Name:

Common Core Algebra II

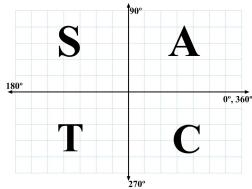
Unit 8

Trigonometry and Graphing Trigonometric Functions

Mr. Schlansky



2025



Lesson 1: I can sketch angles on the grid using 0, 90, 180, and 270.

The reference angle is the acute angle made with the x-axis.

Lesson 2: I can find co-terminal angles by adding or subtracting 360.

We always want an angle between 0 and 360. If it is not, add or subtract 360 until it is.

Lesson 3: I can convert between degrees and radians by multiplying by multiplying by $\frac{\pi}{180}$

or $\frac{180}{\pi}$. Degrees to radians: Multiply by $\frac{\pi}{180}$ Radians to degrees: Multiply by $\frac{180}{\pi}$ *If you are converting to degrees 180 is on top. If you are converting to radians π is on top.

Lesson 4: I can sketch radian angles on the grid by converting to degrees by multiplying by $\frac{180}{\pi}$ and using 0, 90, 180, and 270.

1) Convert radians to degrees by multiplying by $\frac{180}{\pi}$.

2) Graph that angle on the grid.

*If multiple choice, convert the radians to degrees first, whether the radians are in the problem or the answers.

Lesson 5: I can find six trig ratios using SOHCAHTOA and the reciprocal trig functions. To find trig ratios: SOHCAHTOA

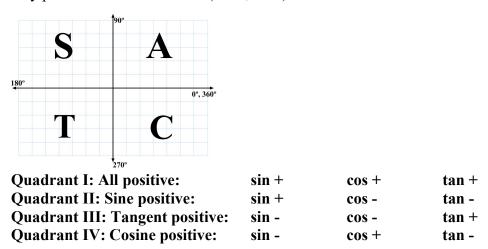
$\sin\theta = \frac{opp}{hyp}$	$\cos\theta = \frac{adj}{hyp}$	$\tan \theta = \frac{opp}{adj}$
$\csc \theta = \frac{hyp}{opp}$	$\sec \theta = \frac{hyp}{adj}$	$\cot \theta = \frac{adj}{opp}$

Reciprocal trig function pairs:

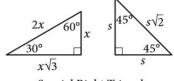
$\csc \theta$	$\sec \theta$	$\tan \theta$
$\sin heta$	$\cos heta$	$\cot \theta$

Lesson 6: I can determine the signs of each trig function in each quadrant using ASTC.

Unit Circle: A circle with a center at the origin and a radius of 1. Any point on the unit circle is $(\cos \theta, \sin \theta)$



Lesson 7: I can find the sides of special right angles using their given ratios



Special Right Triangles

45, 45, 90: The legs are the same, the hypotenuse is the same with a $\sqrt{2}$ after it. 30, 60, 90: The hypotenuse is double the small leg, the large leg is the same as the small leg with a $\sqrt{3}$ after it.

		30	45	60
	Sin	1	$\sqrt{2}$	$\sqrt{3}$
		2	2	2
Lesson 8: I can evaluate special angles using QSFR	Cos	$\sqrt{3}$	$\sqrt{2}$	1
To evaluate special angles (multiples of 30, 45, 60):		2	2	2
1) Sketch the angle on the grid and find the reference angle	tan	-	1	$\sqrt{3}$
2) List quadrant, sign, trig function, reference angle horizontally	60,699,698	3		ų.
3) Evaluate the given expression	l	5		

Lesson 9: I can find a missing side of a right triangle using Pythagorean Theorem of		
Pythagorean Triples.	thagorean Triples. Know your Pythagorean Triple	
$a^2 + b^2 = c^2$	3,4,5	
<i>a</i> and <i>b</i> are the legs	5,12,13	
<i>c</i> is the hypotenuse	7,24,25	
	8,15,17	
	9,40,41	

If it is a Pythagorean Triple, use the triple to find the missing side.

If it is not a Pythagorean Triple, use $a^2 + b^2 = c^2$ to find the third side.

*If using Pythagorean Theorem, leave your answer as a radical if it is not a perfect square.

Lesson 10: I can rationalize a denominator by multiplying the numerator and denominator by the radical.

