

Sketching Rational Functions with Long Division

1. $y = \frac{4x-8}{2x-3}$

$$\begin{array}{r} 2x-3 \overline{) 4x-8} \\ \underline{4x-6} \\ -2 \end{array}$$

$y = 2 - \frac{2}{2x-3}$

VA: $2x-3=0$
 $x = \frac{3}{2}$
 $2x = 3$
 $x = \frac{3}{2}$

y-int: $y = 2 - \frac{2}{2(0)-3}$
 $y = 2 + \frac{2}{3}$
 $y = \frac{8}{3} (0, \frac{8}{3})$

2. $y = \frac{12x+8}{6x+1}$

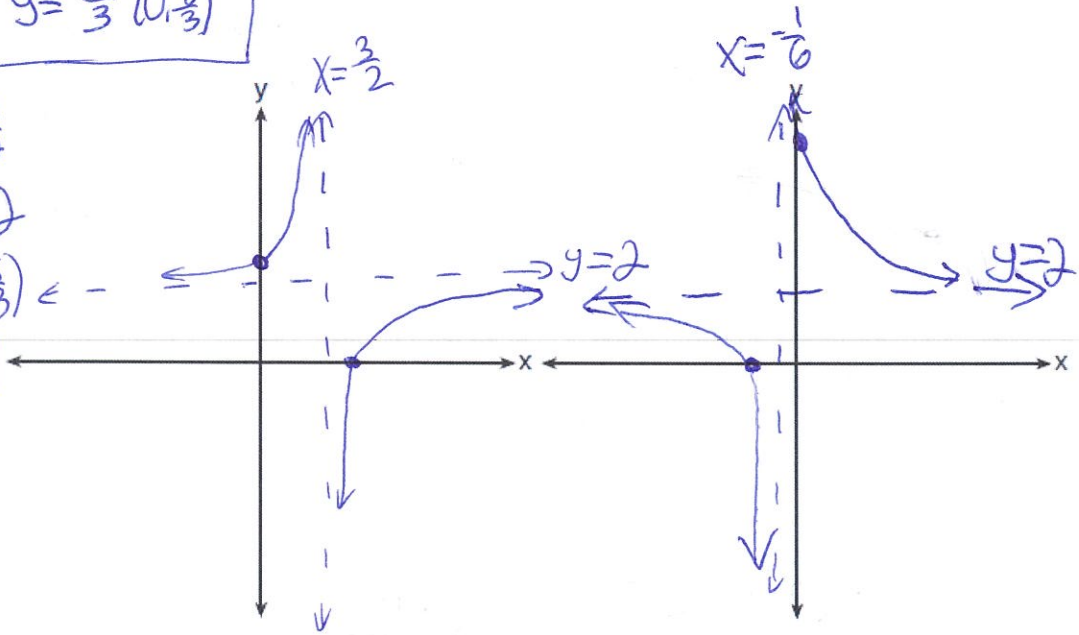
VA: $6x+1=0$
 $6x = -1$
 $x = -\frac{1}{6}$

$$\begin{array}{r} 6x+1 \overline{) 12x+8} \\ \underline{12x+2} \\ 6 \end{array}$$

$y = 2 + \frac{6}{6x+1}$

y-int: $y = 2 + \frac{6}{6(0)+1}$
 $y = 2 + \frac{6}{1}$
 $y = 8$
 $(0, 8)$

VA: $x = \frac{3}{2}$
HA: $y = 2$
y-int: $(0, \frac{8}{3})$
x-int: $(2, 0)$
Q II and IV



VA: $x = -\frac{1}{6}$
HA: $y = 2$
y-int: $(0, 8)$
x-int: $(-\frac{2}{3}, 0)$
Q I and III

X-int:

$$0 = 2 - \frac{2}{2x-3}$$

$$2x-3 \cdot 2 = (-2) \cdot (2x-3)$$

$$-2(2x-3) = -2$$

$$-4x+6 = -2$$

$$-4x = -8$$

$$x = 2$$

$(2, 0)$

X-int:

$$0 = 2 + \frac{6}{6x+1}$$

$$6x+1 \cdot (-2) = (-2) \cdot (6x+1)$$

$$-2(6x+1) = 6$$

$$-12x-2 = 6$$

$$-12x = 8$$

$$x = -\frac{2}{3}$$

$(-\frac{2}{3}, 0)$

$$y = \frac{3x-4}{3x+2} \quad \begin{array}{r} 3x+2 \overline{) 3x-4} \\ \underline{3x+2} \\ -6 \end{array}$$

