Explicit: arithmetic $a_n = a_1 + (n-1)(d)$; geometric $a_n = a_1(r)^{n-1}$

Recursive: $a_1 =$; $a_n = a_{n-1}$

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Algebra II

Sequence/Series Review Sheet

Write an equation for each of the following sequences explicitly and recursively

1. 329.6, 376.8, 424, 471.2,…

$d = 376.8 - 329.6 = 47.2$

$d = 424 - 376.8 = 47.2$

$a_1 = 329.6$

$d = 47.2$

Explicit: $a_n = 329.6 + (n-1)(47.2)$

Recursive: $a_1 = 329.6$; $a_n = a_{n-1} + 47.2$

2. 120, 192, 307.2, 491.52

$r = \frac{192}{120} = 1.6$

$r = \frac{307.2}{192} = 1.6$

$a_1 = 120$

$r = 1.6$

Explicit: $a_n = 120(1.6)^{n-1}$

Recursive: $a_1 = 120$; $a_n = 1.6a_{n-1}$

3. 5400, 4050, 3037.5, 2278.125

$r = \frac{4050}{5400} = .75$

$r = \frac{3037.5}{4050} = .75$

$a_1 = 5400$

$r = .75$

Explicit: $a_n = 5400(.75)^{n-1}$

Recursive: $a_1 = 5400$; $a_n = .75a_{n-1}$

4. 5205.20, 4208.15, 3211.1, 2214.05

$d = 4208.15 - 5205.20 = -997.05$

$d = 3211.1 - 4208.15 = -997.05$

$a_1 = 5205.20$

$d = -997.05$

Recursive: $a_1 = 5205.20$; $a_n = a_{n-1} - 997.05$

Explicit: $a_n = 5205.20 + (n-1)(-997.05)$

5. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_2 = 3(9) - 4$

$a_2 = 23$

$a_3 = 2(23) - 4$

$a_3 = 65$

$a_4 = 3(65) - 4$

$a_4 = 191$

$a_5 = 3(191) - 4$

$a_5 = 569$

6. Find the 8th term for the sequence where $a_n = 5a_{n-1} + 2$ where $a_5 = 3$

$a_6 = 5(3) + 2$

$a_6 = 17$

$a_7 = 5(17) + 2$

$a_7 = 87$

$a_8 = 5(87) + 2$

$a_8 = 437$

1-.08=.92

7. The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of $75,000, which model is a recursive formula representing the value of the boat n years after it was purchased?

1) $a_n = 75,000(0.08)^n$

2) $a_0 = 75,000$

$a_n = (0.92)^n$

3) $a_n = 75,000(1.08)^n$

4) $a_0 = 75,000$

$a_n = 0.92(a_{n-1})$

1+.035=1.035

8. An initial investment of $5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after n years?

1) $A = 5000(0.035)^n$

2) $a_0 = 5000$

$a_n = a_{n-1}(0.035)$

3) $A = 5000(1.035)^n$

4) $a_0 = 5000$

$a_n = a_{n-1}(1.035)$

9. Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

$a_1 = 6$

$a_n = 1.5a_{n-1}$

$r = \frac{9}{6} = 1.5$

$r = \frac{13.5}{9} = 1.5$

10. Write a recursive formula for the sequence 189, 63, 21, 7, ...

$a_1 = 189$

$a_n = \frac{1}{3}a_{n-1}$

$r = \frac{63}{189} = \frac{1}{3}$

$r = \frac{21}{63} = \frac{1}{3}$

11. Kina earns a $27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

r=1.025

a_1

S_n

$S_n = \frac{a_1 - a_1(r)^n}{1-r}$

$a_1 = 27000$

$r = 1.025$

$n = 8$

$S_8 = \frac{27,000 - 27000(1.025)^8}{1-1.025}$

$S_8 = 235875.13$

12. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

r = 1-.08 = .92

a_1

S_n

n=5

$a_1 = 350$

$r = .92$

$n = 5$

$S_5 = \frac{350 - 350(.92)^5}{1-.92}$

$S_5 = 1491.5$

r=1.1

13. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

$a_1=8$, $r=1.1$, $n=6$

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$

2) $\sum_{n=1}^{6} 8(1.10)^{n}$

3) $\frac{8-8(1.10)^6}{0.90}$

4) $\frac{8-8(0.10)^n}{1.10}$

S_n

Summations: $S_n=\sum a_1(r)^{n-1}$

$\sum_{n=1}^{6} 8(1.1)^{n-1}$

explicit: $S_n=\frac{a_1-a_1(r)^n}{1-r}$

$S_n=\frac{8-8(1.1)^n}{1-1.1}$

$S_n=\frac{8-8(1.1)^n}{-.1}$

r=1.25

14. In his first year running track, Brendon earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

1) $\frac{8-8(0.25)^4}{-.25}$

2) $\sum_{n=1}^{4} 8(0.25)^{n-1}$

3) $\frac{8-8(1.25)^4}{1-.25}$

4) $\sum_{n=1}^{4} 8(1.25)^{n-1}$

Summations: $\sum_{n=1}^{4} 8(1.25)^{n-1}$

explicit: $S_4=\frac{8-8(1.25)^4}{1-1.25}$

$S_4=\frac{8-8(1.25)^4}{-.25}$

$a_1=8$, $r=1.25$, $n=4$

Algebraically solve for all values of x

15. $x=1+\sqrt{x+5}$

$-1\ -1$

$(x-1)^2=(\sqrt{x+5})^2$

$(x-1)(x-1)=x+5$

$x^2-2x+1=x+5$

$-x-5\quad -x-5$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4$ ✓ | $x=-1$ ✗ reject

