Finding the Sum of a Series (Sequence Notation)

1. Write an explicit equation to find the sum of the first n terms of the sequence 3,6,12,24...
Use your formula to find the sum of the first ten terms.

$a_1 = 3$

$r = \frac{a_2}{a_1}$

$r = \frac{6}{3}$

$r = 2$

$$S_n = \frac{a_1 - a_1(r)^n}{1 - r} \qquad S_n = \frac{3 - 3(2)^n}{1 - 2} \qquad \boxed{S_n = \frac{3 - 3(2)^n}{-1}}$$

$$S_{10} = \frac{3 - 3(2)^{10}}{-1} = 3069$$

2. Write an explicit equation to find the sum of the first n terms of the series 3+15+75+375 + ...
Use your formula to find the sum of the first eight terms.

$a_1 = 3$

$r = \frac{a_2}{a_1}$

$r = \frac{15}{3}$

$r = 5$

$$S_n = \frac{a_1 - a_1(r)^n}{1 - r} \qquad S_n = \frac{3 - 3(5)^n}{1 - 5} \qquad S_8 = \frac{3 - 3(5)^8}{-4}$$

$$\boxed{S_n = \frac{3 - 3(5)^n}{-4}} \qquad S_8 = 292968$$

3. Write an explicit equation to find the sum of the first n terms of the sequence 4,-12 ,36,-108...
Use your formula to find the sum of the first twelve terms.

$a_1 = 4$

$r = -\frac{12}{4}$

$r = -3$

$$S_n = \frac{a_1 - a_1(r)^n}{1 - r} \qquad S_n = \frac{4 - 4(-3)^n}{4} \qquad S_{12} = \frac{4 - 4(-3)^{12}}{4}$$

$$S_n = \frac{4 - 4(-3)^n}{1 - -3} \qquad S_{12} = -531440$$

4. Write an explicit equation to find the sum of the first n terms of the series $\frac{1}{4} + \frac{1}{2} + 1 + 2 + ...$
Use your formula to find the sum of the first nine terms.

$a_1 = \frac{1}{4}$

$r = 2$

$$S_n = \frac{a_1 - a_1(r)^n}{1 - r} \qquad \boxed{S_n = \frac{\frac{1}{4} - \frac{1}{4}(2)^n}{-1}} \qquad S_9 = \frac{\frac{1}{4} - \frac{1}{4}(2)^9}{-1}$$

$$S_n = \frac{\frac{1}{4} - \frac{1}{4}(2)^n}{1 - 2} \qquad S_9 = \frac{511}{4}$$

5. Write an explicit equation to find the sum of the first n terms of the sequence 1,-3,9,-27...
Use your formula to find the sum of the first sixteen terms.

$a_1 = 1$

$r = -3$

$$S_n = \frac{a_1 - a_1(r)^n}{1 - r} \qquad \boxed{S_n = \frac{1 - 1(-3)^n}{4}} \qquad S_{16} = \frac{1 - 1(-3)^{16}}{4}$$

$$S_n = \frac{1 - 1(-3)^n}{1 - -3} \qquad S_{16} = -10761680$$

6. Write an explicit equation to find the sum of the first n terms of the series $-4 - 8 - 16 - 32 - \ldots$
Use your formula to find the sum of the first twenty terms.

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_n = \frac{-4 - (-4)(2)^n}{-1}$$

$$S_{22} = \frac{-4 - (-4)(2)^{20}}{-1}$$

$$S_n = \frac{-4 - (-4)(2)^n}{1-2}$$

$$S_{20} = 1.\overline{\quad} \times 10^{?}$$

$$S_{20} = -4194300$$

$a_1 = -4$
$r = 2$

7. Write an explicit equation to find the sum of the first n terms of the sequence $128, 64, 32, 16 \ldots$
Use your formula to find the sum of the first eighteen terms.

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_n = \frac{128 - 128\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$$

$$S_n = \frac{128 - 128\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}$$

$$S_{18} = \frac{128 - 128\left(\frac{1}{2}\right)^{18}}{\frac{1}{2}} = 255.9990234$$

$a_1 = 128$
$r = \frac{64}{128}$
$r = \frac{1}{2}$

8. Write an explicit equation to find the sum of the first n terms of the series $7 - 42 + 252 - 1512 + \ldots$
Use your formula to find the sum of the first fifteen terms.

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_n = \frac{7 - 7(-6)^n}{7}$$

$$S_{15} = \frac{7 - 7(-6)^{15}}{7}$$

$$S_n = \frac{7 - 7(-6)^n}{1 - -6}$$

$$S_{15} = 4.7 \times 10^{11}$$

$a_1 = 7$
$r = \frac{-42}{7}$
$r = -6$

9. Write an explicit equation to find the sum of the first n terms of the sequence $\frac{1}{16}, -\frac{1}{4}, 1, -4 \ldots$
Use your formula to find the sum of the first ten terms.

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_n = \frac{\frac{1}{16} - \frac{1}{16}(-4)^n}{5}$$

$$S_n = \frac{\frac{1}{16} - \frac{1}{16}(-4)^n}{1 - -4}$$

$$S_{10} = \frac{\frac{1}{16} - \frac{1}{16}(-4)^{10}}{5}$$

$$S_{10} = -13107.1875$$

$a_1 = \frac{1}{16}$
$r = -4$

$a_1 = 3$
$r = -4$
10. Write an explicit equation to find the sum of the first n terms of the sequence $3 - 12 + 48 - 192 + \ldots$
Use your formula to find the sum of the first thirteen terms.

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_n = \frac{3 - 3(-4)^n}{5}$$

$$S_n = \frac{3 - 3(-4)^n}{1 - -4}$$

$$S_{13} = \frac{3 - 3(-4)^{13}}{5} = 40265319$$