Finding the Sum of a Series (Summation Notation)

$a_1 = 3$

1. Write an expression in summation form to find the sum of the first n terms of the sequence

$r = \frac{a_2}{a_1}$

$3, 6, 12, 24...$

$r = \frac{6}{3}$

Use your formula to find the sum of the first four terms.

$r = 2$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \boxed{\sum_{n=1}^{n} 3(2)^{n-1}} \qquad \sum_{n=1}^{4} 3(2)^{n-1}$$

$$3(2)^{1-1} + 3(2)^{2-1} + 3(2)^{3-1} + 3(2)^{4-1}$$
$$3 + 6 + 12 + 24 = \boxed{45}$$

$a_1 = 3$

$r = \frac{a_2}{a_1}$

2. Write an expression in summation form to find the sum of the first n terms of the series

$r = 5$

$3 + 15 + 75 + 375 + ...$

Use your formula to find the sum of the first three terms.

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} 3(5)^{n-1} \qquad \sum_{n=1}^{3} 3(5)^{n-1}$$

$$3(5)^{1-1} + 3(5)^{2-1} + 3(5)^{3-1}$$
$$3 + 15 + 75 = \boxed{93}$$

$a_1 = 4$

3. Write an expression in summation form to find the sum of the first n terms of the sequence

$r = \frac{a_2}{a_1}$

$4, -12, 36, -108...$

$r = \frac{-12}{4}$

Use your formula to find the sum of the first five terms.

$r = -3$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} 4(-3)^{n-1} \qquad \sum_{n=1}^{5} 4(-3)^{n-1}$$

$$4(-3)^{1-1} \quad 4(-3)^{2-1} \quad 4(-3)^{3-1} \quad 4(-3)^{4-1} \quad 4(-3)^{5-1}$$
$$4 + -12 + 36 + -108 + 324$$
$$\boxed{244}$$

$a_1 = \frac{1}{4}$
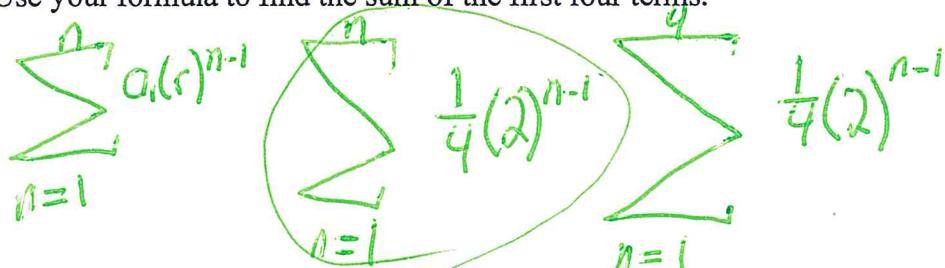
$r = \frac{a_2}{a_1}$

$r = \frac{\frac{1}{2}}{\frac{1}{4}}$

$r = 2$

$\frac{1}{2}(4)$

4. Write an expression in summation form to find the sum of the first n terms of the series

$$\frac{1}{4} + \frac{1}{2} + 1 + 2 + \dots$$

Use your formula to find the sum of the first four terms.

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} \frac{1}{4}(2)^{n-1} \qquad \sum_{n=1}^{4} \frac{1}{4}(2)^{n-1}$$

$$\frac{1}{4}(2)^{1-1} + \frac{1}{4}(2)^{2-1} + \frac{1}{4}(2)^{3-1} + \frac{1}{4}(2)^{4-1}$$

$$\frac{1}{4} + \frac{1}{2} + 1 + 2 = 3\frac{3}{4}$$

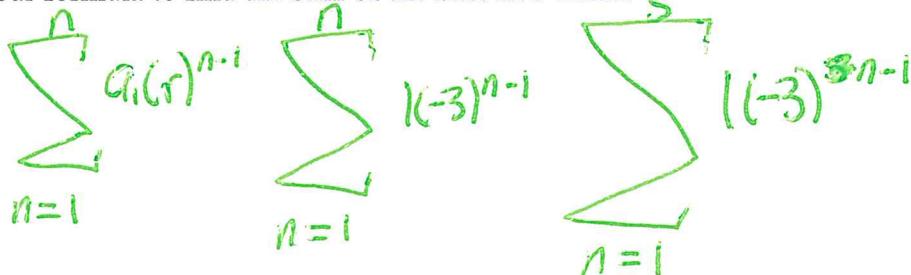5. Write an expression in summation form to find the sum of the first n terms of the sequence $1, -3, 9, -27 \dots$

Use your formula to find the sum of the first five terms.