To rationalize the denominator, multiply top and bottom by the radical

When multiplying a radical by itself, the radical cancels out

Lesson 11: I can find trig ratios using SOHCAHTOA, the reciprocal functions, and ASTC. To find a trig ratio,

1) Sketch a triangle on the grid connecting to the origin and x axis and then:

a) If given $\sin/\cos/\tan = \frac{something}{something}$, use SOHCAHTOA to fill in 2 sides of a triangle

b) If given a point, plot point and create triangle to origin and x axis. The x coordinate is the horizontal leg and the y coordinate is the vertical leg.

c) If given a circle, the radius is the hypotenuse and the x or y coordinate will be given.

d) If given a point on the unit circle, $x = \cos \theta$ and $y = \sin \theta$. Follow procedure *a* from there.

2) Use Pythagorean Triples/Pythagorean Theorem to find the third side.

3) Find the trig ratios using SOHCAHTOA and the reciprocal functions.

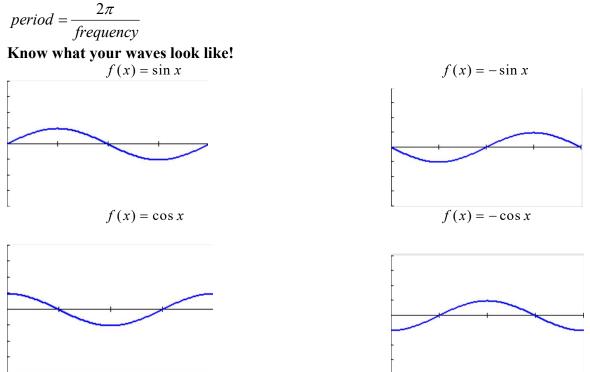
4) Determine the signs using ASTC

*Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

Lesson 12: I can practice finding trig ratios using SOHCAHTOA, the reciprocal functions, and ASTC.

Same notes as lesson 11.

Lesson 13: I can graph sinusoidal functions with different amplitudes, midlines, and frequencies using AMPSINFREQXSHIFT, knowing the four different waves, and using



AMPSINFREQXSHIFT

Amplitude: Distance from the midline to minimum or maximum Frequency: How many waves from 0 to 2π

Period: (Wavelength): How long it takes to make one full cycle

Shift: y value of the midline. The average value of the function.

$$Period = \frac{2\pi}{frequency}, Frequency = \frac{2\pi}{period}$$

To graph:

- 1) Use AMPSINFREQXSHIFT to determine the amplitude, sin or cos (+ or -), frequency, and shift/midline.
- 2) Use $Period = \frac{2\pi}{frequency}$ to find the period.
- 3) Draw in midline and make dashes for max and min on the y axis.
- 4) Make 4 dashes and put the period at the end of the 4^{th} dash on the x axis.
- 5) Draw in the appropriate curve.

Lesson 14: I can practice graphing sinusoidal functions

Same notes as Lesson 13.

Lesson 15: I can graph sinusoidal functions with given domains by stretching or shrinking the waves.

Same notes as lesson 11. If the domain is bigger than the period, you may need to make multiple waves and/or compress them.

Lesson 16: I can write the equation of sinusoidal functions using AMPSINFREQXSHIFT,

 $midline = \frac{\min + \max}{2}$, and $frequency = \frac{2\pi}{period}$.

To write the equation:

- 1) Find midline $(\frac{\min + \max}{2})$
- 2
- 2) Determine the period (the x value where the first full wave ends)
- 3) Use $Frequency = \frac{2\pi}{period}$ to determine the frequency
- 4) Substitute all values into AMPSINFREQXSHIFT

*If multiple choice, cross out incorrect choices using obvious components such as type of wave and midline/shift.

Lesson 17: I can apply the components of sinusoidal functions in different ways by drawing a little picture.

Draw a little picture that contains the midline, min, max, and period to give you structure. Same notes as Lessons 11 - 14.

Lesson 18: I can graph sinusoidal models using AMPSINFREQXSHIFT and drawing a little picture.

Same notes as Lessons 11-15.

