Name:

# **Common Core Algebra II**

# Unit 7

# **Sequences and Series**

# Mr. Schlansky



Lesson 1: I can write the equation of a sequence explicitly using  $a_n = a_1 + (n-1)d$  and

 $a_n = a_1(r)^{n-1}.$ 

Sequences:

Arithmetic: add a constant difference, Geometric: multiply by a common ratio Explicit Formulas (From Reference Sheet)

Arithmetic:  $a_n = a_1 + (n-1)d$  Geometric:  $a_n = a_1(r)^{n-1}$ 

Lesson 2: I can write the equation of a sequence recursively using  $a_1 = a_2 = a_2$ .

**Recursive Formulas** 

Arithmetic:  $a_1 =$  $a_n = a_{n-1} + d$  Geometric:  $a_1 =$  $a_n = ra_{n-1}$ 

Lesson 3: I can evaluate recursive sequences by substituting the previous term in for  $a_{n-1}$ . Evaluating Recursive Sequences

-Start by finding the term after the one you are given.

-Substitute the previous term in for  $a_{n-1}$  to find the current term.

Lesson 4: I can answer recursive sequence multiple choice questions by using  $a_1 = a_n = a_{n-1}$  and

 $r = 1 \pm percent$ .

*r* is the common ratio (what you're multiplying by). If you're increasing or decreasing by a percent, *common ratio* =  $1 \pm rate$ .

For example: Increases by 12% each year, common ratio is 1+.12=1.12Decreases by 20% each year, common ratio is 1-.20=.80

Lesson 5: I can write the equation of a series explicitly using  $S_n = \frac{a_1 - a_1(r)^n}{1 - r}$ . To write a series explicitly:  $S_n = \frac{a_1 - a_1(r)^n}{1 - r}$  where r is the common ratio  $(r = \frac{a_2}{a_1})$ . Lesson 6: I can evaluate summations by substituting the bottom number into the expression and adding until I get to the top number.

#### **Evaluating Summations**

Substitute the bottom number in for the variable.

Substitute the next integer value in for the variable until you get to the top number. Add all of the terms together.

Lesson 7: I can write the equation of a series using  $\sum_{n=1}^{n} a_1(r)^{n-1}$ .

To write a series using summations:  $\sum_{n=1}^{n} a_1(r)^{n-1}$ 

**Lesson 8: I can model series using the formulas and**  $r = 1 \pm percent$  or  $r = \frac{a_2}{a_1}$ )

TOTAL =  $S_n$ 

Use  $S_n = \frac{a_1 - a_1(r)^n}{1 - r}$  unless summations are in the answers and you have to use  $\sum_{n=1}^n a_1(r)^{n-1}$ 

Same notes as lessons 4, 5, and 7.

Lesson 9: I can solve mortgage problems using the given formula and Principal = Total Cost – Down Payment.

### Mortgage/Annuities

P(Principal/Amount Borrowed): P = total cost – down payment M(Mortgage Payment) R(rate): move decimal two places to the left n(# of monthly payments): n=12(# of years) \*The formulas may not use these letters, but they will contain these 4 variables **To find the down payment:** 

- 1) FIND P
- 2) Subtract P from the total cost to find the down payment

### Lesson 10: I can prepare for my sequence/series test by practicing!!

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# Writing Equations of Sequences Explicitly

Write an explicit equation for the following sequence and use the equation to find the tenth term.

1. -1,3,7,11...

2. 19, 16, 13, 10 ...

3. 2,8,32,128,...

4. -509, -503, -497, -491 ...

5. 5, -10, 20, -40, 80, ...

6. 11, 14, 17, 20, 23, ...

7. 63, 57, 51, 45, ...

9. 329.6, 376.8, 424, 471.2,... 10. 120, 192, 307.2, 491.52

11. 5400, 4050, 3037.5, 2278.125 12. 5205.20, 4208.15, 3211.1, 2214.05

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# Writing Equations of Sequences Recursively

 Write a recursive equation for the following sequence.