$x=4$

	x	-1
x	x^2	$-1x$
-1	$-1x$	$+1$

x^2-2x+1

y_1, y_2 Intersect

$y_1=x$

$y_2=1+\sqrt{x+5}$

Intersect

$x=4$

- Isolate the radical
- Square both sides
- Solve equation
- Check!

16. $3=-x+\sqrt{x+5}$

$+x\ +x$

$(x+3)^2=(\sqrt{x+5})^2$

$(x+3)(x+3)=x+5$

$x^2+6x+9=x+5$

$-x-5\quad -x-5$

$x^2+5x+4=0$

$(x+4)(x+1)=0$

$x=-4$ reject | $x=-1$ ✓

$x=-1$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

$y_1=3$

$y_2=-x+\sqrt{x+5}$

intersect

$x=-1$

m = mortgage payment
P = principal amount of loan (total cost − down payment)
r = interest rate (move decimal 2 places to the left)
n = # of monthly payments (12(# of years))

17. Mr. and Mrs. Jenkins just closed on a new home whose purchase price was $380,000. At the closing, they supplied a down payment of $76,000. If on the day of the closing the monthly interest rate was .3125%, determine the Jenkins' monthly mortgage payment, to the *nearest cent*, if they were approved for a 30-year loan.

Use the formula $M = P \bullet \frac{r(1+r)^n}{(1+r)^n - 1}$ where M is the mortgage payment, P is the principal amount of the loan, r is the monthly interest rate, and n is the number of monthly payments.

M = mortgage payment = m
P = principal amount of loan = 380,000 − 76,000 = 304,000
r = monthly interest rate = .003125
n = # of monthly payments = 30(12) = 360

$m = 304{,}000 \cdot .003125(1+.003125)^{360} \over (1+.003125)^{360} - 1$

m = 1407.87

18. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

M = monthly payment = m
P = amount borrowed 120,000
r = annual interest rate .048
n = number of monthly payments 15(12) = 180

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. If they plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

$$m = \frac{120{,}000\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180} - 1}$$

m = 936.50

10 STO→X
15 STO→Y

51512...

19. Which expression is equivalent to $2xy^2\sqrt[3]{x^2y}$?

1) $2x^{\frac{5}{3}}y^{\frac{7}{3}}$ 51512...
3) $2x^{\frac{2}{3}}y^{\frac{2}{3}}$
2) $2xy$
4) $2x^7y^4$

20. Which equation is equivalent to $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$ 2510530.87

1) $P = \sqrt[3]{210x^4y^7}$
3) $P = 210xy^2\sqrt[3]{xy}$ 2510530.87
2) $P = 70xy^2\sqrt[3]{xy}$
4) $P = 210xy^2\sqrt[3]{x^3y^5}$

21. Which is the solution to: $1-2(5)^{2x} = -5$?

1) $\frac{\ln 6}{2\ln 3}$
3) $\frac{2\ln 4}{\ln 3}$
2) $\frac{2\ln 5}{\ln 1}$
4) $\frac{\ln 3}{2\ln 5}$ $-5=-5$ ✓

$\frac{\ln 3}{2\ln 5}$ STO→X

22. Which is the solution to: $5(3)^{2x} = 30$?

1) $\frac{\log 6}{3\log 2}$ $\frac{\log 6}{2\log 3}$ STO→X
3) $\frac{2\log 6}{\log 3}$
2) $\frac{\log 6}{2\log 3}$ $30=30$
4) $\frac{2\log 3}{\log 6}$

Express in simplest form with a rational exponent:

23. $\sqrt[5]{x^2} \cdot \sqrt[2]{x^3}$ $\frac{2}{5}+\frac{3}{2}=\frac{19}{10}$

$x^{\frac{2}{5}} \cdot x^{\frac{3}{2}}$

$x^{\frac{19}{10}}$

24. $\sqrt[4]{a^7} \cdot \sqrt[2]{a^5}$

$a^{\frac{7}{4}} \cdot a^{\frac{5}{2}}$

$a^{\frac{17}{4}}$

- Radicals are fractional exponents
- Get rid of parenthesis
- Negative exponents are fractions
- Clean it up ← multiply, divide/reduce, evaluate

$\frac{7}{4}+\frac{5}{2}=\frac{17}{4}$