1) Graph the wave using AMPSINFREQXSHIFT

2) The first follow up question is usually period $(\frac{2\pi}{frequency})$ and context of the period (the

amount of time it takes to complete one full _____)

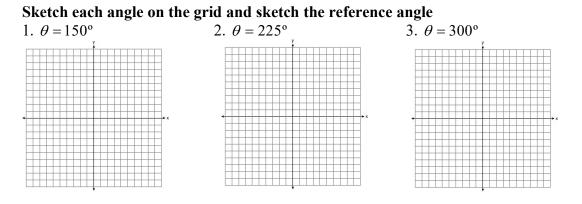
3) The second follow up question can be anything. Don't be afraid to use your calculator.

Lesson 19: I can prepare for my trigonometry test by practicing!

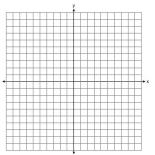


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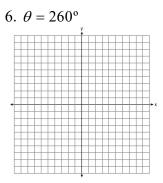
Sketching Angles on the Grid



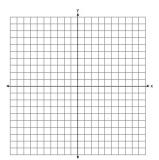
4. $\theta = 190^{\circ}$



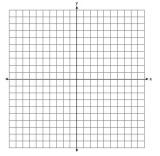
5. $\theta = 220^{\circ}$



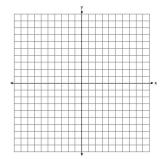
7. $\theta = 100^{\circ}$

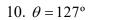






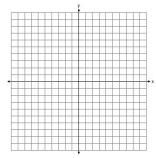
9.
$$\theta = 200^{\circ}$$

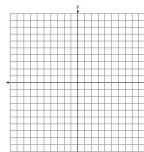


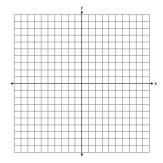


11. $\theta = 83^{\circ}$

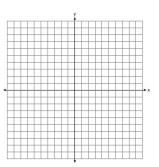




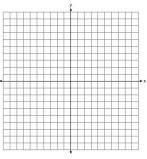




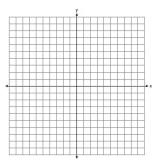
13. $\theta = 230^{\circ}$



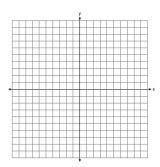
14. $\theta = 190^{\circ}$



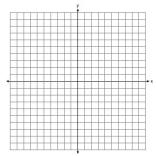




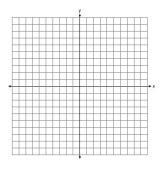
16. $\theta = 202^{\circ}$



17. $\theta = 120^{\circ}$



18. $\theta = 312.5^{\circ}$



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Co-terminal Angles

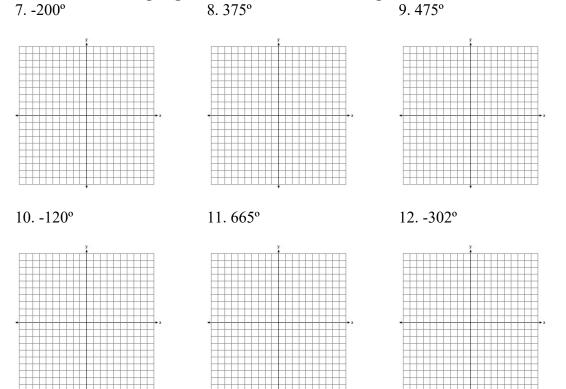
For each angle, state two angles that terminate at the same point.1. 120°2. 250°3. 48°



5. 212°

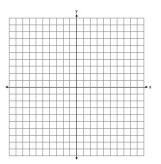
6. 142°

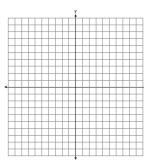
Sketch the following angles and find the reference angle 7. -200° 8. 375° 9.

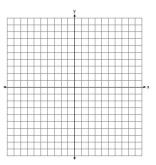


13. 421°

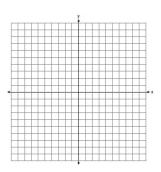
14.-279°



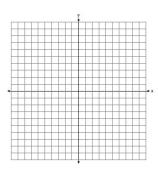




16. 620°

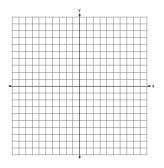


17.-305°

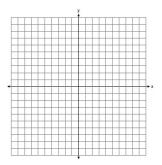


18. 580°

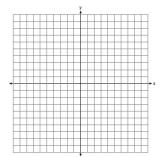
15. 701°



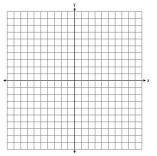
16. 790°



17. 1111°



18. -1060°



Name_ Mr. Schlansky Date _____ Algebra II

Degrees/ Radians Conversions

Convert the following fron 1. 90°	n degrees to radians. Leave y 2. 30°	Your answers in terms of π . 3. 60°
4. 135°	5. 150°	6. 330°
7. 225°	8. 330°	9. 450°
10120°	11. 45°	12300°
13. 67.5°	14. 337.5°	15420°



