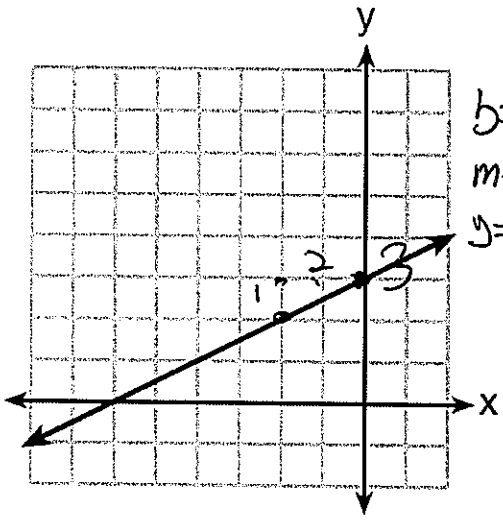


$b=3$

$m=\frac{-1}{2}$

$y=-\frac{1}{2}x+3$

5.

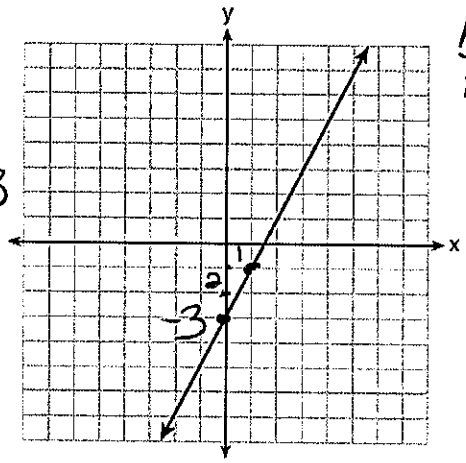


$b=3$

$m=\frac{1}{2}$

$y=\frac{1}{2}x+3$

6.

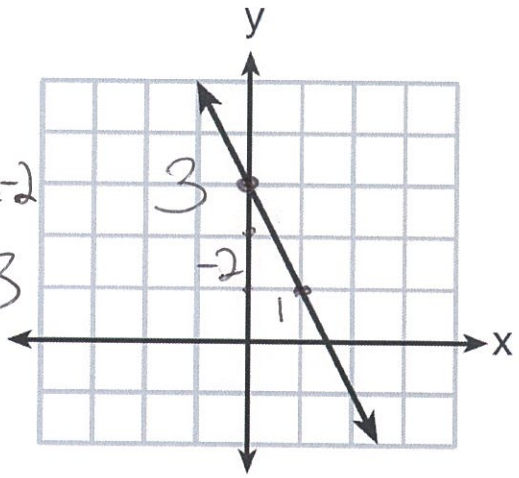


$b=-3$

$m=\frac{2}{1}=2$

$y=2x-3$

7.

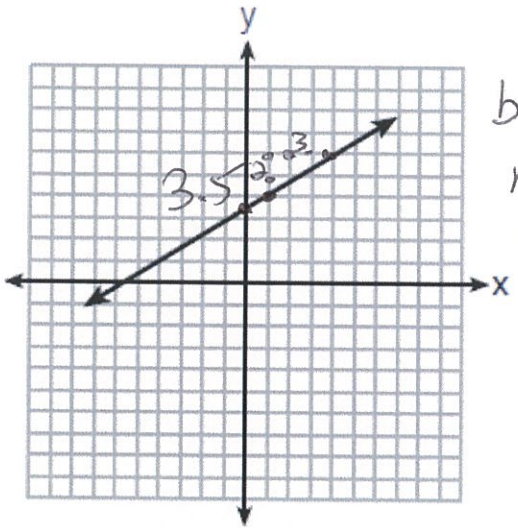


$b=3$

$m=\frac{-2}{1}=-2$

$y=-2x+3$

8.



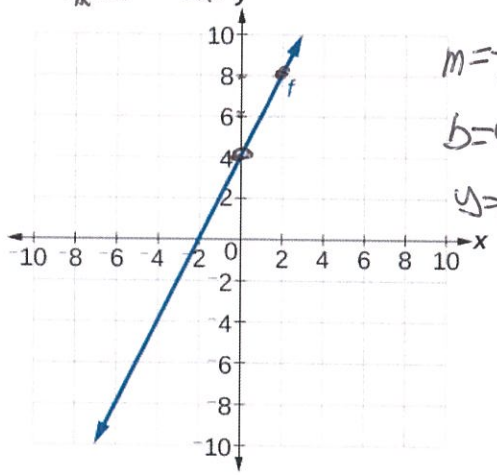
$b=3.5$

$m=\frac{2}{3}$

$y=\frac{2}{3}x+3.5$

9.

*each box is worth 2

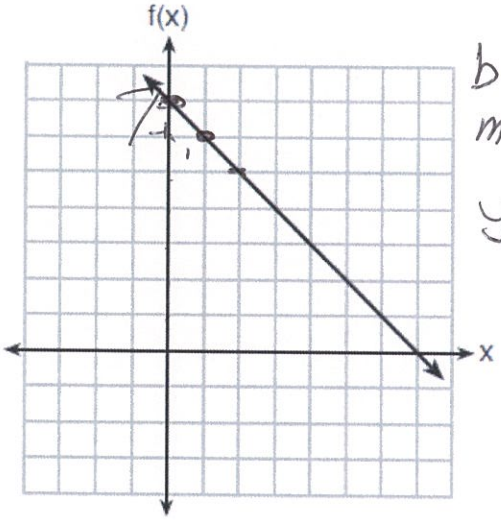


$m=\frac{4}{2}=2$

$b=4$

$y=2x+4$

10.

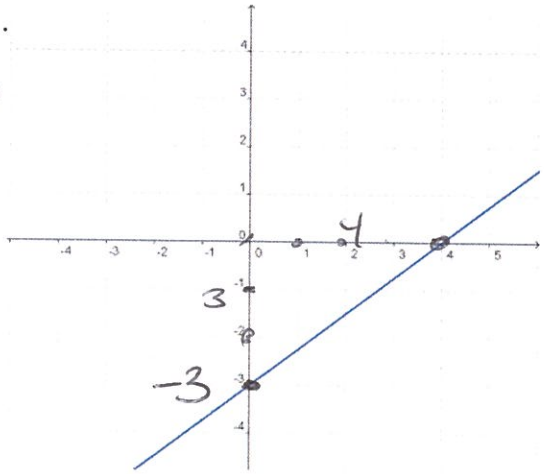


$b=7$

$m=\frac{-1}{1}=-1$

$y=-x+7$

11.

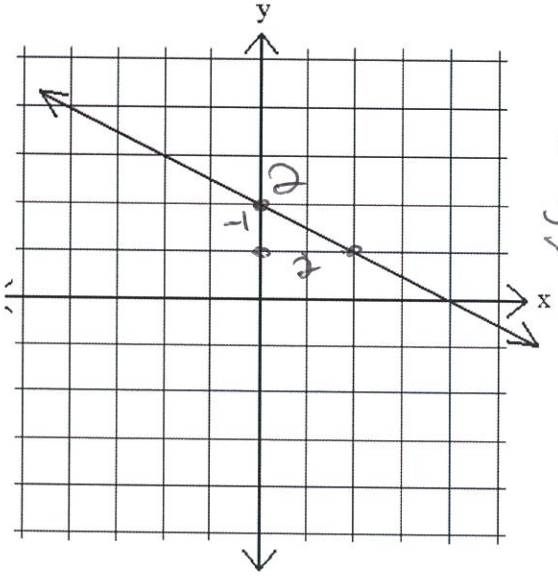


$b=-3$

$m=\frac{3}{4}$

$y=\frac{3}{4}x-3$

12.



$b=2$

$m=\frac{-1}{2}$

$y=-\frac{1}{2}x+2$

Equation of a line through a point

Slope-Intercept Formula: $y = mx + b$ where m is the slope and b is the y-intercept.

1) Find slope

To find slope:

If given 2 points or a table of values, use slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

If given parallel: Parallel lines have the same slope.

If given perpendicular: Perpendicular lines have negative reciprocal slopes (flip/negate)

2) Substitute slope (m), x, and y (the x and y coordinates of any point on the line) into $y = mx + b$ and solve for b.

1. What is an equation for the line that passes through the coordinates (0,0) and (2,4)?

xy

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{4-0}{2-0}$ $\quad m = \frac{4}{2}$ $\quad m = 2$

$y = mx + b$ $\quad 0 = 2(0) + b$ $\quad 0 = 0 + b$ $\quad 0 = b$

$m = 2$ $\quad x = 0$ $\quad y = 0$

$y = 2x + 0$ $\quad y = 2x$

2. What is an equation for the line that passes through the coordinates (-4,6) and (-1,0)?

xy

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{0-6}{-1+4}$ $\quad m = \frac{-6}{3}$ $\quad m = -2$

$y = mx + b$ $\quad 0 = -2(-1) + b$ $\quad 0 = 2 + b$ $\quad -2 = b$

$m = -2$ $\quad x = -1$ $\quad y = 0$

$y = -2x - 2$

3. What is an equation for the line that passes through the coordinates (5,7) and (3,1)?

xy

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{1-7}{3-5}$ $\quad m = \frac{-6}{-2}$ $\quad m = 3$

$y = mx + b$ $\quad 1 = 3(3) + b$ $\quad 1 = 9 + b$ $\quad -8 = b$

$y = 3x - 8$

4. Write an equation for the linear function $f(x)$, which passes through the points (-7,6) and (0,2). Use your equation to determine $f(14)$.

xy

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{2-6}{0+7}$ $\quad m = -\frac{4}{7}$

$y = mx + b$ $\quad 2 = -\frac{4}{7}(0) + b$ $\quad 2 = 0 + b$ $\quad 2 = b$

$m = -\frac{4}{7}$ $\quad x = 0$ $\quad y = 2$

$y = -\frac{4}{7}x + 2$

$f(x) = -\frac{4}{7}x + 2$

$f(14) = -\frac{4}{7}(14) + 2$

$f(14) = -6$

5. Write an equation for the linear function $f(x)$, which passes through the points (6,1) and (8,–1). Use your equation to determine $f(-4)$.

x, y

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-2}{2}$ $\quad m = -1$

$m = \frac{-1 - 1}{8 - 6}$

$y = mx + b$ $\quad m = -1$, $x = 6$, $y = 1$ $\quad y = -1x + 7$

$1 = -1(6) + b$

$1 = -6 + b$

$+6 \quad +6$

$7 = b$

6. The number of people who attended a school's last six basketball games increased as the team neared the state sectional games. The table below shows the data. Write a linear function that represents this data.

Game	13 (x_1)	14 (x_2)	15	16	17	18
Attendance	348 (y_1)	435 (y_2)	522	609	696	783

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{435 - 348}{14 - 13}$ $\quad m = \frac{87}{1}$ $\quad m = 87$

$y = mx + b$ $\quad m = 87$, $x = 13$, $y = 348$

$348 = 87(13) + b$

$348 = 1131 + b$

$-1131 \quad -1131$

$-783 = b$

$y = 87x - 783$

7. Write a linear function for the data in the table below. Use your equation to determine $f(11)$.

x	f(x)
1	12
2	19
3	26
4	33

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{19 - 12}{2 - 1}$ $\quad m = \frac{7}{1}$ $\quad m = 7$

(1, 12) (2, 19) (4, 33)

$y = mx + b$ $\quad m = 7$, $x = 1$, $y = 12$

$12 = 7(1) + b$

$12 = 7 + b$

$-7 \quad -7$

$5 = b$

$y = 7x + 5$

$f(x) = 7x + 5$

$f(11) = 7(11) + 5$

$f(11) = 82$

8. For a given function, $f(x)$, $f(-2) = 5$ and $f(4) = 17$. Find $f(1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{17 - 5}{4 + 2}$ $\quad m = \frac{12}{6}$ $\quad m = 2$

$y = mx + b$ $\quad m = 2$, $x = 4$, $y = 17$

$17 = 2(4) + b$

$17 = 8 + b$

$-8 \quad -8$

$9 = b$

$y = 2x + 9$

$f(x) = 2x + 9$

$f(1) = 2(1) + 9$

$f(1) = 11$

Substitute each choice into the equation of the line to see if it equals out.

9. A line passes through points (1,6) and (3,10). Which of the following points is also on the line?

1) (2,7)
2) (6,–2)
3) (–1,2)
4) (–3,0)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{10-6}{3-1}$ $\quad m = \frac{4}{2}$ $\quad m = 2$

$y = mx + b$ $\quad 6 = 2(1) + b$ $\quad 6 = 2 + b$ $\quad -2 \; -2$ $\quad 4 = b$

$m = 2$, $x = 1$, $y = 6$ $\quad y = 2x + 4$

$y = 2x + 4$ $\quad 2 = 2(-1) + 4$ $\quad 2 = 2$ ✓

10. $f(x)$ defines the function displayed by the table below. Which point is *not* on the graph of $f(x)$?

x	$f(x)$
-2	-5
1	1
5	9
6	11

1) (10,19)
2) (–5,–11)
3) (3,4)
4) (–3,–7)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{9-1}{5-1}$ $\quad m = \frac{8}{4}$ $\quad m = 2$

$y = mx + b$ $\quad 1 = 2(1) + b$ $\quad -3 = b$

$m = 2$, $x = 1$, $y = 1$ $\quad y = 2x - 3$

$1 = 2 + b$ $\quad -2 \; -2$ $\quad -1 = b$

$4 \neq 2(3) - 1$ $\quad 4 \neq 6 - 1$ $\quad 4 \neq 5$ ✗

11. A line passes through points (6,2) and (9,3). If the point (*a*,5) is also on the line, what must the value of *a* be?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{3-2}{9-6}$ $\quad m = \frac{1}{3}$

$y = mx + b$ $\quad 2 = \frac{1}{3}(6) + b$ $\quad 2 = 2 + b$ $\quad -2 \; -2$ $\quad 0 = b$

$y = \frac{1}{3}x$ $\quad 3(5) = \left(\frac{1}{3}a\right)3$ $\quad 15 = a$

12. $f(x)$ defines the function displayed by the table below. If (4,*b*) is also a point on $f(x)$, what is the value of *b*?

x	$f(x)$
-1	-1
2	8
6	20
7	23

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{20-8}{6-2}$ $\quad m = \frac{12}{4}$ $\quad m = 3$

$y = mx + b$ $\quad 8 = 3(2) + b$ $\quad 8 = 6 + b$ $\quad -6 \; -6$ $\quad 2 = b$

$y = 3x + 2$ $\quad b = 3(4) + 2$ $\quad b = 14$

13. What is an equation of the line that passes through the point $(-2, 5)$ and is parallel to the line whose equation is $y = \frac{1}{2}x + 5$?

x y same slope

$m = \frac{1}{2}$

$m\parallel = \frac{1}{2}$

$x = -2$

$y = 5$

$y = mx + b$

$5 = \frac{1}{2}(-2) + b$

$5 = -1 + b$

$+1 \quad +1$

$6 = b$

$y = \frac{1}{2}x + 6$

14. What is an equation of the line that contains the point $(3, -1)$ and is perpendicular to the line whose equation is $y = -3x + 2$?

x y negative reciprocal slope

$m = -3$

$m\perp = \frac{1}{3}$

$x = 3$

$y = -1$

$y = mx + b$

$-1 = \frac{1}{3}(3) + b$

$-1 = 1 + b$

$-1 \quad -1$

$-2 = b$

$y = \frac{1}{3}x - 2$

15. What is the equation of a line that passes through the point $(6, -5)$ and is perpendicular to the line whose equation is $y + 2x = 5$?

x y negative reciprocal slope

$-2x \quad -2x$

$y = -2x + 5$

$m = -2$

$m\perp = \frac{1}{2}$

$x = 6$

$y = -5$

$y = mx + b$

$-5 = \frac{1}{2}(6) + b$

$-5 = 3 + b$

$-3 \quad -3$

$-8 = b$

$y = \frac{1}{2}x - 8$

16. Write an equation of the line that passes through the point $(6, -5)$ and is parallel to the line whose equation is $2x - 3y = 11$.