 1. -1,3,7,11...
 2. 19, 16, 13, 10 ...

3. 2,8,32,128,...

4. -509, -503, -497, -491 ...

5. 5, -10, 20, -40, 80, ... 6. 11, 14, 17, 20, 23, ...

7. 2, 6, 18, 54, ...

8. 63, 57, 51, 45, ...

9. 3, -12, 48, -192, ...

10. -5, -15, -45, -135, ...

11. -99, -92, -85, -78, ...

12. 9.4, 8.1, 6.8, 5.5, ...

13. 329.6, 376.8, 424, 471.2,... 14. 120, 192, 307.2, 491.52

15. 5400, 4050, 3037.5, 2278.125 16. 5205.20, 4208.15, 3211.1, 2214.05

17. 51024, 51669.27, 52314.54, 52959.81 18. 197.56, 217.32, 239.05, 262.96

19. 3, 8, 23, 68, ...

20. 100, 60, 40, 30, ...

Write each of the following equations recursively:

21.  $a_n = 16 \left(\frac{1}{2}\right)^{n-1}$  22.  $a_n = 8(2)^{n-1}$ 

23. 
$$a_n = 3n + 6$$
 24.  $a_n = 4n - 3$ 

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### **Evaluating Recursive Sequences**

1. Find  $a_4$  of the sequence  $a_n = 2a_{n-1} + 3$  where  $a_1 = 1$ .

2. Find  $a_5$  of the sequence  $a_n = 4a_{n-1} - 2$  where  $a_2 = -3$ .

3. Find 
$$a_7$$
 sequence  $\frac{a_4 = -2}{a_n = -3a_{n-1} + 4}$ 

4. If 
$$a_n = \frac{a_{n-1}}{2} + 2$$
 and  $a_2 = 16$ , find  $a_5$ 

5. If  $a_n = (a_{n-1})^2 - 4$  and  $a_4 = 2$ , find  $a_7$ 

- 6. Find the first four terms of the recursive sequence defined below.  $a_1 = -3$
- $\alpha_n = \alpha_{(n-1)} n$

7. Find the 8<sup>th</sup> term for the sequence where  $a_n = 5a_{n-1} + 2n$  where  $a_5 = 3$ 

8. A sequence is defined recursively by  $a_1 = 16$  and  $a_n = a_{n-1} - 4n$ . Find  $a_4$ 

9. The recursive formula to describe a sequence is shown below.

State the first four terms of this sequence. Can this sequence be represented using an explicit geometric formula? Justify your answer.

$$a_1 = 3$$

 $\alpha_n = 1 + 2\alpha_{n-1}$ 

10. What is the fourth term of the sequence defined by  $a_1 = 3xy^5$ 

$$a_n = \left(\frac{2x}{y}\right)a_{n-1}?$$

- 1)  $12x^3y^3$
- 2)  $24x^2y^4$
- 3)  $24x^4y^2$
- 4)  $48x^5y$



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## **Recursive Sequences Regents Practice**

1. The formula below can be used to model which scenario?

 $a_1 = 3000$ 

 $a_n = 0.80a_{n-1}$ 

- 1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
- 2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
- 3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.
- 4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