Convert the following from radians to degrees. Round to the nearest thousandth if necessary

16.
$$\frac{\pi}{4}$$
 radians17. $\frac{3\pi}{2}$ radians18. $\frac{12\pi}{4}$ radians19. $\frac{6\pi}{5}$ radians20. $\frac{2\pi}{5}$ radians21. $\frac{4\pi}{3}$ radians22. $\frac{3\pi}{8}$ radians23. $\frac{\pi}{10}$ radians24. $\frac{5\pi}{8}$ radians25. 4 radians26. 3 radians27. 1.5 radians28. 2.75 radians29. 3.6 radians30. 5 radians

31. 5.25 radians

32. 4.8 radians

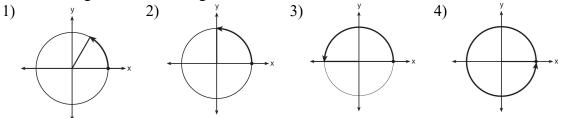
33. 2.1 radians

Sketching Radian Angles on the Grid

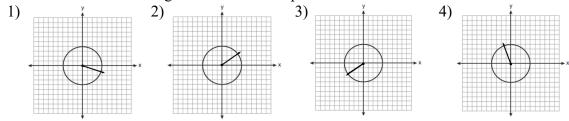
Date _____ Algebra II

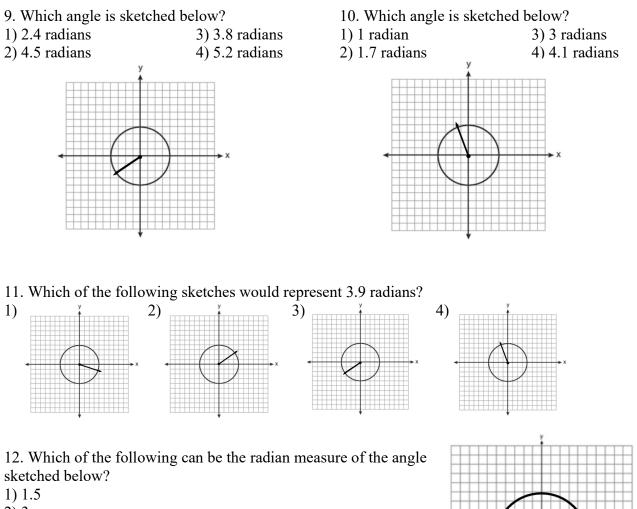
1. $\theta = \frac{5\pi}{3}$ 2. $\theta = \frac{7\pi}{4}$ 3. $\theta = 2$ 1. $\theta = \frac{5\pi}{3}$ 4. $\theta = 4.1$ 5. $\theta = -\frac{\pi}{6}$ 6. $\theta = 9.2$ 1. $\theta = 9.2$ 1. $\theta = \frac{1}{2}$ 1. $\theta =$

7. Which diagram shows an angle rotation of 1 radian on the unit circle?



8. Which of the following sketches would represent 6 radians?



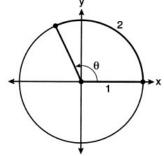


- 2) 3
- 3) 3.8
- 4) 5

13. An angle, Θ , is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below. y

Which value represents the radian measure of angle θ ?

