

## Solving Radical Equations

Solve the following radical equations and CHECK each solution

1.  $(\sqrt{x-4})^2 = 6^2$

$$\begin{array}{r} x-4 = 36 \\ +4 \quad +4 \\ \hline x = 40 \end{array}$$

✓

2.  $5\sqrt{4x-8} + 2 = 12$

$$\begin{array}{r} 5\sqrt{4x-8} = 10 \\ \frac{5\sqrt{4x-8}}{5} = \frac{10}{5} \\ \sqrt{4x-8} = 2 \\ (\sqrt{4x-8})^2 = (2)^2 \\ 4x-8 = 4 \\ +8 \quad +8 \\ \hline 4x = 12 \\ \frac{4x}{4} = \frac{12}{4} \\ x = 3 \end{array}$$

✓

3.  $5 + \sqrt[3]{x+5} = 7$

$$\begin{array}{r} \sqrt[3]{x+5} = 2 \\ (\sqrt[3]{x+5})^3 = (2)^3 \\ x+5 = 8 \\ -5 \quad -5 \\ \hline x = 3 \end{array}$$

4.  $(\sqrt{x})^3 = (x)^3$

$$\begin{array}{r} \sqrt{x} = x^3 \\ -x \quad -x \\ \hline 0 = x^3 - x \\ \frac{0}{x} = \frac{x^3 - x}{x} \\ 0 = x(x^2 - 1) \\ 0 = x(x+1)(x-1) \\ x=0 \quad x=-1 \quad x=1 \end{array}$$

5.  $1 - \sqrt{2x-5} = 1$

$$\begin{array}{r} -\sqrt{2x-5} = -3 \\ \frac{-\sqrt{2x-5}}{-1} = \frac{-3}{-1} \end{array}$$

6.  $(\sqrt{x^2+x})^2 = (\sqrt{4x+10})^2$

$(\sqrt{2x-5})^2 = (3)^2$

$$\begin{array}{r} 2x-5 = 9 \\ +5 \quad +5 \\ \hline 2x = 14 \\ \frac{2x}{2} = \frac{14}{2} \\ x = 7 \end{array}$$

✓

$$\begin{array}{r} x^2 + x = 4x + 10 \\ -4x - 10 \quad -4x - 10 \\ \hline x^2 - 3x - 10 = 0 \end{array}$$

$$\begin{array}{r} x^2 - 3x - 10 = 0 \\ (x-5)(x+2) = 0 \\ \hline x = 5 \quad x = -2 \end{array}$$

✓ ✓

$$7. (x)^2 = (\sqrt{7x-12})^2$$

$$x^2 = 7x - 12$$

$$-7x + 12 \quad -7x + 12$$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$$x=4 \quad x=3$$

$$8. (x+4)^2 = (\sqrt{x+6})^2$$

$$SBT \quad (x+4)^2 = x+6$$

$$x^2 + 8x + 16 = x + 6$$

$$-x - 6 \quad -x - 6$$

$$x^2 + 7x + 10 = 0$$

$$(x+5)(x+2) = 0$$

$$x = -5 \quad x = -2$$

extraneous solution

$$9. x = 1 + \sqrt{x+5}$$

$$-1 \quad -1$$

$$SBT \quad (x-1)^2 = (\sqrt{x+5})^2$$

$$x^2 - 2x + 1 = x + 5$$

$$-x - 5 \quad -x - 5$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x=4 \quad x=-1$$

extraneous solution

$$10. 3 = -x + \sqrt{x+5}$$

$$+x \quad +x$$

$$SBT \quad (x+3)^2 = (\sqrt{x+5})^2$$

$$x^2 + 6x + 9 = x + 5$$

$$-x - 5 \quad -x - 5$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$$x = -4 \quad x = -1$$

extraneous solution

$$11. x = 2 + \sqrt{x+4}$$

$$-2 \quad -2$$

$$SBT \quad (x-2)^2 = (\sqrt{x+4})^2$$

$$x^2 - 4x + 4 = x + 4$$

$$-x - 4 \quad -x - 4$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x=0 \quad x=5$$