2. The formula below can be used to model which scenario?  $a_0 = 92.2$ 

 $a_n = 1.015 a_{n-1}$ 

- 1) The initial population of a county is 92.2 thousand and it is increasing by 15% each year.
- 2) The initial population of a county is 92.2 thousand and it is increasing by 1.5% each year.
- 3) The population after one year is 92.2 thousand and it is increasing by 15% each year.
- 4) The population after one year is 92.2 thousand and it is increasing by 1.5% each year.
- 3. The sequence defined by  $r_1 = 15$  and  $r_n = 0.75r_{n-1}$  best models which scenario?
- 1) Gerry's \$15 allowance is increased by \$0.75 each week.
- 2) A store that has not sold a \$15 item reduces the price by \$0.25 each week until someone purchases it.
- 3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
- 4) A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.
- 4. The sequence defined by  $a_1 = 20$  and  $a_n = 1.05a_{n-1}$  best models which scenario?
- 1) Jamal scored 20 baskets the first week and scores 5 more baskets each week.
- 2) Julie made \$20 her first month working and earns 5% more each month.
- 3) Samantha creates 20 paintings the first year and makes 50% more paintings each year.
- 4) Jennifer's flower is 20 inches tall on day 1 and increases by .05 inches each day.

- 5. Which situation *cannot* be modeled by the formula  $a_n = a_{n-1} + 20$  with  $a_1 = 10$ ?
- 1) Nancy put \$10 in her piggy bank on the first day and then added \$20 daily to her piggy bank.
- 2) Jay has a box of ten crayons and his teacher gives him twenty new crayons each month for good behavior.
- 3) Buzz has ten apples and that number increases by 20% per week.
- 4) Teresa has a block of metal that is 10°F and she heats it up at a rate of 20°F per minute.
- 6. Which situation *can* be modeled by the formula  $a_n = 1.025a_{n-1}$  with  $a_0 = 100$ ?
- 1) Devin has \$100 saved and he will increase that amount by \$2.50 each week.
- 2) Catherine has 100 Pokemon cards and gets 25% more each week.
- 3) Lucas has 100 points and each week increases by 2.5%.
- 4) Olivia's plant is 100 cm tall and it grows .025 cm each week.
- 7. Which situation *cannot* be modeled by the formula  $a_n = a_{n-1} 6$  with  $a_0 = 1000$ ?

1) A bank account with an initial balance of \$1000 increases by 6% each year.

2) Taylor is assigned 1000 SAT problems and completes 6 each day.

3) The starting population of fish in a pond is 1000 and the population decreases by 6% each day.

4) Jessica has \$1000 saved and saves an additional \$6 each week.

8. The height of Jenny's sunflower when she planted it was 6 inches. The sunflower grows by 0.25 inches per day. Which formula can be used to determine the height, in inches, of Jenny's sunflower on day *n*?

(1) 
$$\begin{array}{l} h_0 = 6 \\ h_n = 0.25a_{n-1} \end{array}$$
 (3)  $\begin{array}{l} h_0 = 6 \\ h_n = h_{n-1} + 0.25 \end{array}$   
(2)  $\begin{array}{l} h_0 = 6 \\ h_n = 6 + 0.25h_{n-1} \end{array}$  (4)  $\begin{array}{l} h_0 = 6 \\ h_n = 6h_{n-1} + 0.25 \end{array}$ 

9. A population of bacteria triples every day. If on the first day there are 300 bacteria in a Petri dish, which recursive sequence can be used to determine the population on day *n*?

1) 
$$b_1 = 300$$
  
 $b_n = 3b_{n-1}$   
2)  $b_1 = 300$   
 $b_n = b_{n-1} + 3$   
3)  $b_1 = 300$   
 $b_n = 300(3b_{n-1})$   
4)  $b_1 = 300$   
 $b_n = \frac{1}{3}b_{n-1}$ 

- 10. A lumber yard has 1500 2" by 4" pieces of wood that need to be transported to a construction site. A truck can take 100 pieces of wood per trip. Which sequence can be used to determine the number of pieces of wood left at the lumberyard after *n* trips?
  - (1)  $\begin{array}{l} a_0 = 1500 \\ a_n = a_{n-1} 100 \end{array}$  (3)  $\begin{array}{l} a_0 = 1500 \\ a_n = 1500 100a_{n-1} \end{array}$ (2)  $\begin{array}{l} a_0 = 1500 \\ a_n = 100 - a_{n-1} \end{array}$  (4)  $\begin{array}{l} a_0 = 1500 \\ a_n = 100 - 1500a_{n-1} \end{array}$
- 11. Daniela invested \$2000 in a stock that increases by 1.6% each week. Which of the following recursive sequences represents the value of her stock after n weeks?
- 1)  $a_0 = 2000$   $a_n = a_{n-1} + 1.6$ 2)  $a_0 = 2000$   $a_n = a_{n-1} + 1.016$ 3)  $a_0 = 2000$   $a_n = 1.6a_{n-1}$   $a_0 = 2000$  $a_n = 1.016a_{n-1}$