- 1) 1 3) 65.4
- 2) 2 4) 114.6

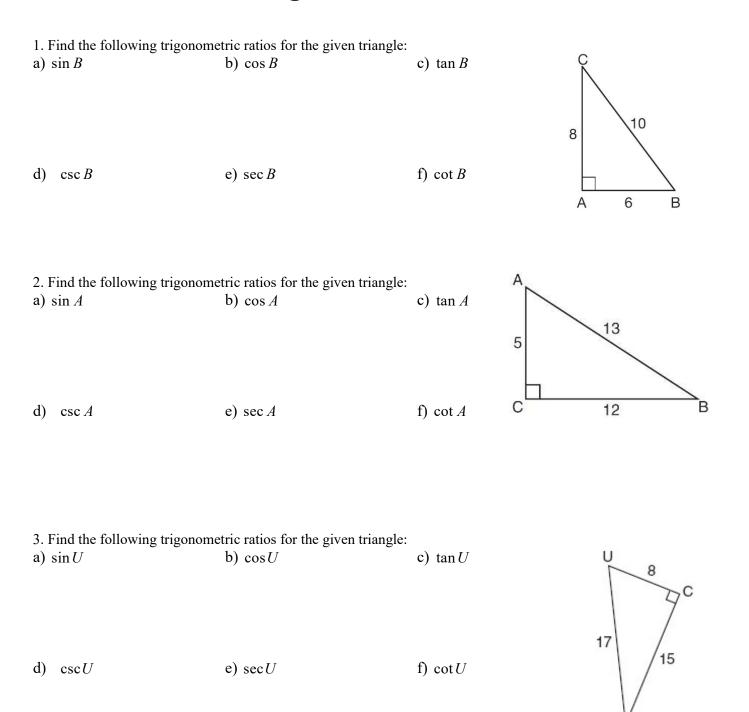




Date _____ Geometry

Name_____ Mr. Schlansky

Basic Trigonometric Ratios



P

Date _____ Algebra II



Signs of Quadrants

In which quadrant does \mathcal{G} lie if:

1. $\cos \theta < 0$, $\tan \theta < 0$ 2. $\sin \theta > 0$, $\tan \theta > 0$ 3. $\cos \theta < 0$, $\sin \theta < 0$

4.
$$\cos \theta > 0$$
, $\cot \theta < 0$ 5. $\csc \theta > 0$, $\tan \theta < 0$ 6. $\tan \theta < 0$, $\csc \theta < 0$

7. $\sin \theta < 0$, $\sec \theta > 0$ 8. $\tan \theta > 0$, $\sin \theta < 0$ 9. $\cos \theta < 0$, $\csc \theta > 0$

10. $\sec \theta < 0$, $\tan \theta > 0$ 11. $\sin \theta > 0$, $\sec \theta < 0$ 12. $\tan \theta < 0$, $\csc \theta > 0$

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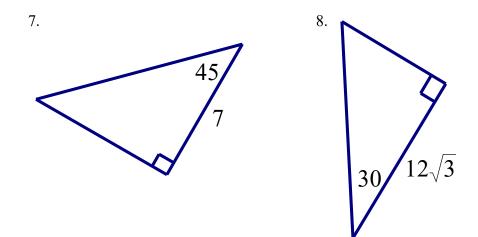
Name_____ Mr. Schlansky

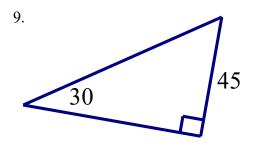
Date _____ Algebra II

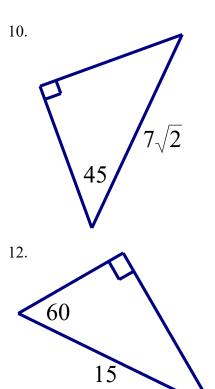
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Special Right Triangles

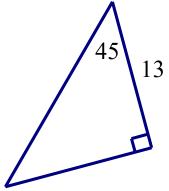
Fill in the two missing sides of each of the following triangles. 2. 1. 45 45 7 45 45 3 3. 4. 45 30 $10\sqrt{2}$ 45 60 5 5. 6. 30 30 20 $8\sqrt{3}$ 60







11.





Evaluating Special Angles

Date

Pre Calculus

1.
$$\sin(30)$$
 2. $\cos(\frac{\pi}{4})$ 3. $\tan(60)$

4.
$$\cos\left(\frac{5\pi}{6}\right)$$
 5. $\sin(120)$ 6. $\tan\left(\frac{3\pi}{4}\right)$

7.
$$\tan(210)$$
 8. $\sin\left(\frac{5\pi}{4}\right)$ 9. $\cos(240)$

10.
$$\cos\left(\frac{11\pi}{6}\right)$$
 11. $\sin(315)$ 12. $\tan\left(\frac{5\pi}{3}\right)$