x y same slope

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x + 11}{-3}$

$y = \frac{2}{3}x - \frac{11}{3}$

$m = \frac{2}{3}$

$m\parallel = \frac{2}{3}$

$x = 6$

$y = -5$

$y = mx + b$

$-5 = \frac{2}{3}(6) + b$

$-5 = 4 + b$

$-4 \quad -4$

$-9 = b$

$y = \frac{2}{3}x - 9$

sinθ = O/H cosθ = A/H tanθ = O/A

Trigonometric Ratios (SOHCAHTOA)

1) Label each side with H, A, and O

2) Use SOHCAHTOA ($\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$)

1. Find the following trig ratios for the given triangle.

$\sin A$ $\frac{8}{17}$

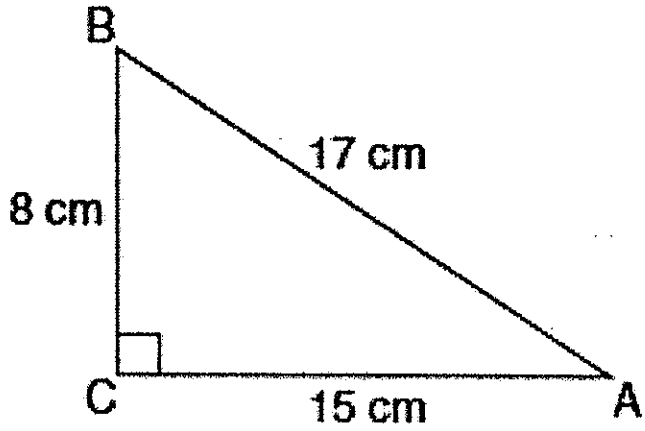
$\cos A$ $\frac{15}{17}$

$\tan A$ $\frac{8}{17}$

$\sin B$ $\frac{15}{17}$

$\cos B$ $\frac{8}{17}$

$\tan B$ $\frac{17}{8}$



2. Find the following trig ratios for the given triangle.

$\sin J$ $\frac{7}{25}$

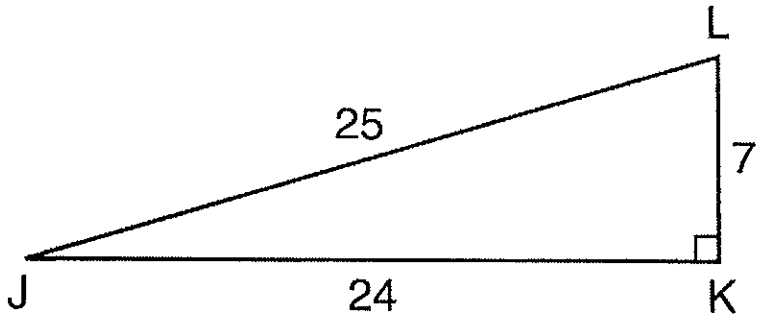
$\cos J$ $\frac{24}{25}$

$\tan J$ $\frac{7}{24}$

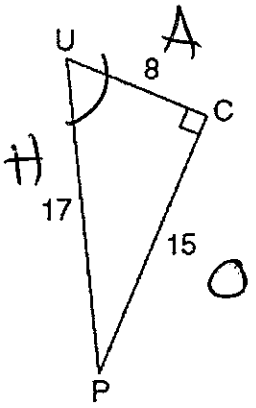
$\sin L$ $\frac{24}{25}$

$\cos L$ $\frac{7}{25}$

$\tan L$ $\frac{24}{7}$



3. The diagram below shows right triangle UPC.



Which ratio represents the sine of $\angle U$?

1) $\frac{15}{8}$
2) $\frac{15}{17}$
3) $\frac{8}{15}$
4) $\frac{8}{17}$

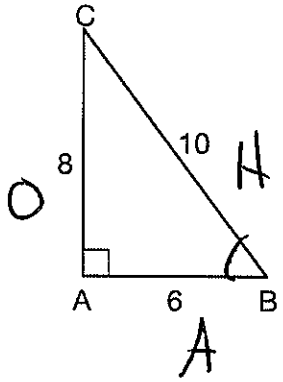
$\sin V = \frac{O}{H}$

$\sin V = \frac{15}{17}$

4. In ΔABC below, the measure of $\angle A = 90°$, $AB = 6$, $AC = 8$, and $BC = 10$.

Which ratio represents the sine of $\angle B$?

1) $\frac{10}{8}$
2) $\frac{8}{6}$
3) $\frac{6}{10}$
4) $\frac{8}{10}$

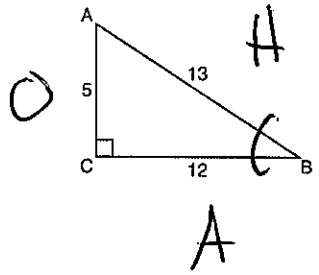


$\sin B = \frac{O}{H}$

$\sin B = \frac{8}{10}$

5. Which ratio represents the tangent of $\angle ABC$?

1) $\frac{5}{13}$
2) $\frac{5}{12}$
3) $\frac{12}{13}$
4) $\frac{12}{5}$



~~Sin A~~ $\tan B = \frac{O}{A}$

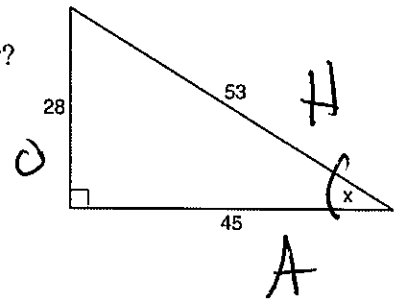
$\tan B = \frac{5}{12}$

6. Which ratio represents $\sin x$ in the right triangle shown below?

1) $\frac{28}{53}$
2) $\frac{28}{45}$
3) $\frac{45}{53}$
4) $\frac{53}{28}$

$\sin X = \frac{O}{H}$

$\sin X = \frac{28}{53}$



Acute Angles in a Right Triangle

$\sin A = \cos B$

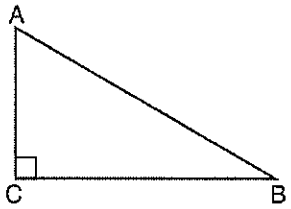
$\sin X = \cos(90 - x)$ **: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**

$A + B = 90$ **: The two acute angles in a right triangle are complementary**

1. In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

Which equation is always true?

1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$



2. In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?

1) $\sin(x) = \cos(x)$
2) $\sin(90 - x) = \cos x$
3) $\sin(x) = \tan(90 - x)$
4) $\sin(90 - x) = \cos(90 - x)$

3. Which expression is always equivalent to $\sin x$ when $0° < x < 90°$?

1) $\cos(90° - x)$
2) $\cos(45° - x)$
3) $\cos(2x)$
4) $\cos x$

4. In a right triangle, one angle measures x, where $\cos x = \frac{3}{4}$. What is $\sin(90 - x)$?

$\sin(90 - A) = \frac{3}{4}$

5. Given: Right triangle ABC with right angle at C. If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.

increased because $\sin A = \cos B$

6. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

$\sin A = \cos B$

7. In right triangle ABC, $m\angle C = 90°$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1) $\cos A$
2) $\cos B$
3) $\tan A$
4) $\tan B$

$\sin A = \cos B$

8. In right triangle XYZ with the right angle at Y, $\sin X = 2x + 5$ and $\cos Z = 4x + 1$. Determine and state the value of x. Explain your answer.

$\sin A = \cos B$

$2x+5 = 4x+1$
$-2x \quad -2x$
$5 = 2x+1$
$-1 \quad -1$
$4 = 2x$
$\frac{4}{2} = \frac{2x}{2}$
$2 = x$

9. In right triangle NBC with the right angle at B, $\cos N = 6x + 5$ and $\sin C = 4x + 20$. Determine and state the value of x. Explain your answer.

$\sin A = \cos B$

$6x+5 = 4x+20$
$-4x \quad -4x$
$2x+5 = 20$
$-5 \quad -5$
$\frac{2x}{2} = \frac{15}{2}$
$x = 7.5$

10. If $\sin(2x + 7)° = \cos(4x - 7)°$, what is the value of x?

1) 7
2) 15
3) 21
4) 30

$\sin A = \cos B$
$A + B = 90$
$2x+7+4x-7 = 90$
$\frac{6x}{6} = \frac{90}{6}$
$x = 15$

11. In a right triangle, $\sin(40 - x)° = \cos(3x)°$. What is the value of x?

1) 10
2) 15
3) 20
4) 25

$\sin A = \cos B$
$A + B = 90$
$40 - x + 3x = 90$
$2x + 40 = 90$
$-40 \quad -40$
$\frac{2x}{2} = \frac{50}{2}$
$x = 25$

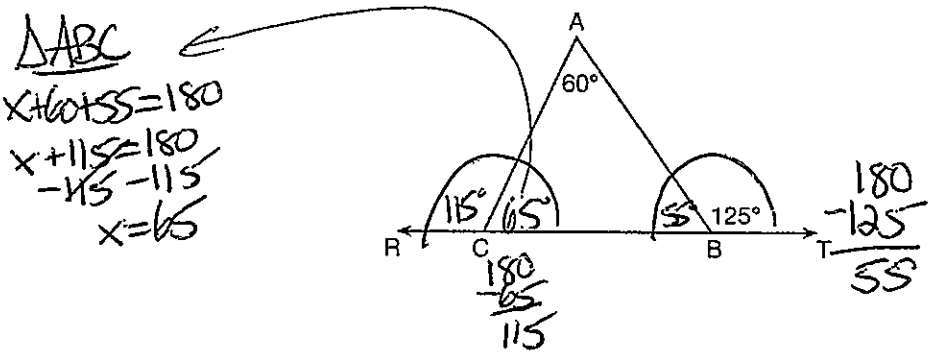
Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

1) The three angles of a triangle add to equal 180°. **Look for triangles.**
   *The four angles of a quadrilateral add to 360°.
2) Linear pairs add to 180°. **Look for linear pairs.**
3) Vertical angles are congruent. Look for an X (intersecting lines).
4) **Given congruent sides:** Isosceles triangle has congruent angles opposite congruent sides.
5) **Given equilateral triangle:** Equilateral triangle has angles 60, 60, 60.
6) **Given angle bisector:** An angle bisector cuts an angle into two congruent halves.
7) **Given parallel**: Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).

1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown with $m\angle A = 60$ and $m\angle ABT = 125$.

What is $m\angle ACR$?

1) 125
2) 115
3) 65
4) 55



△ABC
x+60+55=180
x+115=180
−115 −115
x=65

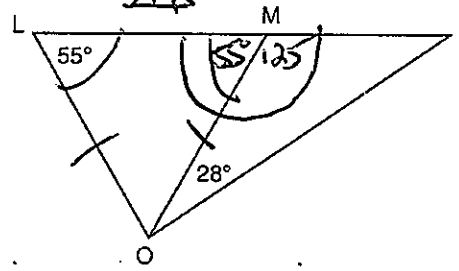
180
−125
55

180
−65
115

2. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.

If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?

1) 27
2) 28
3) 42
4) 70

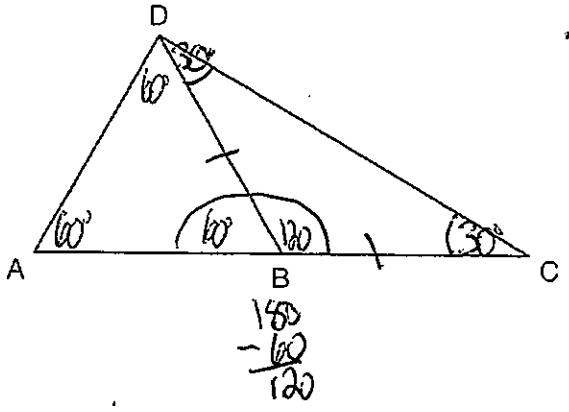


180
−55

△NMO
125+28+x=180
153+x=180
−153 −153
x=27

3. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.

60,60,60

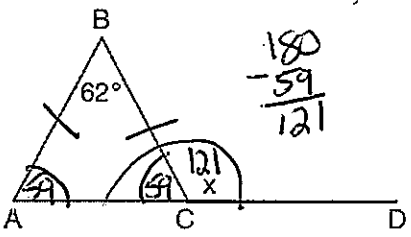


180
−60
120

△DBC
x+x+120=180
2x+120=180
−120 −120
2x=60
x=30

m∠C=30°

4. Given $\triangle ABC$ with $m\angle B = 62°$ and side $\overline{AC}$ extended to D, as shown below.



180 − 59 = 121

$\triangle ABC$

$x + x + 62 = 180$

$2x + 62 = 180$

$-62 \quad -62$

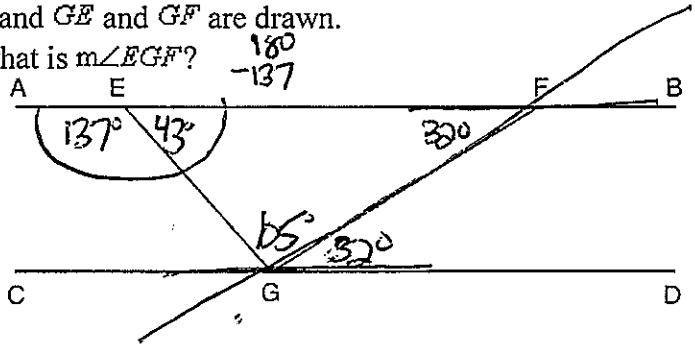
$\frac{2x}{2} = \frac{118}{2}$

$x = 59$

Which value of x makes $\overline{AB} \cong \overline{CB}$?

1) 59°
2) 62°
3) 118°
4) 121°

5. In the diagram below, $\overline{AEFB} \parallel \overline{CGD}$, and $\overline{GE}$ and $\overline{GF}$ are drawn. If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?



180 − 137 = 43

1) 11°
2) 43°
3) 75°
4) 105°

$\triangle EFG$

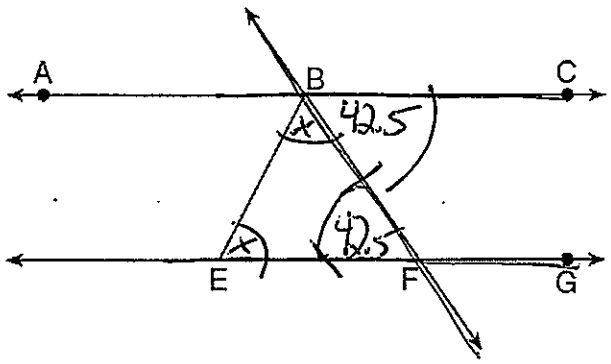
$43 + 32 + x = 180$

$75 + x = 180$

$-75 \quad -75$

$x = 105$

6. As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$. If $m\angle CBF = 42.5°$, then $m\angle EBF$ is



1) 42.5°
2) 68.75°
3) 95°
4) 137.5°

$\triangle EBF$

$x + x + 42.5 = 180$

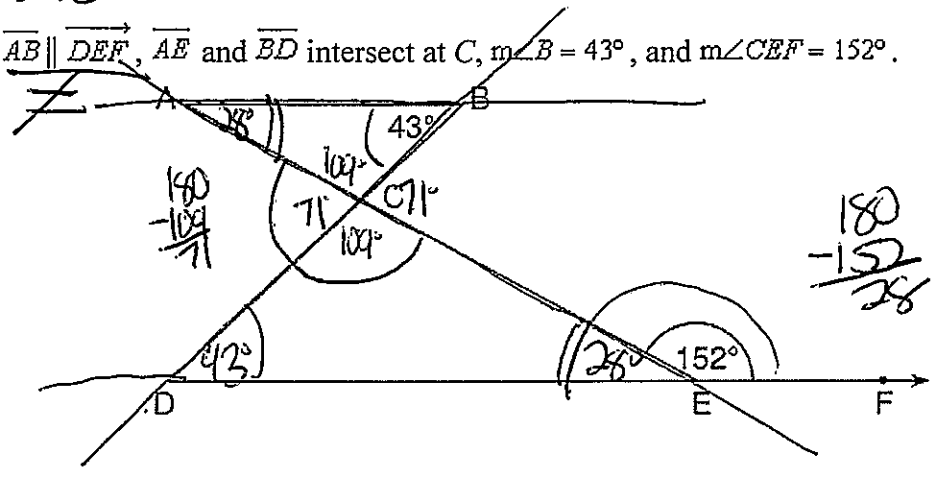
$2x + 42.5 = 180$

$-42.5 \quad -42.5$

$\frac{2x}{2} = \frac{137.5}{2}$

$x = 68.75$

7. In the diagram below, $\overline{AB} \parallel \overrightarrow{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at C, $m\angle B = 43°$, and $m\angle CEF = 152°$. Which statement is true?