$a_1 = 1$

$r = \frac{a_2}{a_1}$

$r = \frac{-3}{1}$

$r = -3$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} 1(-3)^{n-1} \qquad \sum_{n=1}^{5} 1(-3)^{n-1}$$

$$1(-3)^{1-1} + 1(-3)^{2-1} + 1(-3)^{3-1} + 1(-3)^{4-1} + 1(-3)^{5-1}$$

$$1 + -3 + 9 + -27 + 81 = \boxed{61}$$

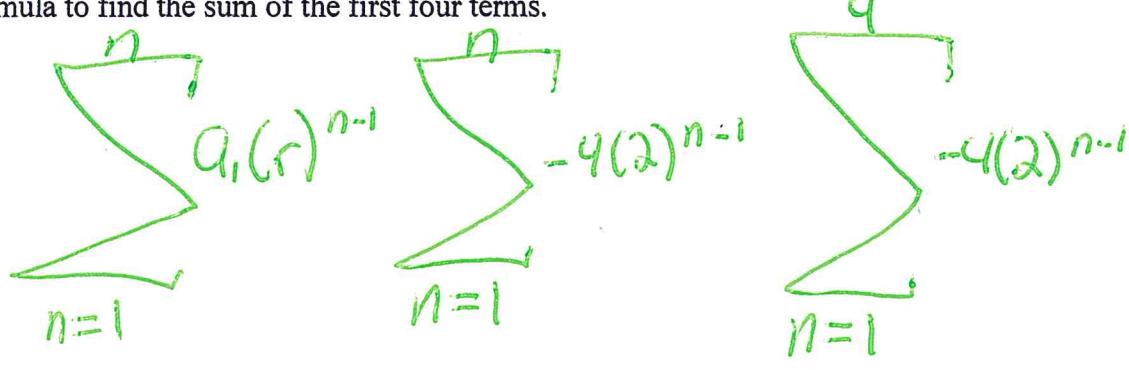6. Write an expression in summation form to find the sum of the first n terms of the series $-4, -8, -16, -32 - \dots$

Use your formula to find the sum of the first four terms.

$a_1 = -4$

$r = \frac{a_2}{a_1}$

$r = \frac{-8}{-4}$

$r = 2$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} -4(2)^{n-1} \qquad \sum_{n=1}^{4} -4(2)^{n-1}$$

$$-4(2)^{1-1} + -4(2)^{2-1} + -4(2)^{3-1} + -4(2)^{4-1}$$

$$-4 + -8 + -16 + -32$$

$$\boxed{-60}$$

7. Write an expression in summation form to find the sum of the first n terms of the sequence 128, 64, 32, 16...

Use your formula to find the sum of the first four terms.

$a_1 = 128$

$r = \dfrac{a_2}{a_1}$

$r = \dfrac{64}{128}$

$r = \dfrac{1}{2}$

$$\sum_{n=1}^{n} a_1 (r)^{n-1} \qquad \sum_{n=1}^{n} 128\left(\tfrac{1}{2}\right)^{n-1} \qquad \sum_{n=1}^{4} 128\left(\tfrac{1}{2}\right)^{n-1}$$

$$128\left(\tfrac{1}{2}\right)^{1-1} + 128\left(\tfrac{1}{2}\right)^{2-1} + 128\left(\tfrac{1}{2}\right)^{3-1} + 128\left(\tfrac{1}{2}\right)^{4-1}$$

$$128 + 64 + 32 + 16$$

$$\boxed{240}$$

8. Write an expression in summation form to find the sum of the first n terms of the series 7 − 42 + 252 − 1512 + ...

Use your formula to find the sum of the first three terms.

$a_1 = 7$

$r = \dfrac{-42}{7}$

$r = -6$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} 7(-6)^{n-1} \qquad \sum_{n=1}^{3} 7(-6)^{n-1}$$

$$7(-6)^{1-1} + 7(-6)^{2-1} + 7(-6)^{3-1}$$

$$7 + -42 + 252 = \boxed{217}$$

9. Write an expression in summation form to find the sum of the first n terms of the sequence $\dfrac{1}{16}, -\dfrac{1}{4}, 1, -4...$

Use your formula to find the sum of the first five terms.

$a_1 = \dfrac{1}{16}$

$r = \dfrac{a_2}{a_1}$

$r = \dfrac{-\frac{1}{4}}{\frac{1}{16}}$

$r = -4$

$$\sum_{n=1}^{n} a_1(r)^{n-1} \qquad \sum_{n=1}^{n} \tfrac{1}{16}(-4)^{n-1} \qquad \sum_{n=1}^{5} \tfrac{1}{16}(-4)^{n-1}$$

$$\tfrac{1}{16}(-4)^{1-1} + \tfrac{1}{16}(-4)^{2-1} + \tfrac{1}{16}(-4)^{3-1} + \tfrac{1}{16}(-4)^{4-1} + \tfrac{1}{16}(-4)^{5-1}$$

$$\tfrac{1}{16} + \tfrac{-1}{4} + 1 + -4 + 16 = \boxed{\dfrac{205}{16}}$$