13.
$$\cos(120)$$
 14. $\tan\left(\frac{3\pi}{4}\right)$ 15. $\sin(300)$
16. $\tan\left(\frac{5\pi}{6}\right)$ 17. $\cos(315)$ 18. $\cos\left(\frac{7\pi}{6}\right)$

19.
$$\sec(30)$$
 20. $\csc\left(\frac{\pi}{3}\right)$ 21. $\cot(210)$

22.
$$\csc\left(\frac{3\pi}{4}\right)$$
 23. $\cot(300)$ 24. $\sec\left(\frac{5\pi}{6}\right)$

Date _____ Pre Calculus



The Unit Circle

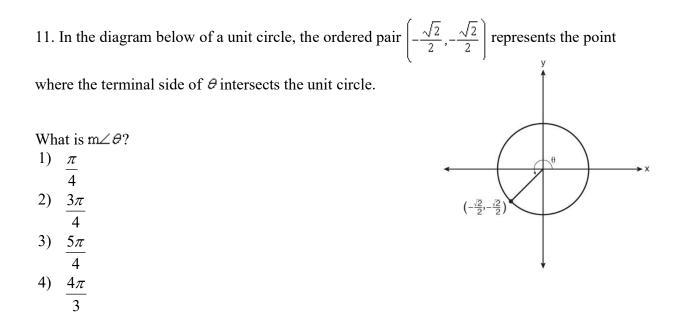
Find the exact value of the coordinate on the unit circle for each of the following

1. $\theta = 30^{\circ}$ 2. $\theta = \frac{\pi}{3}$ 3. $\theta = 45^{\circ}$

4.
$$\theta = \frac{5\pi}{3}$$
 5. $\theta = 300^{\circ}$ 6. $\theta = \frac{7\pi}{6}$

7.
$$\theta = 330^{\circ}$$
 8. $\theta = \frac{5\pi}{4}$ 9. $\theta = 120^{\circ}$

10. In the diagram of a unit circle below, point A, $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, represents the point where the terminal side of θ intersects the unit circle. What is $m \angle \theta$? 1) 30° 2) 120° 3) 135° 4) 150°



12. In the diagram of a unit circle below, a point on the unit circle as coordinates $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. What is $m \angle \theta$? 1) 300° 2) 315° 3) 240° 4) 330° 4) 330°

Pythagorean Theorem

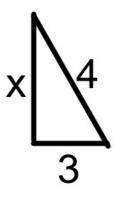
1.

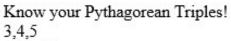
Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$ a and b are the legs c is the hypotenuse

Find the missing side of each right triangle rounding to the nearest tenth

2.

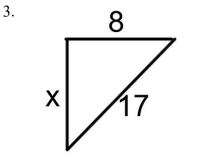
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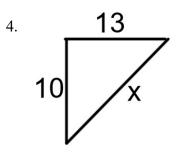




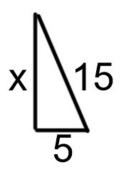
5,12,13 7,24,25 8,15,17 9,40,41



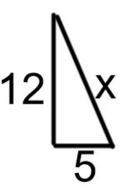




5.



6.



Rationalizing the Denominator

To rationalize the denominator, multiply top and bottom by the radical When multiplying a radical by itself, the radical cancels out

Rationalize the following denominators

1.
$$\frac{2}{\sqrt{5}}$$
 2. $\frac{-7}{\sqrt{11}}$

$$3. \ \frac{3}{\sqrt{2}} \qquad \qquad 4. \ \frac{6}{\sqrt{3}}$$

5. $\frac{4}{\sqrt{6}}$

6.
$$\frac{-5}{\sqrt{10}}$$

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Advanced Trig Ratios

1. If $\cos \theta = \frac{12}{13}$ and θ is in Quadrant I, find: a) $\cos \theta$ b) $\sin \theta$

c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

f) $\cot \theta$

2. If
$$\sin \theta = -\frac{3}{5}$$
 and θ is in Quadrant III, find:
a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

3. If $\tan \theta = \frac{24}{7}$	and θ is in Quadrant III, find:	
a) $\cos \theta$	b) $\sin \theta$	c) $\tan \theta$

d) $\sec \theta$ e) $\csc \theta$ f) $\cot \theta$

4. If
$$\sin \theta = \frac{5}{8}$$
 and θ is in Quadrant II, find:
a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