1) $m\angle D = 28°$
2) $m\angle A = 43°$
3) $m\angle ACD = 71°$
4) $m\angle BCE = 109°$

180 − 109 = 71

180 − 152 = 28

$\triangle ABC$

$28 + 43 + x = 180$

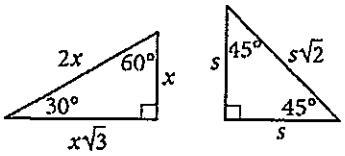
$71 + x = 180$

$-71 \quad -71$

$x = 109$

Special Right Triangles

Taken directly from Reference Sheet on Exam:

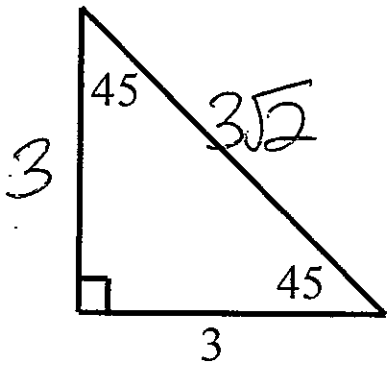


Special Right Triangles

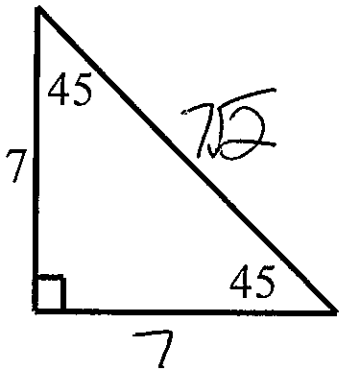
Once you have one side, you have all three following the given rules.

Fill in the two missing sides of each of the following triangles.

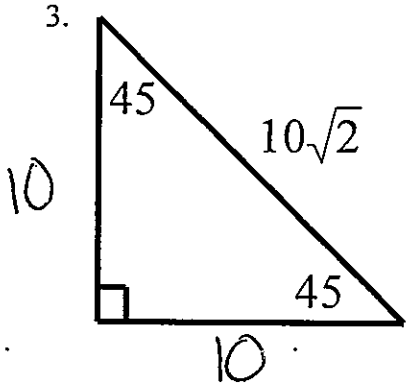
1.



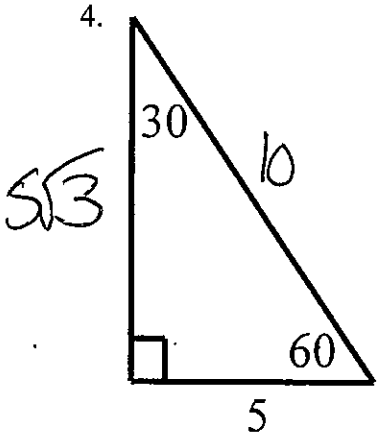
2.



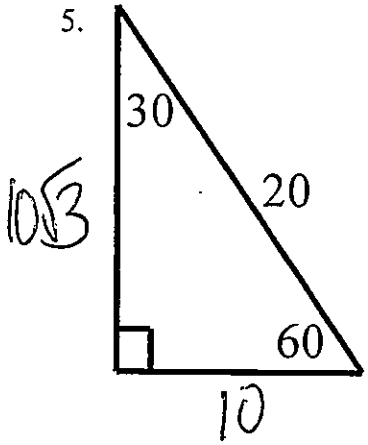
3.



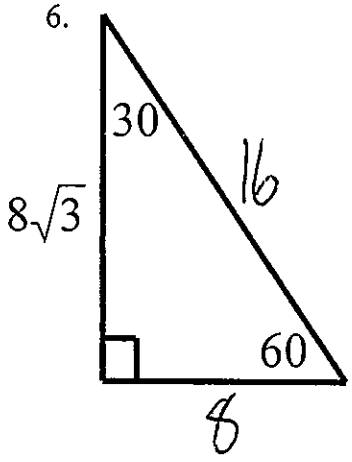
4.



5.



6.



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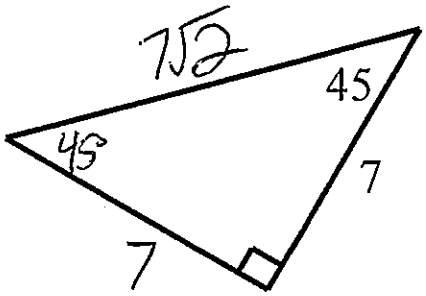
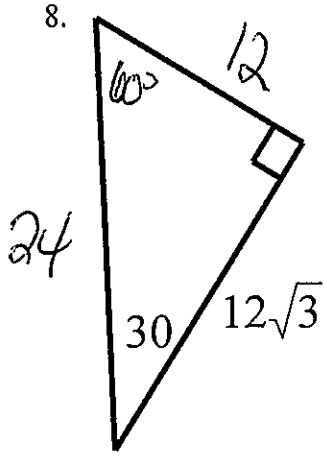
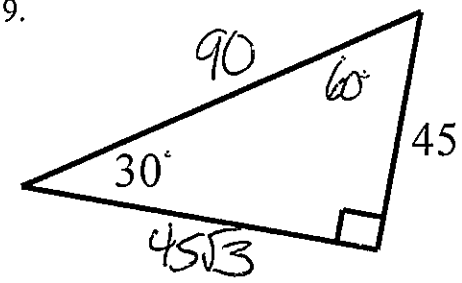

7√2
45
45
7
7

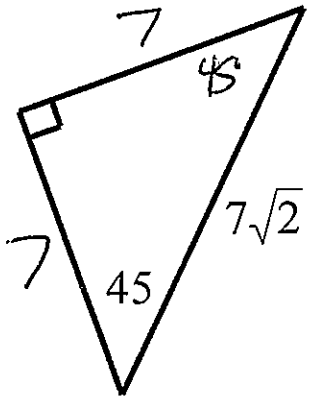
8.


60
12
24
30
12√3

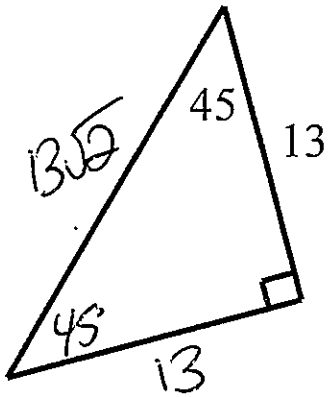
9.


90
60
30
45
45√3

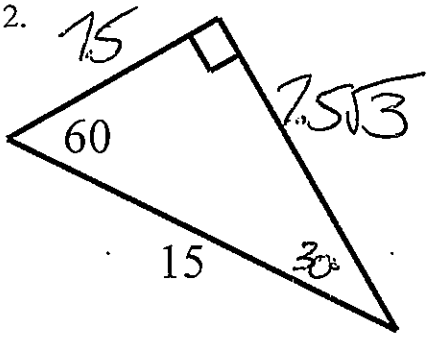
10.


7
45
7
7√2
45

11.


13√2
45
13
45
13

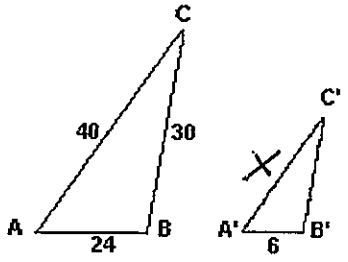
12.


15
7.5√3
60
15
30

Similar Triangles

1) Separate the triangles is necessary.
2) Create a proportion by putting the corresponding sides on top of each other.
3) Cross multiply and solve.

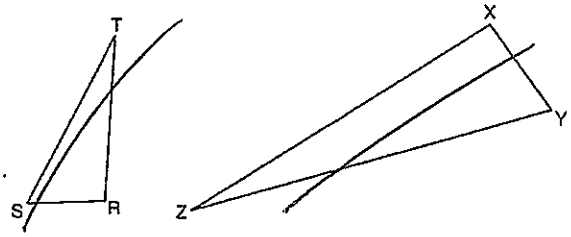
1. In the diagram, ΔABC is similar to $\Delta A'B'C'$, $AB = 24$, $BC = 30$, and $CA = 40$. If the shortest side of $\Delta A'B'C'$ is 6, find the length of the longest side of $\Delta A'B'C'$.

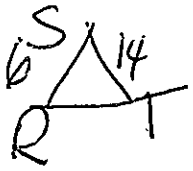


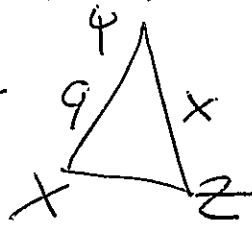
the longest sides correspond to each other

$\frac{40}{X} = \frac{24}{6}$ $\quad \frac{24X}{24} = \frac{240}{24}$ $\quad X = 10$

2. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, find YZ.





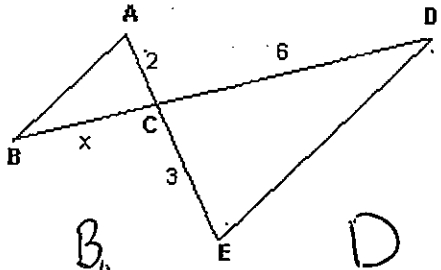


$\frac{6}{9} = \frac{14}{X}$

$\frac{6X}{6} = \frac{126}{6}$

$X = 21$

3. In the diagram: $\overline{AB} \parallel \overline{DE}$. If $AC = 2$, $CD = 6$, and $CE = 3$, what is BC?

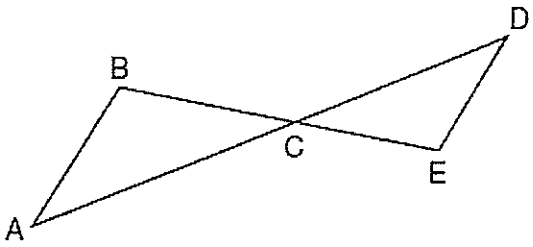


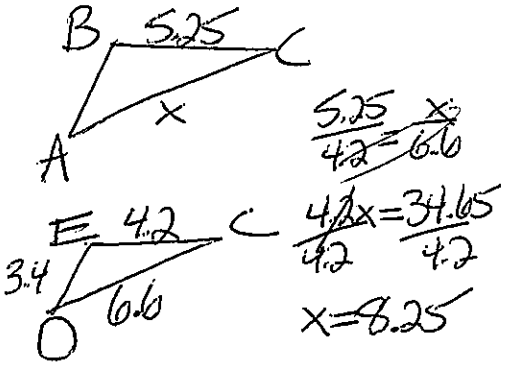
$\frac{X}{6} = \frac{2}{3}$

$\frac{3X}{3} = \frac{12}{3}$

$X = 4$

4. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.



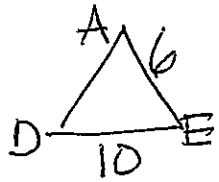


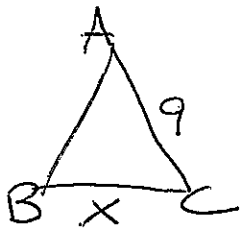
If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?

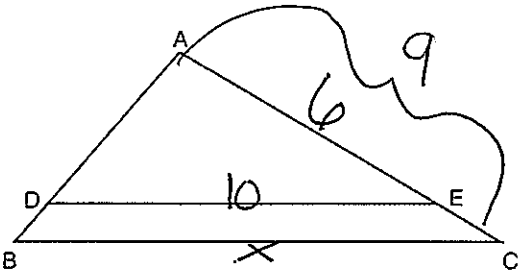
1) 2.70
2) 3.34
3) 5.28
4) 8.25

5. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$.

If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find $\overline{BC}$



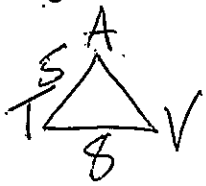


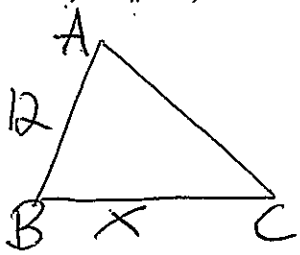


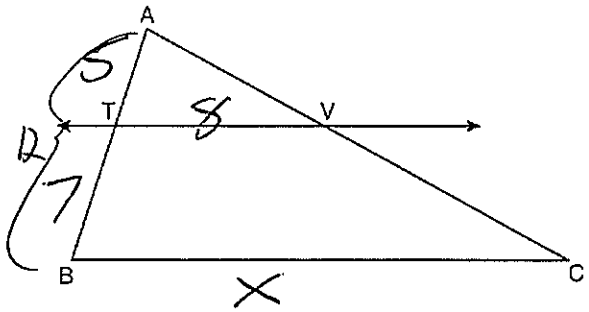
$\frac{6}{9} = \frac{10}{x}$ $\quad \frac{6x = 90}{6 \quad 6}$

$x = 15$

6. In the diagram below of $\triangle ABC$, $\overleftrightarrow{TV} \parallel \overline{BC}$, $\overline{AT} = 5$, $\overline{TB} = 7$, and $\overline{TV} = 8$, find $\overline{BC}$







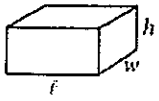
$\frac{5}{12} = \frac{8}{x}$

$\frac{5x = 96}{5 \quad 5}$

$x = 19.2$

Volume

Taken directly from reference sheet on exam:

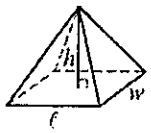


$V = \ell wh$

$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$

$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}\ell wh$

1. A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*? Type π in

$V = \pi r^2 h$

$V = \pi(5)^2(2.3)$

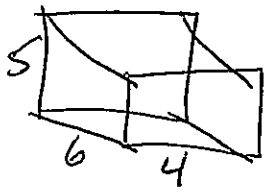
$V = 180.6 \text{ in}^3$

2. What is the volume of a rectangular prism whose length is 4 cm, width is 6 cm, and height is 5 cm?

$V = \ell wh$

$V = 4(6)(5)$

$V = 120 \text{cm}^3$

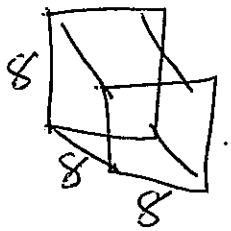


3. What is the volume of a cube if each side of the cube measures 8 in?

$V = \ell wh$

$V = 8(8)(8)$

$V = 512 \text{ in}^3$



4. What is the volume of a cylinder whose height is 12 inches and whose diameter is 20 inches in terms of π? Don't type π in

$V = \pi r^2 h$

$V = \pi(10)^2(12)$

$V = 1200\pi$

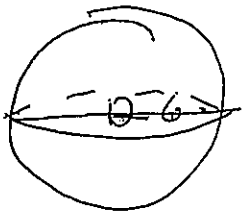
Don't type π in

5. Find the volume of a sphere that has a diameter of 12 in in terms of π.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(6)^3$

$V = 288\pi$

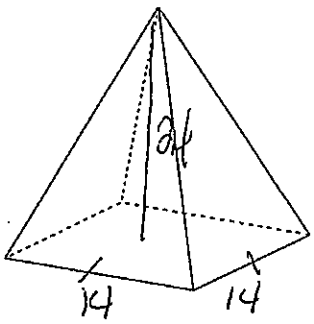


6. A regular pyramid has a square base with an edge length of 14 and an altitude of 24. Find its volume.

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(14)(14)(24)$

$V = 1568$



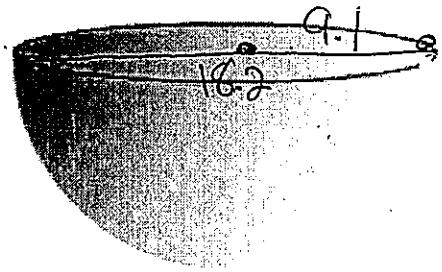
7. Find the volume of a cone with a height of 12 in and a diameter of 8 in rounded to the nearest hundredth. Type π in

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(12)$

$V = 201.06\ in^3$

8. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter*. Type π in



hemisphere = $\frac{1}{2}$ sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(9.1)^3\right)$

$V = 1578\ m^3$

Volume with Algebra
Substitute into appropriate volume formula
Solve the equation
***To get rid of a fraction, multiply by the denominator**
***To get rid of cubed, take the cubed root (final step)**

1. A brick in the shape of a rectangular prism has a base that measures 3 inches by 5 inches. If the volume of the brick is 90 cubic inches, what is the height of the brick?