$$y = 1 - \frac{6}{3x+2}$$

VA: $3x+2=0$
 $x = -\frac{2}{3}$

y-int:
 $y = 1 - \frac{6}{3(0)+2}$
 $y = 1 - \frac{6}{2}$
 $y = -2$
 $(0, -2)$

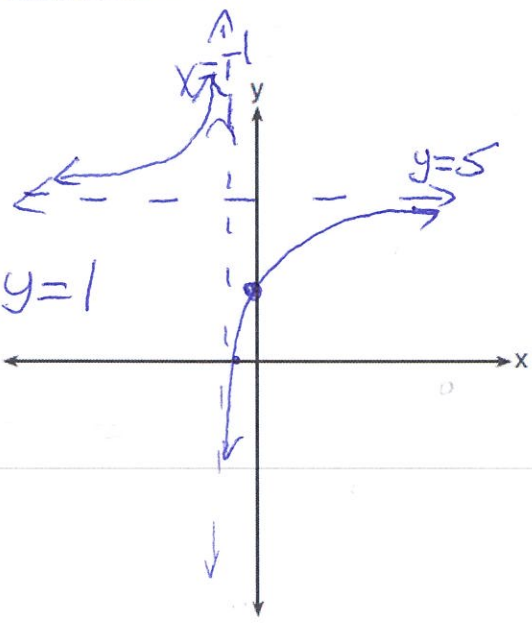
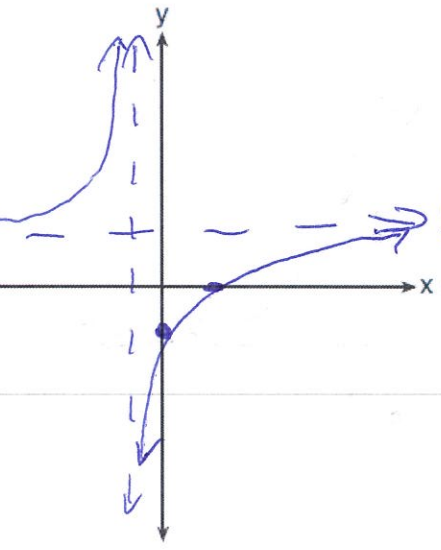
$$y = \frac{10x+5}{2x+2} \quad \begin{array}{r} 2x+2 \overline{) 10x+5} \\ \underline{10x+10} \\ -5 \end{array}$$

$$y = 5 - \frac{5}{2x+2}$$

y-int:
 $y = 5 - \frac{5}{2(0)+2}$
 $y = 5 - \frac{5}{2}$
 $y = 2.5$
 $(0, 2.5)$

VA: $2x+2=0$
 $x = -1$

VA: $x = -\frac{2}{3}$
 HA: $y = 1$
 y-int: $(0, -2)$
 x-int: $(\frac{4}{3}, 0)$
 Q II and IV



VA: $x = -1$
 HA: $y = 5$
 y-int: $(0, 2.5)$
 x-int: $(-0.5, 0)$
 Q II and IV

X-int:
 $0 = 1 - \frac{6}{3x+2}$
 $(-1) = \left(-\frac{6}{3x+2}\right) \cdot (3x+2)$
 $(\frac{4}{3}, 0)$

$$\begin{aligned} -1(3x+2) &= -6 \\ -3x-2 &= -6 \\ +2 &+2 \\ -3x &= -4 \\ \frac{-3x}{-3} &= \frac{-4}{-3} \\ x &= \frac{4}{3} \end{aligned}$$

X-int:
 $0 = 5 - \frac{5}{2x+2}$
 $(-5) = \left(-\frac{5}{2x+2}\right) \cdot (2x+2)$
 $-5(2x+2) = -5$

$$\begin{aligned} -10x-10 &= -5 \\ +10 &+10 \\ -10x &= 5 \\ \frac{-10x}{-10} &= \frac{5}{-10} \\ x &= -\frac{1}{2} \\ (-0.5, 0) \end{aligned}$$

$$y = \frac{6x-7}{3x-1} = 2 - \frac{5}{3x-1}$$

$$\begin{array}{r} 2 - \frac{5}{3x-1} \\ \frac{6x-7}{3x-1} \\ + \frac{6x+2}{3x-1} \\ \hline -5 \end{array}$$