$ec\theta$

e) $\csc \theta$

5. Angle θ is in standard position and (3,4) is a point on the terminal side of θ . Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

f) $\cot \theta$

6. Angle θ is in standard position and (4, -7) is a point on the terminal side of θ . Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

7. Angle θ is in standard position and (-5, -12) is a point on the terminal side of θ . Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

f) $\cot \theta$

8. Angle θ is in standard position and (-2,3) is a point on the terminal side of θ . Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

9. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ , intercepts the circle in Quadrant I at point C. The y-coordinate of point C is 8. Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

f) $\cot \theta$

10. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ , intercepts the circle in Quadrant II at point *P*. The *x*-coordinate of point *P* is 2. Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

11. A circle centered at the origin has a radius of 6 units. The terminal side of an angle, θ , intercepts the circle in Quadrant VI at point *P*. The *x*-coordinate of point *P* is 2. Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc \theta$

f) $\cot \theta$

12. A circle centered at the origin has a radius of 9 units. The terminal side of an angle, θ , intercepts the circle in Quadrant II at point *P*. The *x*-coordinate of point *P* is 7. Find: a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$

e) $\csc\theta$

13. The point
$$\left(\frac{3}{5}, -\frac{4}{5}\right)$$
 lines on the unit circle. Find:
a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec \theta$ e) $\csc \theta$ f) $\cot \theta$

14. The point
$$\left(x, -\frac{2}{3}\right)$$
 lines on the unit circle where $x > 0$. Find:
a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec\theta$

e) $\csc \theta$

Advanced Trig Ratios Regents Practice

Date _____ Algebra II

1. If
$$\cos \theta = -\frac{3}{4}$$
 and θ is in Quadrant III, then $\sin \theta$ is equivalent to
1) $-\frac{\sqrt{7}}{4}$
2) $\frac{\sqrt{7}}{4}$
4) $\frac{5}{4}$

2. If the terminal side of angle θ , in standard position, passes through point (-4, 3), what is the numerical value of $\sin \theta$?

1) $\frac{3}{5}$ 3) $-\frac{3}{5}$ 2) $\frac{4}{5}$ 4) $-\frac{4}{5}$

3 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ , intercepts the circle in Quadrant II at point *C*. The *y*-coordinate of point *C* is 8. What is the value of $\cos \theta$?

$1) - \frac{3}{5}$	3) $\frac{3}{5}$
$(2) - \frac{3}{4}$	4) $\frac{4}{5}$

4. Given $\cos \theta = \frac{7}{25}$, where θ is an angle in standard position terminating in quadrant IV, and $\sin^2 \theta + \cos^2 \theta = 1$, what is the value of $\tan \theta$? 1) $-\frac{24}{25}$ 2) $-\frac{24}{7}$ 3) $\frac{24}{25}$ 4) $\frac{24}{7}$

5. Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$?

1) $\frac{5+\sqrt{2}}{5}$ 2) $\frac{\sqrt{23}}{5}$ 3) $\frac{3\sqrt{3}}{5}$ 4) $\frac{\sqrt{35}}{5}$ 6. Given $\cos A = \frac{3}{\sqrt{10}}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

7. An angle, θ , is in standard position and its terminal side passes through the point (2,-1). Find the *exact* value of sin θ .

8. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ , intercepts the circle in Quadrant III at point *P*. The *x*-coordinate of point *P* is 2. What is the value of $\cos \theta$?

9. The terminal side of θ , an angle in standard position, intersects the unit circle at $P\left(-\frac{1}{3}, -\frac{\sqrt{8}}{3}\right)$.

What is the value of sec
$$\theta$$
?
1) -3
2) $-\frac{3\sqrt{8}}{8}$
3) $-\frac{1}{3}$
4) $-\frac{\sqrt{8}}{3}$