12. At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the *n*th term of this sequence is  $a_n = 25,000 + (n - 1)1000$ . Which rule best represents the equivalent recursive formula?

1)  $a_n = 24,000 + 1000n$ 3)  $a_1 = 25,000, a_n = a_{n-1} + 1000$ 2)  $a_n = 25,000 + 1000n$ 4)  $a_1 = 25,000, a_n = a_{n+1} + 1000$ 

13. The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat n years after it was purchased?

1)  $a_n = 75,000(0.08)^n$ 2)  $a_0 = 75,000$   $a_n = (0.92)^n$ 3)  $a_n = 75,000(1.08)^n$ 4)  $a_0 = 75,000$  $a_n = 0.92(a_{n-1})$ 

14. An initial investment of \$5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after *n* years?

1)  $A = 5000(0.035)^n$ 2)  $a_0 = 5000$   $a_n = a_{n-1}(0.035)$ 3)  $A = 5000(1.035)^n$ 4)  $a_0 = 5000$  $a_n = a_{n-1}(1.035)$  15. MathSchlansky posts a video to his YouTube channel and it receives 4 views on the first day. Each day after that, the number of views increases by 7%. Which sequence can be used to determine the number of views his video receives after *n* days?

1) 
$$a_{1} = 4$$
  
 $a_{n} = a_{n-1} + 7$   
2)  $a_{1} = 4$   
 $a_{n} = a_{n-1} + 1.07$   
3)  $a_{1} = 4$   
 $a_{n} = .07a_{n-1}$   
4)  $a_{1} = 4$   
 $a_{n} = 1.07a_{n-1}$ 

16. A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees,  $a_n$ , after *n* years?

1) 
$$a_1 = 150$$
  
 $a_n = a_{n-1}(0.2) + 80$   
2)  $a_1 = 150$   
 $a_n = a_{n-1}(0.8) + 80$   
3)  $a_n = 150(0.2)^n + 80$   
4)  $a_n = 150(0.8)^n + 80$ 

- 17. A recursive formula for the sequence  $18, 9, 4.5, \dots$  is 1)  $g_1 = 18$
- $g_{n} = \frac{1}{2}g_{n-1}$ 2)  $g_{n} = 18\left(\frac{1}{2}\right)^{n-1}$ 3)  $g_{1} = 18$   $g_{n} = 2g_{n-1}$ 4)  $g_{n} = 18(2)^{n-1}$

18. A recursive formula for the sequence 40, 30, 22.5, ... is

- 1)  $g_{n} = 40 \left(\frac{3}{4}\right)^{n}$ 2)  $g_{1} = 40$   $g_{n} = g_{n-1} - 10$ 3)  $g_{n} = 40 \left(\frac{3}{4}\right)^{n-1}$ 4)  $g_{1} = 40$  $g_{n} = \frac{3}{4}g_{n-1}$
- 19. A recursive formula for the sequence 64, 48, 36,... is

   1)  $a_n = 64(0.75)^{n-1}$  3)  $a_n = 64 + (n-1)(-16)$  

   2)  $a_1 = 64$  4)  $a_1 = 64$ 
   $a_n = a_{n-1} 16$   $a_n = 0.75a_{n-1}$

20. After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days. How can this sequence best be modeled recursively? Day(n) 1 2 3

How can this sequence best be modeled recursively?