extraneous solution

$$12. (\sqrt{4y+3})^2 = (2y)^2$$

$$4y + 3 = 4y^2$$

$$-4y^2 - 3$$

$$0 = 4y^2 - 4y - 3$$

$$0 = y^2 - 4y - 12$$

$$(y-6)(y+2)$$

$$(y-6)(y+2)$$

$$y = \frac{3}{2}$$

$$(2y-3)(2y+1) = 0$$

$$2y-3=0 \quad 2y+1=0$$

$$y = \frac{3}{2} \quad y = -\frac{1}{2}$$

$$y = \frac{3}{2} \quad y = -\frac{1}{2}$$

extraneous solution

13.  $\sqrt{x-5} + x = 7$

SBT

~~$(x-5) = (7-x)^2$~~

$x-5 = 49 - 14x + x^2$   
 $-x+5 \quad +5 \quad -x$   
 $x^2 - 15x + 54 = 0$

extraneous solution

$(x-9)(x-6) = 0$

$x=9 \quad x=6$

14.  $\sqrt{2x-7} + x = 5$

~~$(2x-7) = (5-x)^2$~~

$2x-7 = x^2 - 10x + 25$   
 $-2x+7 \quad -2x+7$   
 $0 = x^2 - 12x + 32$   
 $0 = (x-8)(x-4)$

SBT

extraneous solution

~~$x=8$~~

$x=4$

15. Solve algebraically for all values of x:  $\sqrt{x-4} + x = 6$

~~$(x-4) = (6-x)^2$~~

$x-4 = x^2 - 12x + 36$   
 $-x+4 \quad -x+4$   
 $0 = x^2 - 13x + 40$

$0 = (x-8)(x-5)$

$x=8 \quad x=5$

extraneous solution

MC strategy

16. The solution set for the equation  $\sqrt{x+14} - \sqrt{2x+5} = 1$  is

- 1)  $\{-6\}$
- 2)  $\{2\}$

- 3)  $\{18\}$
- 4)  $\{2, 22\}$

MC strategy

17. The solution set for the equation  $\sqrt{56-x} = x$  is

- 1)  $\{-8, 7\}$
- 2)  $\{-7, 8\}$
- 3)  $\{7\}$
- 4)  $\{\}$

18. Solve algebraically for  $x$ :  $\sqrt{x^2+x-1}+11x=7x+3$

$$\begin{aligned} & \sqrt{x^2+x-1} = (-4x+3) \quad \text{SBT} \\ & \sqrt{x^2+x-1} = 16x^2-24x+9 \\ & -x^2-x+1 \quad -x^2-x+1 \\ & \frac{0}{5} = \frac{15x^2-25x+10}{5} \\ & 0 = 3x^2-5x+2 \quad \text{PT} \\ & x^2-5x+6 \\ & (x-3)(x-2) \\ & \frac{3}{3} \quad \frac{2}{3} \end{aligned}$$

$$\begin{aligned} & \cancel{11x} - \cancel{11x} \\ & 0 = (x-1)(3x-2) \\ & x=1 \quad x=\frac{2}{3} \\ & \text{Extraneous} \\ & \text{Solution} \end{aligned}$$

19. The speed of a tidal wave,  $s$ , in hundreds of miles per hour, can be modeled by the equation  $s = \sqrt{t} - 2t + 6$ , where  $t$  represents the time from its origin in hours. Algebraically determine the time when  $s = 0$ .

$$\begin{aligned} & 0 = \sqrt{t} - 2t + 6 \\ & \sqrt{t} - 6 = -2t + 6 \\ & \text{SBT } (\sqrt{t}-6)^2 = (-2t+6)^2 \\ & 4t^2 - 24t + 36 = 4t^2 - 24t + 36 \\ & -4t^2 + 24t - 36 = -4t^2 + 24t - 36 \\ & 4t^2 - 25t + 36 = 0 \\ & t^2 - 25t + 144 \\ & (t-\frac{16}{4})(t-\frac{9}{4}) \end{aligned}$$

$$\begin{aligned} & (t-4)(4t-9) = 0 \\ & t-4 = 0 \quad 4t-9 = 0 \\ & t=4 \quad t=\frac{9}{4} \\ & \text{Extraneous} \\ & \text{Solution} \end{aligned}$$