$V = lwh$

$90 = 3(5)(x)$

$\frac{90}{15} = \frac{15x}{15}$

$6 = x$

2. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

$V = \pi r^2 h$

$\frac{1000}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{39.7..} = \sqrt{r^2}$

$6.3 = r$

3. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. Find, in centimeters, the height of the pyramid if the volume is 288 cm^3.

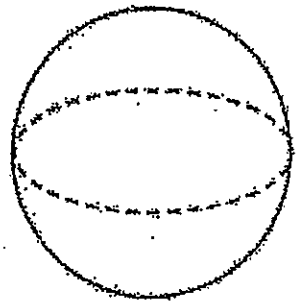
$V = \frac{1}{3}lwh$

$288 = \frac{1}{3}(6)(8)(x)$

$\frac{288}{16} = \frac{16x}{16}$

$x = 18$

4. Find the radius of a sphere with a volume of 576π cubic units. Find the answer to the *nearest tenth of a unit*.



$V = \frac{4}{3}\pi r^3$

$3(576\pi) = \left(\frac{4}{3}\pi r^3\right)3$

$\frac{1728\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{432} = \sqrt[3]{r^3}$

$r = 7.6$

5. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter*.

$V = \pi r^2 h$

$\frac{12,566.4}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{500} = \sqrt{r^2}$

$22.4 = r$

6. The Parkside Packing Company needs a rectangular shipping box. The box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch*, the minimum height of the box such that the volume is *at least* 800 cubic inches.

$V = lwh$

$800 = 11(8)(x)$

$\frac{800}{88} = \frac{88x}{88}$

$9.1 = x$

7. If the volume of a sphere is 36π, what is the radius of the sphere?

(1) 3 (2) 6 (3) 12 (4) 24

$V = \frac{4}{3}\pi r^3$

$3\left(36\pi = \frac{4}{3}\pi r^3\right)3$

$\frac{108\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$3 = r$

8. Find the length of the radius of a cylinder to the *nearest tenth* if it has a volume of 60 cm^3 and a height of 10 *cm*.

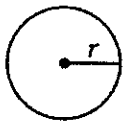
$V = \pi r^2 h$

$\frac{60}{10\pi} = \frac{\pi r^2 (10)}{10\pi}$

$\sqrt{1.9} = \sqrt{r^2}$

$1.4 = r$

Circumference:
Taken directly from reference sheet on exam:



$A = \pi r^2$

$C = 2\pi r$

$$\frac{arc\ length}{2\pi r} = \frac{arc\ area}{\pi r^2} = \frac{central\ angle\ (\text{deg}rees)}{360} = \frac{central\ angle (radians)}{2\pi}$$

Choose the appropriate two columns for your proportion

1. Find the circumference of the circle with a diameter of 12 in terms of π.

$r=6$

$C=2\pi r$
$C=2\pi(6)$
$C=12\pi$

2. Find the circumference of the circle with a radius of 8 in terms of π.

$C=2\pi r$
$C=2\pi(8)$
$C=16\pi$

3. Find the circumference of the circle with a diameter of 24 rounded to the nearest tenth.

$r=12$

$C=2\pi r$
$C=2\pi(12)$
$C=75.4$

4. Find the circumference of the circle with a radius of 10 rounded to the nearest tenth.

$C=2\pi r$
$C=2\pi(10)$
$C=62.8$

5. If the circumference of a circle is 18π, find the radius.

$C = 2\pi r$

$\frac{18\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$9 = r$

6. If the circumference of a circle is 100π, find the diameter.

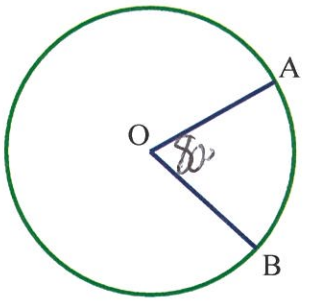
$C = 2\pi r$

$\frac{100\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$50 = r$

$d = 2(50) = 100$

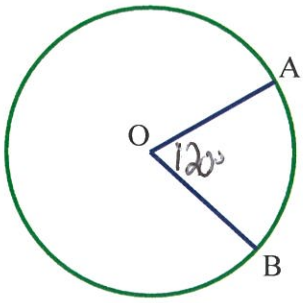
7. In the diagram below, the measure of $m\angle AOB = 80$. What fraction of the area of the circle is the area of sector AOB?



$\frac{x}{360} = \frac{\text{area sector}}{\text{total area}}$

$\frac{80}{360} = \boxed{\frac{2}{9}}$

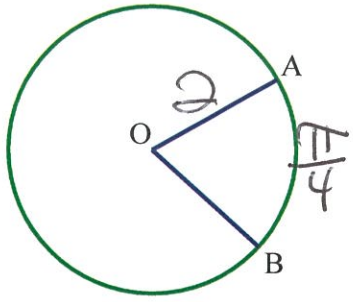
8. In the diagram below, the measure of $m\angle AOB = 120$. What fraction of the circumference of the circle is the arc length of AB?



$\frac{x}{360} = \frac{\text{arc length}}{\text{circumference}}$

$\frac{120}{360} = \boxed{\frac{1}{3}}$

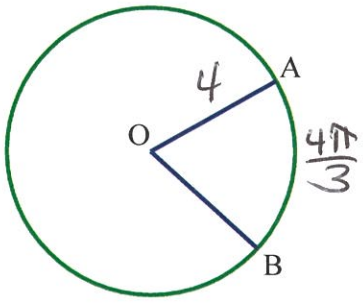
9. Points A and B lie on a circle with a radius of 2 and arc AB has length $\frac{\pi}{4}$. What fraction of the circumference of the circle is the length of arc AB?



$\frac{\text{arc length}}{\text{circumference}}$ $2\pi r$

$\frac{\frac{\pi}{4}}{2\pi(2)}$ $\frac{\frac{\pi}{4}}{\frac{4\pi}{1}}$ $\frac{\pi}{4}\cdot\frac{1}{4\pi}$ $\boxed{\frac{1}{16}}$

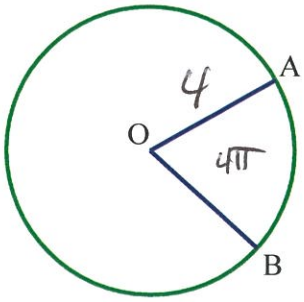
10. Points A and B lie on a circle with a radius of 4 and arc AB has length $\frac{4\pi}{3}$. What fraction of the circumference of the circle is the length of arc AB?



$\frac{\text{arc length}}{\text{circumference}}$ $2\pi r$

$\frac{\frac{4\pi}{3}}{2\pi(4)}$ $\frac{\frac{4\pi}{3}}{\frac{8\pi}{1}}$ $\frac{4\pi}{3}\cdot\frac{1}{8\pi} = \boxed{\frac{1}{6}}$

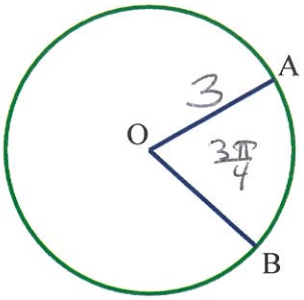
11. Points A and B lie on a circle with a radius of 4 and area of sector AOB is 4π. What fraction of the area of the circle is the area of sector AOB?



$\frac{\text{area sector}}{\text{total area}}$ πr^2

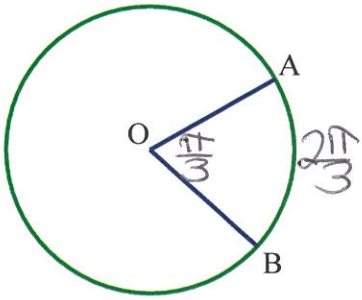
$\frac{4\pi}{\pi(4)^2}$ $\frac{4\pi}{16\pi}$ $\boxed{\frac{1}{4}}$

12. Points A and B lie on a circle with a radius of 3 and area of sector AOB is $\frac{3\pi}{4}$. What fraction of the area of the circle is the area of sector AOB?



$\frac{\text{area sector}}{\text{total area}}$ $\quad \frac{\frac{3\pi}{4}}{\pi r^2}$ $\quad \frac{\frac{3\pi}{4}}{\pi(3)^2}$ $\quad \frac{\frac{3\pi}{4}}{\frac{9\pi}{1}}$ $\quad \frac{3\pi}{4}\cdot\frac{1}{9\pi}$ $\quad \frac{3}{36}=\frac{1}{12}$

13. Points A and B lie on a circle and arc AB has length $\frac{2\pi}{3}$. If $m\angle AOB=\frac{\pi}{3}$, find the circumference of the circle.

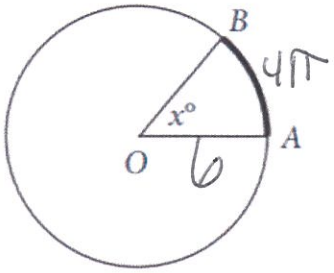


$\frac{\text{central } \angle}{2\pi}=\frac{\text{arc length}}{\text{circumference}}$

$\frac{\frac{1\pi}{3}}{2\pi}=\frac{\frac{2\pi}{3}}{x}$ $\quad 3\left(\frac{1}{3}\right)x=\left(\frac{4\pi}{3}\right)3$

$x=4\pi$

14. In circle O below, $\overline{AO}=6$ and arc AB = 4π. Find the degree measure of x.



$\frac{\angle}{360}=\frac{\text{arc length}}{2\pi r}$

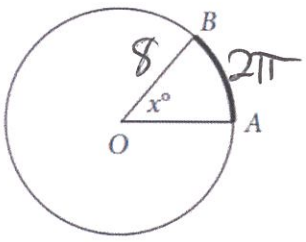
$\frac{x}{360}=\frac{4\pi}{2\pi(6)}$

$\frac{x}{360}=\frac{4}{12}$

$\frac{12x}{12}=\frac{1440}{12}$

$x=120°$

14. In circle O below, $\overline{OB} = 8$ and arc AB = 2π. Find the degree measure of x.



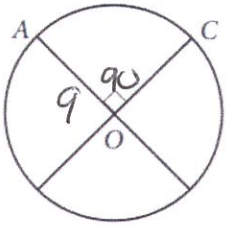
$\frac{x}{360} = \frac{\text{arc length}}{2\pi r}$

$\frac{x}{360} = \frac{2\pi}{2\pi(8)} = \frac{1}{8}$

$\frac{8x}{8} = \frac{360}{8}$

$x = 45°$

15. The circle below with center O has $\overline{AO} = 9$. What is the length of arc AC?



$\frac{x}{360} = \frac{\text{arc length}}{2\pi r}$

$\frac{90}{360} = \frac{x}{2\pi(9)} = \frac{1}{4}$

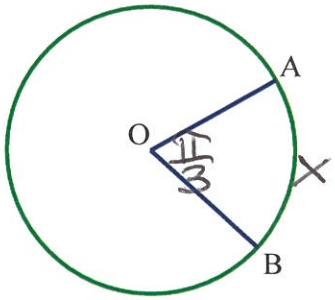
$\frac{1}{4} = \frac{x}{18\pi}$

$4x = 18\pi$

$\frac{4x}{4} = \frac{18\pi}{4}$

$x = \frac{9\pi}{2}$

16. In the circle below, $m\angle AOB = \frac{\pi}{3}$. If the circumference of the circle is 18π, find arc AB.



$\frac{x}{2\pi} = \frac{\text{arc length}}{\text{circumference}}$ $\quad 2\pi r$

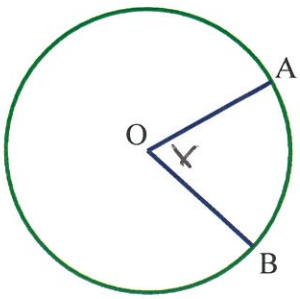
$\frac{\frac{\pi}{3}}{2\pi} = \frac{x}{18\pi}$

$3\pi = x$

$\frac{18\pi}{3} = 2x$

$\frac{6\pi}{2} = \frac{2x}{2}$

18. In the circle below, arc AB is $\frac{3}{4}$ the circumference of the circle. What is the degree measure of $\angle AOB$.



$\frac{x}{360} = \frac{\text{arc}}{\text{circumference}}$

$\frac{x}{360} = \frac{3}{4}$

$\frac{1080}{4} = \frac{4x}{4}$

$270° = x$

1 is imaginary power
2 is imaginary root

Fractional Exponents

Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$)

Rewrite the following as radicals

1. $x^{\frac{2}{3}}$ $\sqrt[3]{x^2}$

2. $x^{\frac{3}{4}}$ $\sqrt[4]{x^3}$

3. $x^{\frac{5}{6}}$ $\sqrt[6]{x^5}$

4. $x^{\frac{1}{3}}$ $\sqrt[3]{x}$

5. $x^{\frac{3}{2}}$ $\sqrt{x^3}$

6. $x^{\frac{1}{2}}$ $\sqrt{x}$

6. $x^{\frac{4}{5}}$ $\sqrt[5]{x^4}$

8. $x^{\frac{1}{7}}$ $\sqrt[7]{x}$

9. $x^{\frac{5}{2}}$ $\sqrt{x^5}$

Rewrite the following using fractional exponents

10. $\sqrt[3]{x^4}$ $x^{\frac{4}{3}}$

11. $\sqrt[5]{x^3}$ $x^{\frac{3}{5}}$

12. $\sqrt[4]{x^7}$ $x^{\frac{7}{4}}$

13. $\sqrt{x^3}$ $x^{\frac{3}{2}}$

14. $\sqrt[6]{x^5}$ $x^{\frac{5}{6}}$

15. $\sqrt{x}$ $x^{\frac{1}{2}}$

15. $\sqrt[8]{x^3}$ $x^{\frac{3}{8}}$

16. $\sqrt[5]{x^3}$ $x^{\frac{3}{5}}$

17. $\sqrt[3]{x}$ $x^{\frac{1}{3}}$

Probability with Two Way Tables

Conditional Probabilities: Circle the row/column that contains the condition. Condition always comes after the phrase given that. You will not always see the phrase given that.
"And" is not conditional.

1.	Sports	No Sports	Total
Music	3	7	10
No Music	5	6	11
Total	8	13	21

What is the probability that a student chosen at random? Express your answer as a fraction, decimal (nearest thousandth) and percent (nearest percent).

Plays music and sports

$\frac{3}{21} = .143 = 14\%$

Plays music but not sports

$\frac{7}{21} = .333 = 33\%$

Plays sports but not music

$\frac{5}{21} = .238 = 24\%$

Does not play sports or music

$\frac{6}{21} = .286 = 29\%$

Plays sports

$\frac{8}{21} = .381 = 38\%$

Does not play sports

$\frac{13}{21} = .619 = 62\%$

Plays music

$\frac{10}{21} = .476 = 48\%$

Does not play music

$\frac{11}{21} = .524 = 52\%$

Plays music given that they play sports

$\frac{3}{8} = .375 = 38\%$

Plays sports given that they play music

$\frac{3}{10} = .3 = 30\%$

Plays music given that they don't play sports

$\frac{7}{13} = .538 = 54\%$

Doesn't play sports given that they don't play music

$\frac{6}{11} = .545 = 55\%$

2. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table.