$$y = 2 - \frac{5}{3x-1}$$

$$VA: 3x-1=0$$

$$\frac{3}{3}x = \frac{1}{3}$$

$$x = \frac{1}{3}$$

$$y-int: y=2 - \frac{5}{-1}$$

$$y = 2 + 5 = 7$$

$$(0, 7)$$

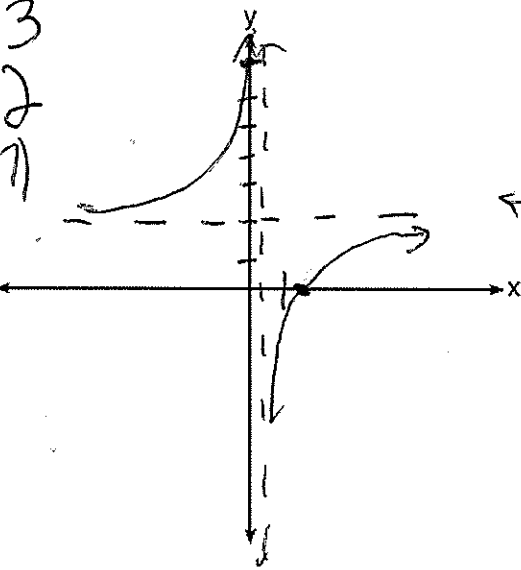
$$VA: x = \frac{1}{3}$$

$$HA: y = 2$$

$$y-int: (0, 7)$$

$$x-int: (\frac{7}{6}, 0)$$

negative
II and IV



$$y = \frac{8x-4}{2x-6}$$

$$y = 4 + \frac{20}{2x-6}$$

$$VA: 2x-6=0$$

$$\frac{2}{2}x = \frac{6}{2}$$

$$x = 3$$

$$y-int: y = 4 + \frac{20}{-6}$$

$$y = \frac{2}{3} \quad 0 = 4 + \frac{20}{2x-6}$$

$$2x-6(-4) = \frac{20}{2x-6}$$

$$-4(2x-6) = 20$$

$$-8x+24 = 20$$

$$-24 - 24$$

$$-8x = -4$$

$$\frac{-8}{-8}x = \frac{-4}{-8}$$

$$x = \frac{1}{2}$$

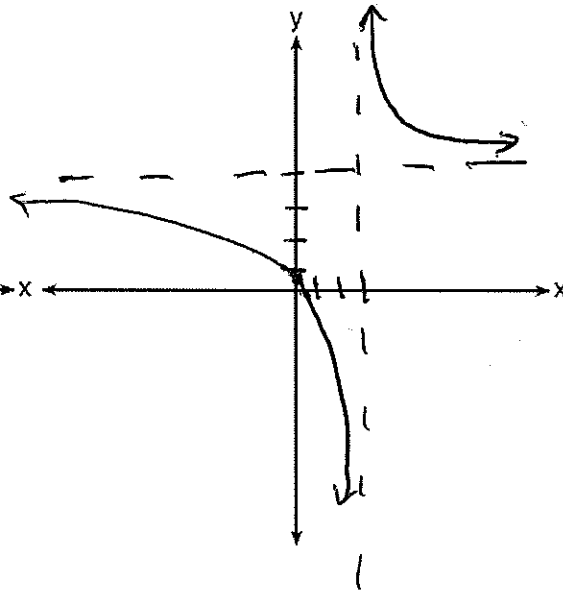
$$VA: x = 3$$

$$HA: y = 4$$

$$y-int: (0, \frac{2}{3})$$

$$x-int: (\frac{1}{2}, 0)$$

negative: II and IV



$$x-int: 0 = 2 - \frac{5}{3x-1}$$

$$\begin{array}{r} -2 - 2 \\ \frac{5}{3x-1} \\ \hline -5 \end{array}$$

$$-6x+2 = -5$$

$$-6 - 2$$

$$-6x = -7$$

$$\frac{-6}{-6}x = \frac{-7}{-6}$$

$$x = \frac{7}{6}$$

$$y = \frac{7x-1}{7x+2} \quad \frac{7x+2 \sqrt{7x-1}}{+7x+2} \quad 1 - \frac{3}{7x+2}$$

$$y = \frac{15x+3}{5x+2}$$

$$3 - \frac{3}{5x+2} \quad \frac{5x+2 \sqrt{15x+3}}{+15x+6}$$

$$y = 1 - \frac{3}{7x+2}$$

VA: $7x+2=0$ y -int: $y = 1 - \frac{3}{2}$

$7x = -2$ x -int: $0 = 1 - \frac{3}{7x+2}$

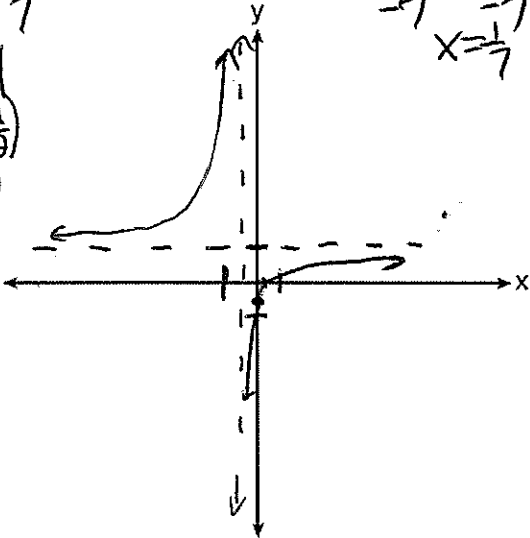
$x = -\frac{2}{7}$ $7x+2(-1) = (-\frac{3}{7x+2})$

$-7x-2 = -3$
 $-7x = -1$
 $x = \frac{1}{7}$

VA: $x = -\frac{2}{7}$

HA: $y = 1$
 y -int: $(0, \frac{1}{2})$
 x -int: $(\frac{1}{7}, 0)$

Negative:
 II and IV



VA: $5x+2=0$ y -int: $y = 3 - \frac{3}{2}$ x -int: $0 = 3 - \frac{3}{5x+2}$

$5x = -2$ $y = \frac{3}{2}$ $(-3) = -(\frac{3}{5x+2})$
 $x = -\frac{2}{5}$

$-15x-6 = -3$

$-15x = 3$
 $x = -\frac{1}{5}$

VA: $x = -\frac{2}{5}$

HA: $y = 3$

y -int: $(0, \frac{3}{2})$

x -int: $(-\frac{1}{5}, 0)$

Negative:
 II and IV