1)	w = 2000	3)	··· _ 2000	<b>Dosage</b> (m)	2000	1680	1411.2	1185.4
1)	<i>m</i> <sub>1</sub> = 2000	5)	m1 - 2000					
	$m_{-} = m_{-} - 320$		$m_{\rm m} = (0.84)m_{\rm m}$					

 $m_n = m_{n-1} - 520 \qquad \qquad m_n = (0.84)m_{n-1}$ 2)  $m_n = 2000(0.84)^{n-1} \qquad \qquad 4) \qquad m_n = 2000(0.84)^{n+1}$ 

21. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: 250,000 250,937 251,878 252,822 How can this sequence be recursively modeled? 1)  $j_n = 250,000(1.00375)^{n-1}$  3)  $j_n = 250,000 + 937^{(n-1)}$ 2)  $j_1 = 250,000$  4)  $j_1 = 250,000$  $j_n = 1.00375j_{n-1}$   $j_n = j_{n-1} + 937$ 

22. Write a recursive formula for the sequence 6, 9, 13.5, 20.25, ...

23. Write a recursive formula for the sequence 189, 63, 21, 7, ....



24. Write a recursive formula,  $a_n$ , to describe the sequence graphed below.

25. The explicit formula  $a_n = 6 + 6n$  represents the number of seats in each row in a movie theater, where *n* represents the row number. Rewrite this formula in recursive form.

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## Finding the Sum of a Series (Explicit Notation)

1. Write an explicit equation to find the sum of the first n terms of the sequence 3,6,12,24... Use your formula to find the sum of the first ten terms.

2. Write an explicit equation to find the sum of the first n terms of the series 3+15+75+375 + ... Use your formula to find the sum of the first eight terms.

3. Write an explicit equation to find the sum of the first n terms of the sequence 4,-12,36,-108... Use your formula to find the sum of the first twelve terms.

4. Write an explicit equation to find the sum of the first n terms of the series  $\frac{1}{4} + \frac{1}{2} + 1 + 2 + \dots$ Use your formula to find the sum of the first nine terms.

5. Write an explicit equation to find the sum of the first n terms of the sequence 1,-3,9,-27... Use your formula to find the sum of the first sixteen terms. 6. Write an explicit equation to find the sum of the first n terms of the series -4 - 8 - 16 - 32 - ... Use your formula to find the sum of the first twenty terms.

7. Write an explicit equation to find the sum of the first n terms of the sequence 128, 64, 32, 16...Use your formula to find the sum of the first eighteen terms.

8. Write an explicit equation to find the sum of the first n terms of the series 7 - 42 + 252 - 1512 + ...Use your formula to find the sum of the first fifteen terms.

9. Write an explicit equation to find the sum of the first n terms of the sequence  $\frac{1}{16}, -\frac{1}{4}, 1, -4...$ Use your formula to find the sum of the first ten terms.

10. Write an explicit equation to find the sum of the first n terms of the sequence 3-12+48-192+... Use your formula to find the sum of the first thirteen terms.

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# **Evaluating Summations**

1. What is the value of  $\sum_{k=0}^{2} 3(2)^{k}$ ?

2. What is the value of 
$$\sum_{k=1}^{3} (2-k)^2$$
 ?

3. What is the value of 
$$\sum_{n=2}^{\infty} \frac{n}{2}$$
?

4. What is the value of the expression 
$$\sum_{n=0}^{2} n^2 + 2^n$$
?

5. Evaluate: 
$$\sum_{n=1}^{3} n^4 - n$$

6. Evaluate: 
$$\sum_{n=4}^{7} \frac{1}{2} n^2 - 1$$

7. Evaluate: 
$$\sum_{k=1}^{5} k^3 - 1$$

8. Evaluate: 
$$\sum_{x=2}^{4} x^2 - 5$$

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## Finding the Sum of a Series (Summation Notation)

1. Write an expression in summation form to find the sum of the first n terms of the sequence 3, 6, 12, 24...

Use your formula to find the sum of the first four terms.