	In Favor	Against	
Male	15	45	60
Female	4	36	40
	19	81	100

P(male and in favor)

$\frac{15}{100} = .15 = 15\%$

P(female and in favor)

$\frac{4}{100} = .04 = 4\%$

P(male and against)

$\frac{45}{100} = .45 = 45\%$

P(female and against)

$\frac{36}{100} = .36 = 36\%$

P(male)

$\frac{60}{100} = .6 = 60\%$

P(in favor)

$\frac{19}{100} = .19 = 19\%$

P(female)

$\frac{40}{100} = .4 = 40\%$

P(against)

$\frac{81}{100} = .81 = 81\%$

P(male given in favor)

$\frac{15}{19} = .789 = 79\%$

P(against given female)

$\frac{36}{40} = .9 = 90\%$

P(in favor given male)

$\frac{15}{60} = .25 = 25\%$

P(female given against)

$\frac{36}{81} = .444 = 44\%$

3. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion	
21–40	30	12	8	50
41–60	20	40	15	75
Over 60	25	35	15	75
	75	87	38	200

Find the probability that a voter is between 21 and 60.

50+75 $\frac{125}{200} = .625 = 63\%$

Find the probability that a voter is not for the candidate.

87+38 $\frac{125}{200} = .625 = 63\%$

Find the probability that a voter over 60 is against the candidate.

$\frac{35}{200} = .175 = 18\%$

4. A radio station did a survey to determine what kind of music to play by taking a sample of middle school, high school, and college students. They were asked which of three different types of music they prefer on the radio: hip-hop, alternative, or classic rock. The results are summarized in the table below.

What percentage of college students prefer classic rock?

$\frac{14}{50} = .28 =$ 50%

	Hip-Hop	Alternative	Classic Rock	
Middle School	28	18	4	50
High School	22	22	6	50
College	16	20	14	50
	66	60	24	150

What is the probability that a student is in middle school or high school?

50+50 $\frac{100}{150}$

What is the probability that a college student does not like classic rock?

16+20 = 36 $\frac{36}{50}$

What is the probability that a student likes hip hop given that they are not in middle school?

22+16 $\frac{38}{66}$

5. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama	
Male	70	35	105
Female	48	42	90
	118	77	195

What is the probability that a male student would prefer comedy?

$\frac{70}{105}$

What is the probability that a student is female and likes drama?

$\frac{42}{195}$

6. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

	Favorite Type of Program			
	Sports	**Reality Show**	**Comedy Series**	
Senior	83	110	67	260
Freshmen	119	103	54	276
	202	213	121	536

What is the probability the student response is from a freshman, given the student prefers to watch reality shows on television?

$\frac{103}{213}$

What is the probability the student response is from a senior and that it's not sports?

$110 + 67$ $\qquad \frac{177}{536}$

Scatter Plots

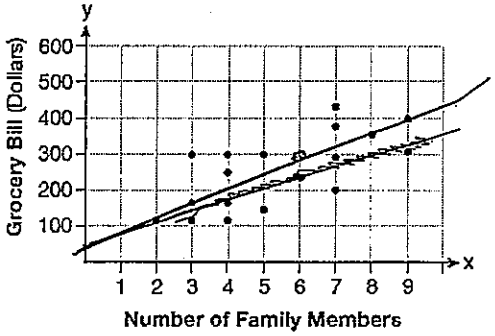
The line of best fit is the line which best fits the points in the scatter plot. It should pass through the middle and have points above and below it.

The line of best fit is the expected values. The points themselves are the actual values. To find the difference, find each value and subtract them.

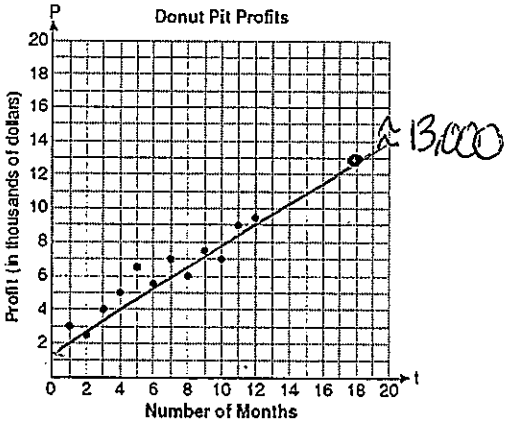
1. The scatter plot below shows the relationship between the number of members in a family and the amount of the family's weekly grocery bill.

The most appropriate prediction of the grocery bill for a family that consists of six members is

1) $100
2) $300
3) $400
4) $500



2. Megan and Bryce opened a new store called the Donut Pit. Their goal is to reach a profit of $20,000 in their 18th month of business. The scatter plot below represent the profit, *P*, in thousands of dollars, that they made during the first 12 months. Draw a reasonable line of best fit. Using the line of best fit, predict whether Megan and Bryce will reach their goal in the 18th month of their business. Justify your answer.

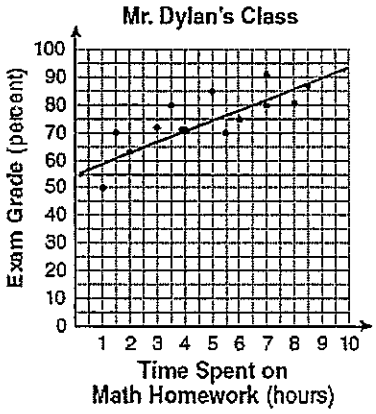


No, they will have a profit of approximately $13,000

3. The number of hours spent on math homework each week and the final exam grades for twelve students in Mr. Dylan's algebra class are plotted below.

Based on a line of best fit, which exam grade is the best prediction for a student who spends about 4 hours on math homework each week?

1) 62
2) 72
3) 82
4) 92

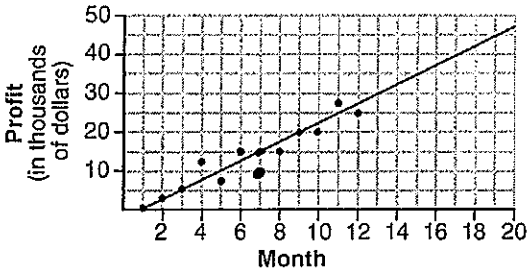


4. The scatter plot below shows the profit, by month, for a new company for the first year of operation. Kate drew a line of best fit, as shown in the diagram.
Approximately how much lower was the actual profit compared to the expected profit after 7 months?

1) $5,000
2) $10,000
3) $15,000
4) $20,000

actual profit = 10,000
expected profit = 15,000

15,000
−10,000
5,000

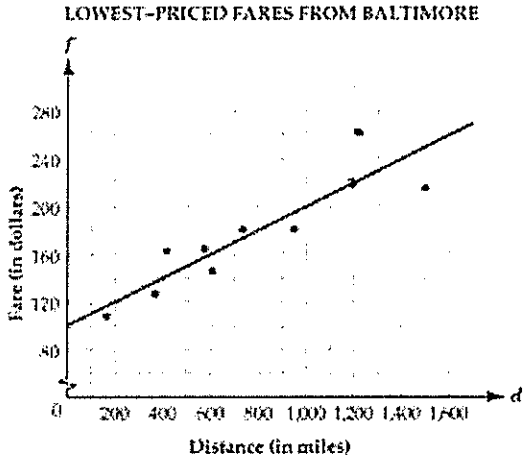


5. The following scatter plot shows the data for train tickets purchased from Baltimore. Approximately how much higher is the actual value for a ticket to travel 1,200 miles than the predicted value?

1) 220
2) 40
3) 260
4) 72

actual value = 260
expected value = 220

260 − 220 = 40

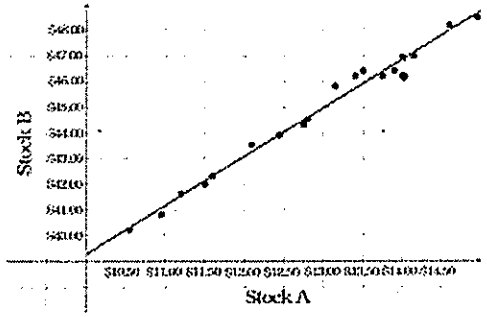


6. The following scatter plot compares the value of two stocks. Approximately how much lower was the actual value compared to the predicted value when Stock A was worth $14.00?

1) $46.03
2) $3.24
3) $0.89
4) $23.04

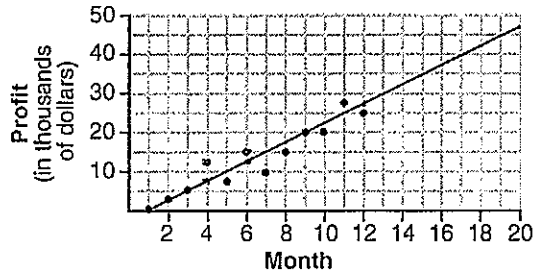
actual value = 46.10
expected value = 47.00

47 − 46.10 = .90



7. The scatter plot below shows the profit, by month, for a new company for the first year of operation. Kate drew a line of best fit, as shown in the diagram. Which of the following months had the greatest difference between actual profit and expected profit?

1) Month 2
2) Month 4
3) Month 6
4) Month 12



Measures of Central Tendency
Stat, Edit
Stat, Calc, 1-Var Stats
*If there is a frequency column, put L_2 into FreqList
$\bar{x}$ = *mean*
σ = *s tan dard deviation*
Med = Median
Range = maximum – minimum
Mode = number that occurs the most (not given in calculator)

1. Sara's test scores in mathematics were 64, 80, 88, 78, 60, 92, 84, 76, 86, 78, 72, and 90. Determine the mean, the median, lower quartile, upper quartile, range, interquartile range, standard deviation, and the mode of Sara's test scores.

$\bar{X}=79$ $Q3=87$ $IQR=Q3-Q1$ $Sx=9.96$ mode=78
Med=79 range=max-min $=87-74$ $\sigma x=9.54$
$Q1=74$ $92-60=32$ $=13$

2. Mickayla's test scores in social studies were 84, 77, 63, 72, 90, 71, 75, 76, 77, 81, 78, and 80. Determine the mean, the median, lower quartile, upper quartile, range, interquartile range, standard deviation, and the mode of Mickayla's test scores.

$\bar{X}=77$ $Q3=80.5$ $IQR=Q3-Q1$ $Sx=6.78$ mode=77
Med=77 range=max-min $=80.5-73.5$ $\sigma x=6.49$
$Q1=73.5$ $90-63=27$ $=7$

3. Which statement is true about the data set 3, 4, 5, 6, 7, 7, 10?
1) mean = mode $6=7$ X
2) mean > mode $6>7$ X
3) mean = median $6=6$ ✓
4) mean < median $6<6$ X

$\bar{X}=6$
Med = 6
mode = 7

4. The following are Regents scores in a math class.

59	56	64	69	55
67	55	57	55	68
64	69	65	71	45

Which of the following statement is true?
1) mean < mode X
2) mode = lower quartile ✓
3) standard deviation = range X
4) interquartile range = standard deviation X

$\bar{X}=61.26$
mode = 55
$Q1=55$
$Sx=7.34$
range = 71 – 45 = 26
$IQR=68-55=13$

5. The two sets of data below represent the number of runs scored by two different youth baseball teams over the course of a season.

Team *A*: 4, 8, 5, 12, 3, 9, 5, 2
Team *B*: 5, 9, 11, 4, 6, 11, 2, 7

A: $\bar{x}=6$, $S_x=3.38$ B: $\bar{x}=6.875$, $S_x=3.27$

Which set of statements about the mean and standard deviation is true?

1) mean *A* < mean *B* ✓
   standard deviation *A* > standard deviation *B* ✓
2) mean *A* > mean *B* ✓
   standard deviation *A* < standard deviation *B* ✗
3) mean *A* < mean *B* ✓
   standard deviation *A* < standard deviation *B* ✗
4) mean *A* > mean *B* ✗
   standard deviation *A* > standard deviation *B* ✓

6. Christopher looked at his quiz scores shown below for the first and second semester of his Algebra class.

Semester 1: 78, 91, 88, 83, 94
Semester 2: 91, 96, 80, 77, 88, 85, 92

S1: $\bar{x}=86.8$, Med = 88, Q3 = 92.5, IQR = 92.5 − 80.5 = 12

S2: $\bar{x}=87$, Med = 88, Q3 = 92, IQR = 92 − 80 = 12

Which statement about Christopher's performance is correct?

1) The interquartile range for semester 1 is greater than the interquartile range for semester 2. ✗
2) The median score for semester 1 is greater than the median score for semester 2. ✗
3) The mean score for semester 2 is greater than the mean score for semester 1. ✓
4) The third quartile for semester 2 is greater than the third quartile for semester 1. ✗

7. Each member of a class reported the number of books he or she read during the first half of the school years. The data is organized in the table below.
What is the mean, median, mode, standard deviation, range, and interquartile range for this set of data? How many students are in the class?

L_1 Books	L_2 Frequency
8	4
7	0
6	2
5	6
4	2
3	7
2	3
1	1

25 total students

Put L_2 in freqlist

$\bar{x}=4.4$
Med = 4
Mode = 3 (the # with the highest frequency)
$S_x=2.06$
$\sigma_x=2.02$
Range = max − min
8 − 1 = 7
IQR = Q3 − Q1
5.5 − 3 = 2.5

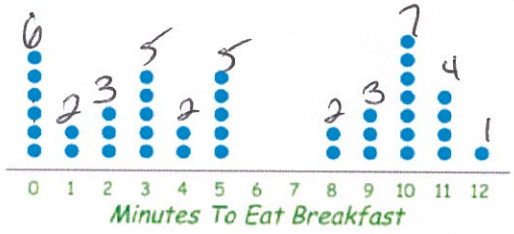
8. In the table, the data indicates the heights, in inches, of basketball players.
What is the mean, median, mode, standard deviation, range, and interquartile range for this set of data? How many students are on this basketball team?

Height (Inches) L1	Frequency L2
77	2
76	0
75	5 mode
74	3
73	4
72	2
71	1

17

median =
mean = 74
mode = 75
standard deviation = 1.7
range = max - min = 77 - 71 = 6
IQR = Q3 - Q1 = 75 - 73 = 2
n = 17 total students

9. The table below represents the time taken, in minutes, to eat breakfast. For this set of data, find the mean, median, mode, standard deviation, range, and interquartile range. How many people were involved in this study?



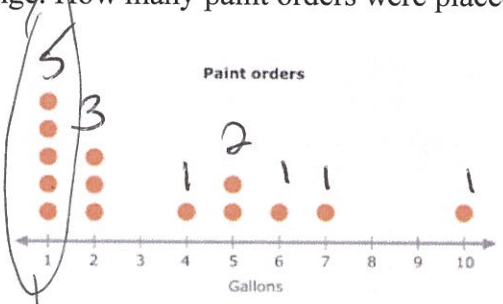
mode

type the frequencies into L2

0	6
1	2
2	3
3	5
4	2
5	5
6	0
7	0
8	2
9	3
10	7
11	4
12	1

mean = 5.625
median = 5
mode = 7
standard deviation = 4.1
range = max - min = 12 - 0 = 12
IQR = Q3 - Q1 = 10 - 2 = 8
n = 40 total ppl

10. The following data represents the number of gallons of paint in a paint order in a given day. For this set of data, find the mean, median, mode, standard deviation, range, and interquartile range. How many paint orders were placed on this day?



mode

1	5
2	3
3	0
4	1
5	2
6	1
7	1
8	0
9	0
10	1

mean = 3.428...
median = 2
mode = 1
standard deviation = 2.8
range = max - min = 10 - 1 = 9
IQR = Q3 - Q1 = 5 - 1 = 4
n = 14 paint order

Mean Problems

$$\frac{total\ points}{number\ of\ data\ points} = mean$$

1. On Jessica's first four math tests, she scored 74, 89, 80, and 82. What would Jessica have to score on her fifth math test to bring her average to an 84?

x = 5th test

$\frac{74+89+80+82+x}{5} = 84$

$\frac{\text{total points}}{\#\text{ of tests}} = \text{mean}$

$5\left(\frac{325+x}{5}\right) = (84)5$

$325 + x = 420$

$-325 \quad -325$

$x = 95$

2. The Giants scored 14 points, 23 points, and 6 points in their first three games of the season. How many points will they have to score in their fourth game to average 18 points per game?