2. Write an expression in summation form to find the sum of the first n terms of the series 3+15+75+375+...

Use your formula to find the sum of the first three terms.

3. Write an expression in summation form to find the sum of the first n terms of the sequence 4, -12, 36, -108...

Use your formula to find the sum of the first five terms.

4. Write an expression in summation form to find the sum of the first n terms of the series  $\frac{1}{4} + \frac{1}{2} + 1 + 2 + \dots$ 

Use your formula to find the sum of the first four terms.

5. Write an expression in summation form to find the sum of the first n terms of the sequence 1, -3, 9, -27...

Use your formula to find the sum of the first five terms.

6. Write an expression in summation form to find the sum of the first n terms of the series -4, -8, -16, -32 - ...

Use your formula to find the sum of the first four terms.

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# **Modeling Series**

1. Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula,  $S_n$ , for Alexa's total earnings over *n* years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

2. Ross has a hobby of collecting comic books. He currently has 50 comic books and each year, he will increase his collection by 15%. Write a geometric series formula,  $S_n$ , for Ross' total amount of comic books after *n* years. Use this formula to find the total number of comic books Ross will have 12 years from now.

3. Dee is planning on decreasing the amount of time she eats fast food per month. After the first month, she ate fast food 42 times. Each month, she eats at fast food restaurants 10% less than the previous month. Write a geometric series formula,  $S_n$ , for the total amount of fast food Dee eats after *n* months. Using your formula, how many total times does she eat fast food in the first four months? Round your answer to the nearest integer.

4. Kina earns a \$27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

5. Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the *nearest thousandth*.

6. A 7-year lease for office space states that the annual rent is \$85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?

7. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

8. A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

9. Your parents want you to do some work around the house. You get them to agree to pay you \$.01 on the first day, \$.02 on the second day, \$.04 on the third day, and so on. At the end of the 30-day month, what is the total amount of money your parents have paid you, to the *nearest cent*?

10. On Sunday, the first day of the week, Natasha does 5 pushups. Each day, she doubles the amount of pushups she does. How many total pushups will Tasha complete at the end of the 7 day week?

11. Samantha logged her weekly running distances in the table below. If the continues increasing her distance at this rate, what is the total amount of miles Samantha will have ran after 10 weeks to the nearest tenth of a mile?

Week	Distance (In Miles)
1	12
2	14.4
3	17.28
4	20.736

12. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

13. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) 
$$\sum_{n=1}^{6} 8(1.10)^{n-1}$$
  
2) 
$$\sum_{n=1}^{6} 8(1.10)^{n}$$
  
3) 
$$\frac{8 - 8(1.10)^{6}}{0.90}$$
  
4) 
$$\frac{8 - 8(0.10)^{n}}{1.10}$$

14. In his first year running track, Brendon earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

1) 
$$\frac{8-8(0.25)^4}{-.25}$$
  
2)  $\sum_{n=1}^{4} 8(0.25)^{n-1}$   
3)  $\frac{8-8(1.25)^4}{1-.25}$   
4)  $\sum_{n=1}^{4} 8(1.25)^{n-1}$ 

15. A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

Year	Amount		
	Saved		
	(in dollars)		
1	59,000		
2	64,900		
3	71,390		
4	78,529		
5	86,381.9		

Which expression determines the total amount of money saved by the company over 5 years?

1) 
$$\frac{59,000-59,000(1.1)^3}{1-1.1}$$
  
2)  $\frac{59,000-59,000(0.1)^5}{1-0.1}$   
3)  $\sum_{n=1}^{3} 59,000(1.1)^n$   
4)  $\sum_{n=1}^{5} 59,000(0.1)^{n-1}$ 

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## Mortgage Problems

1. Jim is looking to buy a vacation home for \$172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is  $M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$  where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With a \$20,000 down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$900.

2. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the *nearest cent*.

$$P_n = PMT\left(\frac{1 - (1 + i)^{-n}}{i}\right)$$

 $P_n$  = present amount borrowed n = number of monthly pay periods PMT = monthly payment i = interest rate per month

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

3. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment P = amount borrowed r = annual interest rate n = number of monthly payments

The Banks family would like to purchase a home for \$220,000. They qualified for an annual interest rate of 4.8%. If they put make a down payment of \$100,000 and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

If they want their monthly payment to be \$1500, what would their down payment have to be?

4. Mr. and Mrs. Jenkins just closed on a new home whose purchase price was \$380,000. At the closing, they supplied a down payment of \$76,000. If on the day of the closing the monthly interest rate was .3125%, determine the Jenkins' monthly mortgage payment, to the *nearest cent*, if they were approved for a 30-year loan.

Use the formula  $M = P \bullet \frac{r(1+r)^n}{(1+r)^n - 1}$  where M is the mortgage payment, P is the principal amount

of the loan, r is the monthly interest rate, and n is the number of monthly payments.

Algebraically determine and state the down payment, to the *nearest dollar*, Mr. and Mrs. Jenkins would need to initially supply in order to bring their monthly mortgage payment down to \$1200.

5. Malia wants to renovate the kitchen in her house and estimates that it will cost \$39,000 to do so. She plans to make a down payment of \$5,000 and then finance the rest at 0.25% interest per month over a ten-year period.

Use the following formula to determine Malia's monthly payment to the nearest cent.

$$P_n = PMT\left(\frac{1 - (1 + i)^{-n}}{i}\right)$$

 $P_n$  = present amount borrowed n = number of monthly pay periods PMT = monthly payment i = interest rate per month

Malia can reasonably only afford a monthly payment of \$275. What must her down payment be in order to accomplish this monthly payment?

6. Astrid just purchased a new car for \$30,000. She traded in her old car and used the money she received from it to make a \$4,000 down payment on the car. To the *nearest cent*, what will be Astrid's monthly payment on her new car if her loan has an interest rate of 0.05% per month and

the life of the loan is ten years? Use the formula  $A = R\left(\frac{1-(1+i)^{-n}}{i}\right)$  where A = present amount

borrowed, R = monthly payment, n = number of monthly pay periods, and I = monthly interest rate.

Astrid knows that she cannot afford a monthly payment of more than \$200 for the same time period. What must her down payment be for her monthly payment to be \$200?

7. The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment P = amount borrowed r = annual interest rate n = total number of monthly payments

8. Monthly mortgage payments can be found using the formula below, where M is the monthly payment, P is the amount borrowed, r is the annual interest rate, and n is the total number of monthly payments. If Adam takes out a 15-year mortgage, borrowing \$240,000 at an annual interest rate of 4.5%, What will his monthly payment be?

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

9. Robert is buying a car that costs \$22,000. After a down payment of \$4000, he borrows the remainder from a bank, a six year loan at 6.24% annual interest rate. The following formula can be used to calculate his monthly loan payment. What will Robert's monthly payment be?

$$R = \frac{(P)(i)}{1 - (1 + i)^{-t}}$$

$$R = \text{monthly payment}$$

$$P = \text{loan amount}$$

$$i = \text{monthly interest rate}$$

$$t = \text{time, in months}$$

Date \_\_\_\_\_ Algebra II

### Sequence/Series Review Sheet

 Write an equation for each of the following sequences explicitly and recursively

 1. 329.6, 376.8, 424, 471.2,...
 2. 120, 192, 307.2, 491.52

3. 5400, 4050, 3037.5, 2278.125

4. 5205.20, 4208.15, 3211.1, 2214.05

5. If  $a_n = 3a_{n-1} - 4$  and  $a_1 = 9$ , find  $a_5$ 

6. Find the 8<sup>th</sup> term for the sequence where  $a_n = 5a_{n-1} + 2$  where  $a_5 = 3$ 

7. The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat *n* years after it was purchased?

1)	$a_n = 75,000(0.08)^n$	3)	$a_n = 75,000(1.08)^n$
2)	a <sub>0</sub> = 75,000	4)	a <sub>0</sub> = 75,000
	$a_n = (0.92)^n$		$a_n = 0.92(a_{n-1})$

8. An initial investment of \$5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after *n* years?

1)	$A = 5000(0.035)^n$	3)	$A = 5000(1.035)^n$
2)	$a_0 = 5000$	4)	$a_0 = 5000$
	$a_n = a_{n-1}(0.035)$		$a_n = a_{n-1}(1.035)$

9. Write a recursive formula for the sequence 6, 9, 13.5, 20.25, ...

10. Write a recursive formula for the sequence 189, 63, 21, 7, ....

11. Kina earns a \$27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

12. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

13. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) 
$$\sum_{n=1}^{6} 8(1.10)^{n-1}$$
  
2) 
$$\sum_{n=1}^{6} 8(1.10)^{n}$$
  
3) 
$$\frac{8 - 8(1.10)^{6}}{0.90}$$
  
4) 
$$\frac{8 - 8(0.10)^{n}}{1.10}$$

14. In his first year running track, Brendon earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

1) 
$$\frac{8-8(0.25)^4}{-.25}$$
  
2)  $\sum_{n=1}^{4} 8(0.25)^{n-1}$   
3)  $\frac{8-8(1.25)^4}{1-.25}$   
4)  $\sum_{n=1}^{4} 8(1.25)^{n-1}$ 

Algebraically solve for all values of x 15.  $x = 1 + \sqrt{x+5}$  16. 3

16. 
$$3 = -x + \sqrt{x+5}$$

17. Mr. and Mrs. Jenkins just closed on a new home whose purchase price was \$380,000. At the closing, they supplied a down payment of \$76,000. If on the day of the closing the monthly interest rate was .3125%, determine the Jenkins' monthly mortgage payment, to the *nearest cent*, if they were approved for a 30-year loan.

Use the formula  $M = P \bullet \frac{r(1+r)^n}{(1+r)^n - 1}$  where M is the mortgage payment, P is the principal amount

of the loan, r is the monthly interest rate, and n is the number of monthly payments.

18. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment P = amount borrowed r = annual interest rate n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. If they plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

- 19. Which expression is equivalent to  $2xy^2 \sqrt[3]{x^2y}$ ? 1)  $5 \frac{3}{2x^3y^3}$ 2)  $2x^3y^4$ 3)  $2x^2y^2$ 4)  $2x^7y^4$ 20. Which equation is equivalent to  $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$ 1)  $P = \sqrt[3]{210x^4y^7}$ 3)  $P = 210xy^{23}\sqrt{xy}$ 4)  $P = 210xy^{23}\sqrt{x^3y^5}$
- 2)  $P = 70xy^2 \sqrt[3]{xy}$
- 21. Which is the solution to:  $1 2(5)^{2x} = -5$ ?
- 3)  $\frac{2\ln 4}{\ln 3}$ 1)  $\frac{\ln 6}{2\ln 3}$  $4) \frac{\ln 3}{2\ln 5}$  $2) \ \frac{2\ln 5}{\ln 1}$
- 22. Which is the solution to:  $5(3)^{2x} = 30$ ?
- $1) \frac{\log 6}{3\log 2}$  $3) \ \frac{2\log 6}{\log 3}$  $2) \frac{\log 6}{2\log 3}$  $4) \frac{2\log 3}{\log 6}$

Express in simplest form with a rational exponent: 24.  $\sqrt[4]{a^7} \bullet \sqrt{a^5}$ 23.  $\sqrt[5]{x^2} \cdot \sqrt{x^3}$