$\frac{\text{total points}}{\#\text{ of games}} = \text{mean}$

x = points in 4th game

$\frac{14+23+6+x}{4} = 18$

$4\left(\frac{43+x}{4}\right) = (18)4$

$43 + x = 72$

$-43 \quad -43$

$x = 29$

29 points

3. The attendance at Mr. Schlansky's tutorials over 7 days was 4, 7, 3, 11, 2, 8, 4. How many students would have to show up to tutorial on the 8th day for the average number of students at tutorial to be 6?

$\frac{\text{total attendance}}{\#\text{ of tutorials}} = \text{mean}$

x = 8th tutorial attendance

$\frac{4+7+3+11+2+8+4+x}{8} = 6$

$8\left(\frac{39+x}{8}\right) = (6)8$

$39 + x = 48$

$-39 \quad -39$

$x = 9$

9 students

4. Jessica is taking 6 classes. Her grades for the first 5 are 92, 79, 83, 86, and 91. What would she need to get in her 6th class for her quarter average to be an 85?

$\frac{\text{total points}}{\#\text{ of classes}} = \text{mean}$

x = # of points in 6th class

$\frac{92+79+83+86+91+x}{6} = 85$

$6\left(\frac{431+x}{6}\right) = (85)6$

$431 + x = 510$

$-431 \quad -431$

$x = 79$

5. The Sabres scored 4, 0, 3, 1, and 2 goals during the first 5 games. How many total goals do they need to score over their next 3 games for their average over their first 8 games to be 3 goals?

$\frac{\text{total goals}}{\#\text{ of games}} = \text{mean}$ $\quad \frac{4+0+3+1+2+X}{8} = 3$ $\quad 10+X=24$, $-10 \quad -10$, $X=14$

$8\left(\frac{10+X}{8}\right)=(3)8$

$5+3$

X = total goals over next 3 games

6. Micah has made a total of \$435 dollars over the past 6 weeks at his part time job. How much money must he make in the 7th week to make an average of \$74 per week?

$\frac{\text{total money}}{\#\text{ of weeks}} = 74$ $\quad 7\left(\frac{435+X}{7}\right)=(74)7$

$435+X=518$, $-435 \quad -435$, $X=83$

X = money made in 7th week (dollars added)

7. The 7 members of a chess club have an average of 6 wins. The team gains a new member and now has an average of 7 wins. How many wins does the new member have?

$\frac{\text{total wins}}{\#\text{ of members}} = \text{mean}$ $\quad$ total wins $= 7(6) = 42$

$8\left(\frac{42+X}{8}\right)=(7)8$ $\quad 42+X=56$, $-42 \quad -42$, $X=14$ wins

X = # of wins of member

8. Phil collected 6 rocks whose weight in ounces was 4.2, 3.6, 4.7, 4.2, 5.3, and 6.2. Maria collected 6 rocks whose weight in ounces was 3.7, 6.1, 3.5, 4.4, 5.9, but she did not know the weight of the 6th rock. The mean of the masses of the rocks that Maria collected is .2 ounces more than Phil's. What must be the weight of Maria's 6th rock?

X = weight of Maria's 6th rock

Phil

$\frac{\text{total weight}}{\#\text{ of rocks}} = \text{mean}$

$\frac{4.2+3.6+4.7+4.2+5.3+6.2}{6} = \text{mean}$

$\frac{28.2}{6} = 4.7$

Maria

$\frac{3.7+6.1+3.5+4.4+5.9+X}{6} = 4.7+.2$

$6\left(\frac{23.6+X}{6}\right)=(4.9)6$

$23.6+X=29.4$, $-23.6 \quad -23.6$, $X=5.8$

$8(18)=144$ sum

9. The mean of 8 numbers is 18. If the highest number is removed, the mean of the remaining 7 numbers is 16. What is the highest number?

$\frac{\text{Sum of numbers}}{\text{total numbers}} = \text{mean}$

x = highest #

$7\left(\frac{144-x}{7}\right)=(16)7$

$144-x=112$

$-144 \quad -144$

$\frac{-x}{-1}=\frac{-32}{-1}$

$x=32$

$10(12)=120$ total points

10. The mean score for 10 players in a basketball game was 12 points. If the highest individual score is removed, the mean score of the remaining 9 players becomes 11 points. What was the highest score?

$\frac{\text{total points}}{\text{\# of players}} = \text{mean}$

x = highest player score

$9\left(\frac{120-x}{9}\right)=(11)9$

$120-x=99$

$-120 \quad -120$

$\frac{-x}{-1}=\frac{-21}{-1}$

$x=21$

11. There are a total of 18 missing homeworks for a class. A new student joins the class who has 2 missing homeworks. If the average amount of missing homeworks after the new student joined is 2, how many students were in the class before the new student joined?

$\frac{\text{total missing homeworks}}{\text{\# of students}} = \text{mean}$

x = # of current students (before the new student)

$x+1$ = # of current students plus the new student

$(x+1)\left(\frac{18+2}{x+1}\right)=(2)x+1$

$20=2x+2$

$-2 \quad -2$

$\frac{18}{2}=\frac{2x}{2}$

$9=x$

9 students

12. The students in a class have a total of 52 absences. When a new student who has 2 absences joins the class, the class average becomes 5.4 absences. How many students were in the class before the new student joined?

$\frac{\text{total absences}}{\text{\# of students}} = \text{mean}$

x = # of students before new student

$x+1$

$\left(\frac{52+2}{x+1}\right)=(5.4)x+1$

$54=5.4(x+1)$

$54=5.4x+5.4$

$-5.4 \quad -5.4$

$\frac{48.6}{5.4}=\frac{5.4x}{5.4}$

$9=x$

9 students

Mixtures/Solutions

$$\frac{\textit{total amount of specific substance}}{\textit{total amount of product}} = \textit{percent / ratio}$$

1. A class has 16 male students and 12 female students. How many more male students would have to enter the class so that $\frac{3}{5}$ of the students in the class would be male?

$\frac{\text{total male}}{\text{total students}} = \frac{3}{5}$ $5(28+x)$ $\left(\frac{16+x}{28+x}\right) = \left(\frac{3}{5}\right)5(28+x)$

$5(16+x) = 3(28+x)$

$80 + 5x = 84 + 3x$

$-3x \quad -3x$

$80 + 2x = 84$

$-80 \quad -80$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

2. A track team has 24 sprinters and 10 distance runners. How many additional distance runners would have to be added onto the team so that the team would have 40% distance runners?

$\frac{\text{total distance runners}}{\text{total athletes}} = .4$ $34+x$ $\left(\frac{10+x}{34+x}\right) = (.4)34+x$

$10 + x = 13.6 + .4x$

$-.4x \quad -.4x$

$10 + .6x = 13.6$

$-10 \quad -10$

$\frac{.6x}{.6} = \frac{3.6}{.6}$

$x = 6$

3. Jessica's wardrobe has 23 tops and 13 bottoms. After she goes shopping for tops, 74% of her wardrobe is tops. How many tops did she buy?

$\frac{\text{total tops}}{\text{total tops and bottoms}} = .74$ $36+x$ $\left(\frac{23+x}{36+x}\right) = (.74)(36+x)$

$23 + x = 26.64 + .74x$

$-.74x \quad -.74x$

$23 + .26x = 26.64$

$-23 \quad -23$

$\frac{.26x}{.26} = \frac{3.64}{.26}$

$x = 14$

.2x .15(4)

4. How many liters of a 20% saline solution must be added to 4 liters of a 15% saline solution to obtain an 18% saline solution?

$\frac{\text{total salt}}{\text{total solution}} = .18$

$(4+x)\left(\frac{.2x+.15(4)}{4+x}\right) = (.18)(4+x)$

$.2x + .6 = .72 + .18x$

$-.18x \quad -.18x$

$.02x + .6 = .72$

$-.6 \quad -.6$

$\frac{.02x}{.02} = \frac{.12}{.02}$

$x = 6$

.03x 12(.06)

5. Meat A contains 3% fat and Meat B contains 6% fat. How many pounds of Meat A would need to be added to 12 pounds of Meat B to produce a meat product consisting of 4% fat?

$\frac{\text{total fat}}{\text{total pounds of meat}} = .04$

$(12+x)\left(\frac{.03x+12(.06)}{12+x}\right) = (.04)(12+x)$

$.03x + .72 = .48 + .04x$

$-.03x \quad -.03x$

$.72 = .48 + .01x$

$-.48 \quad -.48$

$\frac{.24}{.01} = \frac{.01x}{.01}$

$24 = x$

.3x .7(2)

6. Solution A consists of 30% water and Solution B consists of 70% water. How many gallons of Solution A must be added to 2 gallons of Solution B to produce a solution that contains 60% water?

$\frac{\text{total water}}{\text{total solution}} = .6$

$(2+x)\left(\frac{.3x+.7(2)}{2+x}\right) = (.6)(2+x)$

$.3x + 1.4 = 1.2 + .6x$

$-.3x \quad -.3x$

$1.4 = 1.2 + .3x$

$-1.2 \quad -1.2$

$\frac{.2}{.3} = \frac{.3x}{.3}$

$\frac{2}{3} = x$

Data affecting central tendency
If no outlier: mean is best measure of central tendency
If outlier: median is best measure of central tendency
An outlier affects the range the most, the mean the second most, and the median the least.

1. The values of 11 houses on Washington St. are shown in the table below.
State which measure of central tendency, the mean or the median, *best* represents the values of these 11 houses. Justify your answer. If the $700,000 house was removed, would the mean, median, or range be most affected? Which would be least affected?

Value per House	Number of Houses
$100,000	1
$175,000	5
$200,000	4
$700,000	1

The ~~Mean~~ Median is the best because there is an outlier (700,000).

Range is most affected by outlier.
Mean is second most affected
Median is least affected.

2. In the table, the data indicates the heights, in inches, of basketball players.
State which measure of central tendency, the mean or the median, *best* represents the heights of these basketball players. Justify your answer. If a new member of the team joined who was 82 inches, would the mean, median, or range be most affected? Which would be least affected?

Height (Inches)	Frequency
77	2
76	0
75	5
74	3
73	4
72	2
71	1

The mean is the best because there is no outlier.

the range would be most ~~affect~~affected by the outlier
the mean would be second most affected.
the median would be the least affected.

3. The prices of seven race cars sold last week are listed in the table below.
State which of these measures of central tendency, the mean or the median, best represents the value of the seven race cars. Justify your answer. If the $819,000 race car was removed, would the mean, median, or range be most affected? Which would be least affected?

Price per Race Car	Number of Race Cars
$126,000	1
$140,000	2
$180,000	1
$400,000	2
$819,000	1

the median is the best because there is an outlier ($819,000).

the range would be the most affected by the outlier.
the mean would be the second most affected by the outlier.
the median would be the least affected.

4. The scores on a math test were 27, 66, 69, 72, 76, 76, 81, 86, 89, and 91. Would the mean or median be the better choice for representing the data? Explain your answer. If the 27 test score is removed, would the mean, median, or range be most affected? Which would be least affected?

Median because there is an outlier. (27).
The range would be most affected.
The mean would be the 2nd most affected
the median would be the least affected.

5. The table below shows the annual salaries for the 24 members of a professional sports team in terms of millions of dollars.

0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

The team signs an additional player to a contract worth 10 (outlier) million dollars per year. Which represents the order of which measures of central tendency would be most affected?

1) mean, range, median
2) median, range, mean
3) range, mean, median
4) range, median, mean

6. The height of students in a class in inches are 32, 33, 33, 34, 36, 36, 39, 39, 39, 39, 40, 41, 41, 43. A new student joined the class who is 47 inches tall. Which will change the least when incorporating the new student?

1) Mean
2) Median
3) Range
4) They will all change by the same amount

7. A survey was taken of the value of cars in a community, and it was found that the mean car value was $16,000 and the median car value was $13,500. Which of the following situations could explain the difference between the mean and median car values in the community?

1) The cars have values that are close to each other.
2) There are a few cars that are valued much less than the rest.
3) There are a few cars that are valued much more than the rest. → this caused the mean to be substantially higher
4) Many of the cars have values between $13,500 and $16,000.

Interpreting Results from a statistical study

A good sample should be randomly selected where every member of the population has a chance of being chosen.

A good sample should not include bias. For example, don't ask the Football Team if they like Football. Ask every 10th student walking into the building in the morning.

The results of a statistical study approximate the characteristics of the population if an appropriate sample was used. If 30% of the sample like Football, it can be concluded that approximately 30% of the population likes Football.

1. Which statement(s) about statistical studies is true?

I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying. Random/all students take English

II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school. Bias, 9th graders don't represent the entire school and they don't drive

III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites. Bad sample, HS students don't represent adults

IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math. Bias, calculus students are good at math

(1) I, only 2) II, only 3) I and III 4) III and IV

2. Which survey is *least* likely to contain bias?

(1) surveying a sample of people leaving a movie theater to determine which flavor of ice cream is the most popular Random, People at the movies don't necessarily feel a particular way about ice cream

2) surveying the members of a football team to determine the most watched TV sport Bias, they like football.

3) surveying a sample of people leaving a library to determine the average number of books a person reads in a year Bias, People in a library like to read books.

4) surveying a sample of people leaving a gym to determine the average number of hours a person exercises per week Bias, people at a gym like to exercise.

3. A survey is to be conducted in a small upstate village to determine whether or not local residents should fund construction of a skateboard park by raising taxes. Which segment of the population would provide the most unbiased responses?

1) a club of local skateboard enthusiasts bias, skateboarders want a skateboard park

2) senior citizens living on fixed incomes bias, elderly people don't want to spend money for a park they're not using.

3) a group opposed to any increase in taxes bias, they're all saying no.

(4) every tenth person 18 years of age or older walking down Main St. random

4. A researcher selected 500 people at random from a group of people who indicated that they liked a certain type of music. The 500 people were shown a music video and then asked whether they liked the music video. Of those surveyed, 95% liked the music video. Which of the following inferences can appropriately be drawn from this survey result?

1) It is expected that people who like this type of music will like the music video. 95% of the people who like this music like the video.
2) It is expected that people who like this music video will like this type of music.
3) 95% of all people will this music video. X
4) 95% of people who like music videos will like this music video. X

5. A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is a valid conclusion based on the data?

I: There is a 48% chance that Mayor Ortega will win the election. X
II: The estimate of voters who will vote for Mayor Ortega is 48%. ✓ $\frac{264}{550} = .48$

1) I only
2) II only
3) I and II
4) Neither I nor II

If 48% of the sample will vote for him, approximately 48% of the population will vote for him.

6. In order to determine if a drug is successful in healing headaches, a research study was conducted. From a large population of people with a history of headaches, 400 participants were selected at random. Half of the participants were randomly assigned to receive the drug while the other half received a placebo. The resulting data showed that participants who received the drug had their headaches significantly improved. Based on the design and results of the study, which of the following is an appropriate conclusion?

1) The drug is likely to improve headaches for people who suffer from headaches.
2) The drug is the best cure for people who suffer from headaches. X
3) The drug will improve headache pain for anyone who takes it. X
4) The drug will cause a substantial improvement for headache pain. X

7. A study was done on the weights of tomatoes on a farm. A random sample of tomatoes were chosen and weighed. 40% of the sample of tomatoes weighed more than 6 ounces. Which of the following conclusions is best supported by the sample data?

1) The majority of all of the tomatoes on the farm weigh less than 6 ounces.
2) The average weight of all of the tomatoes is approximately 6 ounces.
3) Approximately 40% of all of the tomatoes on the farm weigh more than 6 ounces.
4) Approximately 40% of all tomatoes weigh more than 6 ounces. the sample was tomatoes on the farm, not all tomatoes.

8. To determine the mean number of pets per household in a community, Chidi randomly surveyed 25 families at a local dog park. For the 25 families surveyed, the mean number of pets per household was 2.8. Which of the following statements must be true?

1) The mean number of pets per household in this community is 2.4.
2) The mean number of pets per household in this community is approximately 2.4.
3) The sampling method is flawed and may produce a biased estimate of the mean number of pets per household in the community. Bias because people at a dog park all have dogs!
4) The sampling method is not flawed because the sample is chosen randomly.

9. The members of student council wanted to assess the opinions of all students about converting a classroom into a video game room for students to use during their free periods. The student council surveyed a random sample of the members of the Video Game Club. The survey showed that the majority of those sampled were in favor of the video game room. Which of the following is true about the student council's survey?

1) It shows that the majority of students are in favor of the video game room.
2) The survey sample should have included more students who play video games.
3) The survey sample should not have included any members of the video game club.
4) The survey sample is biased because it is not representative of all students.

10. A survey was conducted in a high school and it was found that 90% of the sample of students use SnapChat. Which of the following would be a valid conclusion to draw?

1) 90% of the students in the high school use SnapChat. approximately 90, not 90.
2) Approximately 27 students in a randomly chosen English class of 30 use SnapChat. $\frac{27}{30} = .9$
3) 9 out of the first 10 students that walk in the building in the morning use SnapChat.
4) Approximately 90% of SnapChat users are high school students.

Absolute Value Equations

1) Isolate the absolute value bars
2) Create two equations:
   -1st equation: drop the absolute value bars
   2nd equation: drop the absolute value bars and negate the right hand side (the side not with the absolute value)
3) Check!

*Evaluating absolute values always produces a value that is 0 or positive. There will be no solution if the absolute value would need to be negative.

1. $|2k| = 7$

$\frac{2k}{2} = \frac{7}{2}$ $\quad \frac{2k}{2} = \frac{-7}{2}$

$k = 3.5$ $\quad k = -3.5$ ✓ ✓

2. $|x| + 3 = 4$

$-3 \quad -3$

$|x| = 1$

$x = 1$ $\quad x = -1$ ✓ ✓

3. $4|2x+3| - 3 = 17$

$+3 \quad +3$

$\frac{4|2x+3|}{4} = \frac{20}{4}$

$|2x+3| = 5$

$2x+3 = 5$ $\quad 2x+3 = -5$

$-3 \quad -3 \quad -3 \quad -3$

$\frac{2x}{2} = \frac{2}{2}$ $\quad \frac{2x}{2} = \frac{-8}{2}$

$x = 1$ ✓ $\quad x = -4$ ✓

4. $2|4x-1| - 5 = 25$

$+5 \quad +5$

$\frac{2|4x-1|}{2} = \frac{30}{2}$

$|4x-1| = 15$

$4x-1 = 15$ $\quad 4x-1 = -15$

$+1 \quad +1 \quad +1 \quad +1$

$\frac{4x}{4} = \frac{16}{4}$ $\quad \frac{4x}{4} = \frac{-14}{4}$

$x = 4$ ✓ $\quad x = -3.5$ ✓

5. $2|k+4| + 4 = 6$

$-4 \quad -4$

$\frac{2|k+4|}{2} = \frac{2}{2}$

$|k+4| = 1$

$k+4 = 1$ $\quad k+4 = -1$

$-4 \quad -4 \quad -4 \quad -4$

$k = -3$ ✓ $\quad k = -5$ ✓

6. $8|y-3| - 5 = 11$

$+5 \quad +5$

$\frac{8|y-3|}{8} = \frac{16}{8}$

$|y-3| = 2$

$y-3 = 2$ $\quad y-3 = -2$

$+3 \quad +3 \quad +3 \quad +3$

$y = 5$ ✓ $\quad y = 1$ ✓

once the bars are isolated, they can't equal a negative

7. Which of the following equations has no solution?
1) $|x+2|=5$
2) $|x-3|=4$
3) $|x+3|=-4$ → the right hand side is negative
4) $|x-5|=3$

8. Which of the following equations has a solution?
1) $|x+7|=-1$
2) $|x-7|=3$ → the right hand side is not negative
3) $|x-8.5|=-8$
4) $|x-5|=-5$

9. Which of the following equations has no solution?
1) $|x| = 7$
2) $|-x| = 7$
3) $|x| = -7$ → the right hand side is negative
4) $|x-7| = 0$

10. Which of the following equations has a solution?
1) $|x+a|=-2$
2) $|x-a|=-2$
3) $|x+a|=-4$
4) $|x-a|=4$ → the right hand side is positive

11. Which of the following equations has no solution?
1) $|x+a|+4=3$ $\quad |x+a|=-1$ → right hand side is negative
2) $|x-a|+2=3$ $\quad |x-a|=1$
3) $|x+a|-5=-4$ $\quad |x+a|=1$
4) $|x-a|+3=9$ $\quad |x-a|=6$

12. Which of the following expressions is equal to 0 for some value of x?
1) $|x+2|+3$
2) $|2-x|+5$
3) $|x+5|-2$ → the bars must equal positive or 0, so you can't add a positive to a positive/zero to equal 0.
4) $|5-x|+1$

Radical Equations

1) Isolate the radical
2) Square both sides
3) Solve the equation
4) Check!

1. $\sqrt{x-4}=6$

$(\sqrt{x-4})^2=(6)^2$

$x-4=36$

$+4 \quad +4$

$x=40$ ✓

2. $\sqrt{6x-2}=4$

$(\sqrt{6x-2})^2=(4)^2$

$6x-2=16$

$+2 \quad +2$

$\frac{6x}{6}=\frac{18}{6}$

$x=3$ ✓

3. $\sqrt{2x-1}=3$

$(\sqrt{2x-1})^2=(3)^2$

$2x-1=9$

$+1 \quad +1$

$\frac{2x}{2}=\frac{10}{2}$

$x=5$ ✓

4. $\sqrt{5x-1}=8$

$(\sqrt{5x-1})^2=(8)^2$

$5x-1=64$

$+1 \quad +1$

$\frac{5x}{5}=\frac{65}{5}$

$x=13$ ✓

5. $5\sqrt{4x-8}+2=12$

$-2 \quad -2$

$\frac{5\sqrt{4x-8}}{5}=\frac{10}{5}$

$(\sqrt{4x-8})^2=(2)^2$

$4x-8=4$

$+8 \quad +8$

$\frac{4x}{4}=\frac{12}{4}$

$x=3$ ✓

6. $2\sqrt{2x+3}+1=11$

$-1 \quad -1$

$\frac{2\sqrt{2x+3}}{2}=\frac{10}{2}$

$(\sqrt{2x+3})^2=(5)^2$

$2x+3=25$

$-3 \quad -3$

$\frac{2x}{2}=\frac{22}{2}$

$x=11$ ✓

7. $3\sqrt{3x+1}-8=-5$

$+8 \quad +8$

$\frac{3\sqrt{3x+1}}{3}=\frac{3}{3}$

$(\sqrt{3x+1})^2=(1)^2$

$3x+1=1$

$-1 \quad -1$

$\frac{3x}{3}=\frac{0}{3}$

$x=0$ ✓

8. $4\sqrt{2x+3}-1=19$

$+1 \quad +1$

$\frac{4\sqrt{2x+3}}{4}=\frac{20}{4}$

$(\sqrt{2x+3})^2=(5)^2$

$2x+3=25$

$-3 \quad -3$

$\frac{2x}{2}=\frac{22}{2}$

$x=11$ ✓

9. $(x)^2=(\sqrt{7x-12})^2$

$x^2=7x-12$

$-7x+12 \quad -7x+12$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$x-4=0 \quad | \quad x-3=0$

$+4 \quad +4 \quad +3 \quad +3$

$x=4 \quad x=3$ ✓

10. $(x)^2=(\sqrt{5x+14})^2$

$x^2=5x+14$

$-5x-14 \quad -5x-14$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x-7=0 \quad | \quad x+2=0$

$+7 \quad +7 \quad -2 \quad -2$

$x=7 \quad x=-2$ ✓

11. $(x)^2=(\sqrt{6x-8})^2$

$x^2=6x-8$

$-6x+8 \quad -6x+8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x-4=0 \quad | \quad x-2=0$

$+4 \quad +4 \quad +2 \quad +2$

$x=4 \quad x=2$ ✓

12. $(x)^2=(\sqrt{10x+24})^2$

$x^2=10x+24$

$-10x-24 \quad -10x-24$

$x^2-10x-24=0$

$(x-12)(x+2)=0$

$x-12=0 \quad | \quad x+2=0$

$+12 \quad +12 \quad -2 \quad -2$

$x=12$ ✓ $\quad x=-2$

Finding a Common Exponential Base
Multiplication/Division:
Find a common exponential base
Convert each to the common exponential base (multiply when raising a power to a power)
Add exponents (multiplication) or subtract exponents (division)

Variable Exponential Equations with a Common Base

1) Find a common exponential base
2) Convert each side to the common base (multiply when raising a power to a power)
3) Solve equation

Express each of the following as a single term with an exponent:

1. $2^3 \cdot 2^4$

2^7

2. $\frac{3^8}{3^6}$

3^2

3. $5^4 \cdot 125^2$

$5^4 \cdot (5^3)^2$

$5^4 \cdot 5^6$

5^{10}

4. $\frac{49^3}{7^2}$

$\frac{(7^2)^3}{7^2} = \frac{7^6}{7^2} = 7^4$

5. $4^3 \cdot 8^2$

$(2^2)^3 \cdot (2^3)^2$

$2^6 \cdot 2^6 = 2^{12}$

6. $\frac{81^2}{27^2}$

$\frac{(3^4)^2}{(3^3)^2}$ $\frac{3^8}{3^6} = 3^2$

7. $8^{x+1} \cdot 16^{x-2}$

$(2^3)^{x+1} \cdot (2^4)^{x-2}$

$2^{3x+3} \cdot 2^{4x-8}$

2^{7x-5}

8. $\frac{100^{3x-1}}{10^{2x}}$

$\frac{(10^2)^{3x-1}}{10^{2x}}$

$\frac{10^{6x-2}}{10^{2x}} = 10^{4x-2}$

9. $32^{2x} \cdot 16^{2x-1}$

$(2^5)^{2x} \cdot (2^4)^{2x-1}$

$2^{10x} \cdot 2^{8x-4}$

2^{18x-4}

10. $\frac{27^{4x}}{9^{x-2}}$

$\frac{(3^3)^{4x}}{(3^2)^{x-2}} = \frac{3^{12x}}{3^{2x-4}}$

$3^{12x-(12x-4)} = 3^{12x-2x+4} = 3^{10x+4}$

Solve each of the following equations for x:

11. $2^4 = 16^x$

$2^4 = (2^4)^x$

$2^4 = 2^{4x}$

$\frac{4}{4} = \frac{4x}{4}$

$1 = x$

12. $9^{3x} = 3^{3x+1}$

$(3^2)^{3x} = 3^{3x+1}$

$3^{6x} = 3^{3x+1}$

$6x = 3x + 1$

$-3x \quad -3x$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

13. $5^{x+1} = 125^{2x}$

$5^{x+1} = (5^3)^{2x}$

$5^{x+1} = 5^{6x}$

$x + 1 = 6x$

$-x \quad -x$

$\frac{1}{5} = \frac{5x}{5}$

$\frac{1}{5} = x$

14. $27^x = 9^{x+2}$

$(3^3)^x = (3^2)^{x+2}$

$3^{3x} = 3^{4x+4}$

$3x = 4x + 4$

$-3x \quad -3x$

$0 = x + 4$

$-4 \quad -4$

$-4 = x$

15. $4^{2b-3} = 8^{1-b}$

$(2^2)^{2b-3} = (2^3)^{1-b}$

$2^{4b-6} = 2^{3-3b}$

$4b - 6 = 3 - 3b$

$+3b \quad +3b$

$7b - 6 = 3$

$+6 \quad +6$

$\frac{7b}{7} = \frac{9}{7}$

$b = \frac{9}{7}$

16. $64^{x-2} = 256^{2x}$

$(2^6)^{x-2} = (2^8)^{2x}$

$2^{6x-12} = 2^{16x}$

$6x - 12 = 16x$

$-6x \quad -6x$

$\frac{-12}{10} = \frac{10x}{10}$

$-\frac{6}{5} = x$

Completing the Square (Solving Quadratic Equations)

Completing the Square

1) Write the x's together and move constant to the other side

$ax^2 + bx = c$

*Divide away the a value if it is not 1

2) Add $\left(\frac{b}{2}\right)^2$ to both sides

3) Factor the trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

To solve quadratic equations using completing the square:

5) Take the square root of both sides (don't forget $\pm$)

6) Isolate x

*Reduce the radical (if necessary)

1. $x^2 + 4x - 2 = 0$

$\left(\frac{4}{2}\right)^2 = 4$

$+2 \quad +2$

$x^2 + 4x = 2$

$x^2 + 4x + 4 = 2 + 4$

$\sqrt{(x+2)^2} = \sqrt{6}$

$x + 2 = \pm\sqrt{6}$

$-2 \quad -2$

$x = -2 \pm \sqrt{6}$

2. $x^2 - 8x + 4 = 0$

$\left(\frac{-8}{2}\right)^2 = 16$

$-4 \quad -4$

$x^2 - 8x = -4$

$x^2 - 8x + 16 = -4 + 16$

$\sqrt{(x-4)^2} = \sqrt{12}$

$x - 4 = \pm\sqrt{12}$

$+4 \quad +4$

$x = 4 \pm \sqrt{12}$

$\sqrt{4}\sqrt{3}$

$x = 4 \pm 2\sqrt{3}$

3. $x^2 - 6x + 3 = 0$

$\left(\frac{-6}{2}\right)^2 = 9$

$-3 \quad -3$

$x^2 - 6x = -3$

$x^2 - 6x + 9 = -3 + 9$

$\sqrt{(x-3)^2} = \sqrt{6}$

$x - 3 = \pm\sqrt{6}$

$+3 \quad +3$

$x = 3 \pm \sqrt{6}$

4. $x^2 + 10x + 2 = 0$

$\left(\frac{10}{2}\right)^2 = 25$

$-2 \quad -2$

$x^2 + 10x = -2$

$x^2 + 10x + 25 = -2 + 25$

$\sqrt{(x+5)^2} = \sqrt{23}$

$x + 5 = \pm\sqrt{23}$

$-5 \quad -5$

$x = -5 \pm \sqrt{23}$

5. $x^2 - 8x + 1 = 0$

$\left(\frac{-8}{2}\right)^2 = 16$

$-1 \quad -1$

$x^2 - 8x = -1$

$x^2 - 8x + 16 = -1 + 16$

$\sqrt{(x-4)^2} = \sqrt{15}$

$x - 4 = \pm\sqrt{15}$

$+4 \quad +4$

$x = 4 \pm \sqrt{15}$

6. $x^2 + 2x - 2 = 0$

$\left(\frac{2}{2}\right)^2 = 1$

$+2 \quad +2$

$x^2 + 2x = 2$

$x^2 + 2x + 1 = 2 + 1$

$\sqrt{(x+1)^2} = \sqrt{3}$

$x + 1 = \pm\sqrt{3}$

$-1 \quad -1$

$x = -1 \pm \sqrt{3}$

7. $\sqrt{(2x-3)^2} = \sqrt{16}$

$2x-3 = \pm 4$

$2x-3=4$ $\quad +3 \;+3$

$\frac{2x}{2} = \frac{7}{2}$

$x = 3.5$

$2x-3=-4$ $\quad +3 \;+3$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -.5$

8. $\sqrt{(3x+1)^2} = \sqrt{4}$

$3x+1 = \pm 2$

$3x+1=2$ $\quad -1 \;-1$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

$3x+1=-2$ $\quad -1 \;-1$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

9. $\sqrt{(4x-5)^2} = \sqrt{9}$

$4x-5 = \pm 3$

$4x-5=3$ $\quad +5 \;+5$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

$4x-5=-3$ $\quad +5 \;+5$

$\frac{4x}{4} = \frac{2}{4}$

$x = \frac{1}{2}$

10. $\sqrt{(6x-1)^2} = \sqrt{1}$

$6x-1 = \pm 1$

$6x-1=1$ $\quad +1 \;+1$

$\frac{6x}{6} = \frac{2}{6}$

$x = 1/3$

$6x-1=-1$ $\quad +1 \;+1$

$\frac{6x}{6} = \frac{0}{6}$

$x = 0$

11. $\sqrt{(5x-3)^2} = \sqrt{2}$

$5x-3 = \pm\sqrt{2}$

$5x-3=\sqrt{2}$ $\quad +3 \;+3$

$\frac{5x}{5} = \frac{3+\sqrt{2}}{5}$

$x = \frac{3+\sqrt{2}}{5}$

$5x-3=-\sqrt{2}$ $\quad +3 \;+3$

$\frac{5x}{5} = \frac{3-\sqrt{2}}{5}$

$x = \frac{3-\sqrt{2}}{5}$

12. $\sqrt{(8x-2)^2} = \sqrt{3}$

$8x-2 = \pm\sqrt{3}$

$8x-2=\sqrt{3}$ $\quad +2 \;+2$

$\frac{8x}{8} = \frac{2+\sqrt{3}}{8}$

$x = \frac{2+\sqrt{3}}{8}$

$8x-2=-\sqrt{3}$ $\quad +2 \;+2$

$\frac{8x}{8} = \frac{2-\sqrt{3}}{8}$

$x = \frac{2-\sqrt{3}}{8}$

13. $\frac{2}{x-1} = x-1$ $\quad (x-1)$

$\sqrt{2} = \sqrt{(x-1)^2}$

$\pm\sqrt{2} = x-1$ $\quad +1 \;+1$

$1 \pm \sqrt{2} = x$

14. $\frac{5}{x-3} = \frac{x-3}{2}$ $\quad 2(x-3)$

$\sqrt{10} = \sqrt{(x-3)^2}$

$\pm\sqrt{10} = x-3$ $\quad +3 \;+3$

$3 \pm \sqrt{10} = x$

Completing the Square (Vertex Form)
Completing the Square

1) Write the x's together and move constant to the other side

$$y+c=ax^2+bx$$

*Divide away the a value if it is not 1

2) Add $\left(\frac{b}{2}\right)^2$ to both sides

3) Factor the trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

Vertex Form

$y=a(x-v)^2+t$ where (v,t) is the vertex

After you complete the square:

-Isolate y

-Negate what's in the parenthesis for the x coordinate, keep what's outside the parenthesis as the y coordinate.

Put each of the following equations into vertex form and state the vertex

1. $f(x)=x^2+6x+2$ $\left(\frac{6}{2}\right)^2=9$

$-2 \quad -2$

$f(x)-2=x^2+6x$

$f(x)-2+\boxed{9}=x^2+6x+\boxed{9}$

$f(x)+7=(x+3)^2$

$-7 \quad -7$

$f(x)=(x+3)^2-7$

$(-3,-7)$

2. $f(x)=x^2-8x+3$ $\left(\frac{-8}{2}\right)^2=16$

$-3 \quad -3$

$f(x)-3=x^2-8x$

$f(x)-3+\boxed{16}=x^2-8x+\boxed{16}$

$f(x)+13=(x-4)^2$

$-13 \quad -13$

$f(x)=(x-4)^2-13$

$(4,-13)$

3. $f(x)=x^2+12x+30$

$-30 \quad -30$

$f(x)-30=x^2+12x$

$f(x)-30+\boxed{36}=x^2+12x+\boxed{36}$

$f(x)+6=(x+6)^2$

$-6 \quad -6$

$f(x)=(x+6)^2-6$

$(-6,-6)$

4. $y=x^2+2x-5$

$+5 \quad +5$

$y+5=x^2+2x$

$y+5+\boxed{1}=x^2+2x+\boxed{1}$

$y+6=(x+1)^2$

$-6 \quad -6$

$y=(x+1)^2-6$

$(-1,-6)$

5. $f(x) = x^2 + 12x + 2$

$\left(\frac{12}{2}\right)^2 = 36$

$f(x) - 2 = x^2 + 12x$

$f(x) - 2 + \boxed{36} = x^2 + 12x + \boxed{36}$

$f(x) + 34 = (x+6)^2$

$f(x) = (x+6)^2 - 34$

$(-6, -34)$

6. $f(x) = x^2 + 8x + 5$

$\left(\frac{8}{2}\right)^2 = 16$

$f(x) - 5 = x^2 + 8x$

$f(x) - 5 + \boxed{16} = x^2 + 8x + \boxed{16}$

$f(x) + 11 = (x+4)^2$

$f(x) = (x+4)^2 - 11$

$(-4, -11)$

7. $\frac{f(x)}{2} = \frac{2x^2 + 8x + 16}{2}$

$\left(\frac{4}{2}\right)^2 = 4$

$\frac{f(x)}{2} = x^2 + 4x + 8$

$\frac{f(x)}{2} - 8 + \boxed{4} = x^2 + 4x + \boxed{4}$

$\frac{f(x)}{2} - 4 = (x+2)^2$

$f(x) = 2(x+2)^2 + 8$

$(-2, 8)$

8. $\frac{f(x)}{-1} = \frac{-x^2 + 4x + 16}{-1}$

$f(x) = x^2 - 4x - 16$

$\frac{f(x)}{-1} + 16 = x^2 - 4x$

$\frac{f(x)}{-1} + 16 + \boxed{4} = x^2 - 4x + \boxed{4}$

$\frac{f(x)}{-1} + 20 = (x-2)^2$

$-1\left(\frac{f(x)}{-1}\right) = -1\left((x-2)^2 - 20\right)$

$f(x) = -1(x-2)^2 + 20$

$(2, 20)$

9. $\frac{f(x)}{-1} = \frac{-x^2 + 14x + 20}{-1}$

$\left(\frac{-14}{2}\right)^2 = 49$

$\frac{f(x)}{-1} = x^2 - 14x - 20$

$\frac{f(x)}{-1} + 20 = x^2 - 14x$

$\frac{f(x)}{-1} + 20 + \boxed{49} = x^2 - 14x + \boxed{49}$

$\frac{f(x)}{-1} + 69 = (x-7)^2$

$-1\left(\frac{f(x)}{-1}\right) = -1\left((x-7)^2 - 69\right)$

$f(x) = -1(x-7)^2 + 69$

$(7, 69)$

10. $\frac{f(x)}{4} = \frac{4x^2 + 12x - 28}{4}$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\frac{f(x)}{4} = x^2 + 3x - 7$

$\frac{f(x)}{4} + 7 = x^2 + 3x$

$\frac{f(x)}{4} + 7 + \boxed{\frac{9}{4}} = \left(x + \frac{3}{2}\right)^2$

$\frac{f(x)}{4} + \frac{37}{4} = \left(x + \frac{3}{2}\right)^2$

$4\left(\frac{f(x)}{4}\right) = 4\left(\left(x + \frac{3}{2}\right)^2 - \frac{37}{4}\right)$

$f(x) = 4\left(x + \frac{3}{2}\right)^2 - 37$

$\left(-\frac{3}{2}, -37\right)$

Completing the Square (Center-Radius Form)
Completing the Square

1) Write the x's together, y's together, and move constant to the other side

$$x^2+bx+y^2+by=c$$

2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable

3) Factor each trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

To solve quadratic equations using completing the square:

5) Take the square root of both sides (don't forget $\pm$)

6) Isolate x

*Reduce the radical (if necessary)

Center and Radius Form Using Completing the Square

COMPLETE THE SQUARE TWICE

$(x-a)^2+(y-b)^2=r^2$ where (a,b) is the center and r is the radius

To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.

Radius is the square root of the right hand side

1. What are the center and the radius of the circle whose equation is $(x-3)^2+(y+3)^2=36$

(1) center = (3, −3); radius = 6

2) center = (−3, 3); radius = 6

3) center = (3, −3); radius = 36

4) center = (−3, 3); radius = 36

2. The equation of a circle is $x^2+(y-7)^2=16$. What are the center and radius of the circle?

(1) center = (0, 7); radius = 4

2) center = (0, 7); radius = 16

3) center = (0, −7); radius = 4

4) center = (0, −7); radius = 16

3. What are the center and the radius of the circle whose equation is $(x-5)^2+(y+3)^2=16$?

1) (−5, 3) and 16

2) (5, −3) and 16

3) (−5, 3) and 4

(4) (5, −3) and 4

4. A circle is represented by the equation $x^2+(y+3)^2=13$. What are the coordinates of the center of the circle and the length of the radius?

1) (0, 3) and 13

2) (0, 3) and $\sqrt{13}$

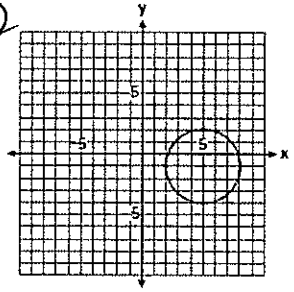
3) (0, −3) and 13

(4) (0, −3) and $\sqrt{13}$

5. Which graph represents a circle with the equation $(x-5)^2+(y+1)^2=9$?

(5, -1) r = 3

(1)



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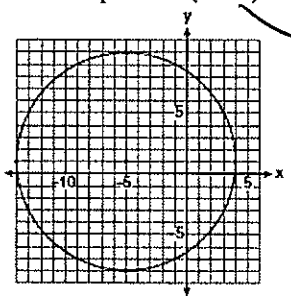

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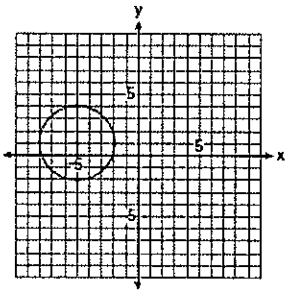

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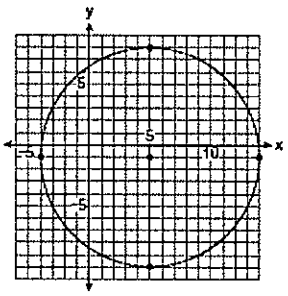
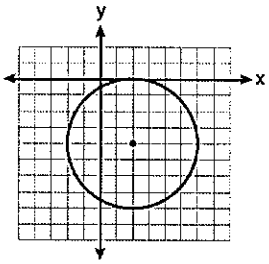


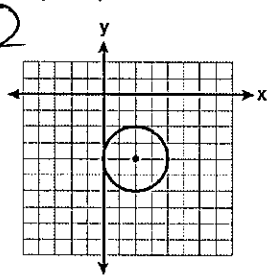
6. The equation of a circle is $(x-2)^2+(y+4)^2=4$. Which diagram is the graph of the circle?

(2, -4) r = 2

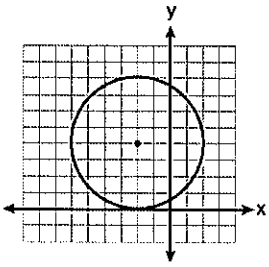
(1)



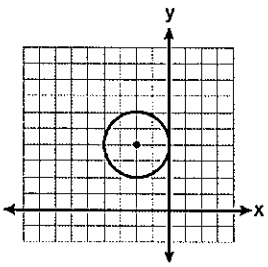
(3)



(2)



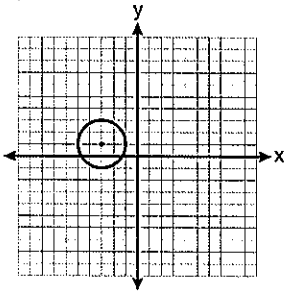
(4)



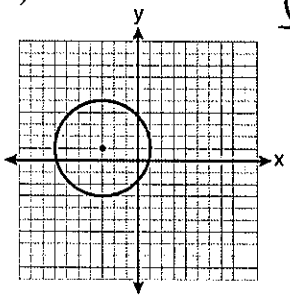
(3, -1) r = 2

7. Which graph represents a circle with the equation $(x-3)^2+(y+1)^2=4$?

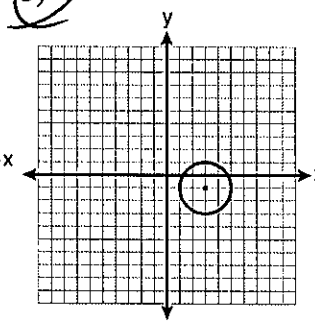
1)



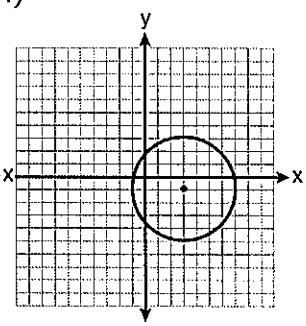
2)



3)

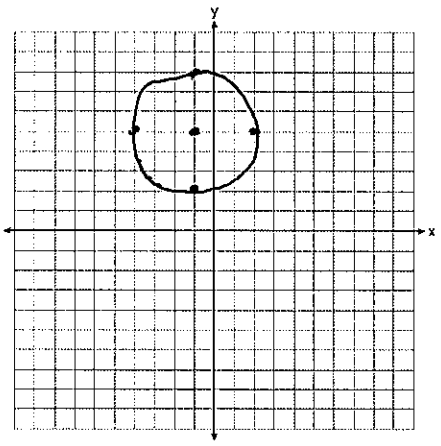


4)

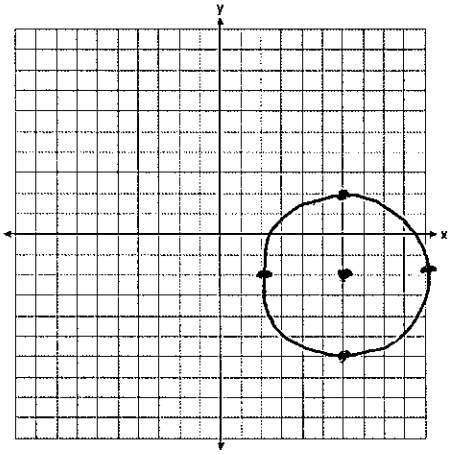


Graph the following circles on the axes provided

8. $(x+1)^2+(y-5)^2=9$ (−1, 5) r = 3



9. $(x-6)^2+(y+2)^2=16$ (6, −2) r = 4



10. Find the center and radius of a circle whose equation is $x^2+y^2-16x+6y+53=0$?

$\left(\frac{-16}{2}\right)^2=64$ $\left(\frac{6}{2}\right)^2=9$

$x^2-16x+y^2+6y=-53$

$-53\ -53$

$x^2-16x+64+y^2+6y+9=-53+64+9$

$(x-8)^2+(y+3)^2=20$

center: (8, −3)

$r=\sqrt{20}$

11. The equation of a circle is $x^2+y^2+6y=7$. What are the coordinates of the center and the length of the radius of the circle?

$\left(\frac{6}{2}\right)^2=9$

1) center (0,3) and radius 4
2) center (0,−3) and radius 4
3) center (0,3) and radius 16
4) center (0,−3) and radius 16

$x^2+y^2+6y+9=7+9$

$x^2+(y+3)^2=16$

(0, −3) r = 4

12. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2+6x+y^2-4y=23$?

1) (3,−2) and 36
2) (3,−2) and 6
3) (−3, 2) and 36
4) (−3, 2) and 6

$x^2+6x+9+y^2-4y+4=23+9+4$

$(x+3)^2+(y-2)^2=36$

$\left(\frac{6}{2}\right)^2=9$ $\left(\frac{-4}{2}\right)^2=4$

(−3, 2)

r = 6

$\left(\frac{-8}{2}\right)^2=16$ $\left(\frac{6}{2}\right)^2=9$

13. What are the coordinates of the center and the length of the radius of the circle whose equation is $x^2+y^2=8x-6y+39$?

1) center $(-4,3)$ and radius 64
2) center $(4,-3)$ and radius 64
3) center $(-4,3)$ and radius 8
4) center $(4,-3)$ and radius 8

$x^2-8x+\boxed{16}+y^2+6y+\boxed{9}=39+\boxed{16}+\boxed{9}$

$(x-4)^2+(y+3)^2=64$

$(4,-3)$ $r=8$

14. $x^2+y^2+16x+6y+69=0$

$\left(\frac{16}{2}\right)^2=64$ $\left(\frac{6}{2}\right)^2=9$

$x^2+16x+\boxed{64}+y^2+6y+\boxed{9}=-69+\boxed{64}+\boxed{9}$

$(x+8)^2+(y+3)^2=4$

$(-8,-3)$ $r=2$

15. $x^2+y^2-12x-14y=-76$

$\left(\frac{-12}{2}\right)^2=36$ $\left(\frac{-14}{2}\right)^2=49$

$x^2-12x+\boxed{36}+y^2-14y+\boxed{49}=-76+\boxed{36}+\boxed{49}$

$(x-6)^2+(y-7)^2=9$

$(6,7)$ $r=3$