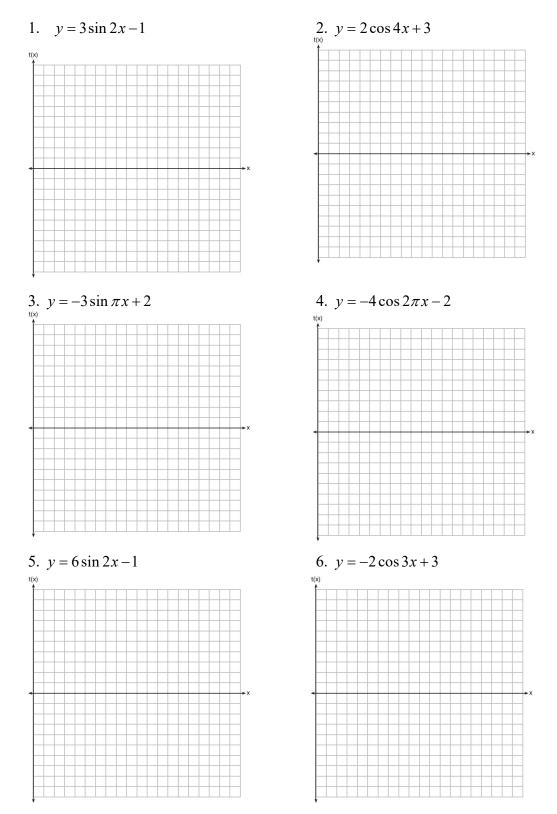
10. Point $M\left(t, \frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of *t*.

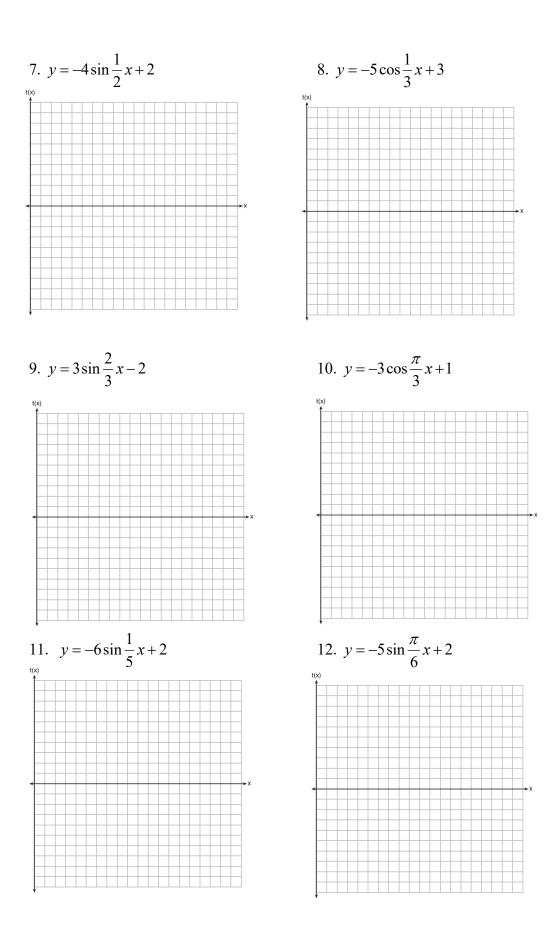
Date _____ Algebra II



Graphing Sinusoidal Curves

Graph one full wave of the following trigonometric functions



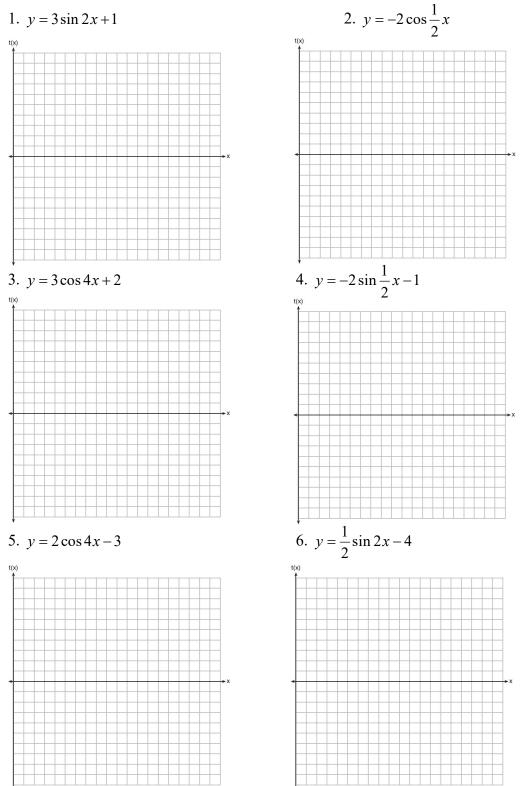


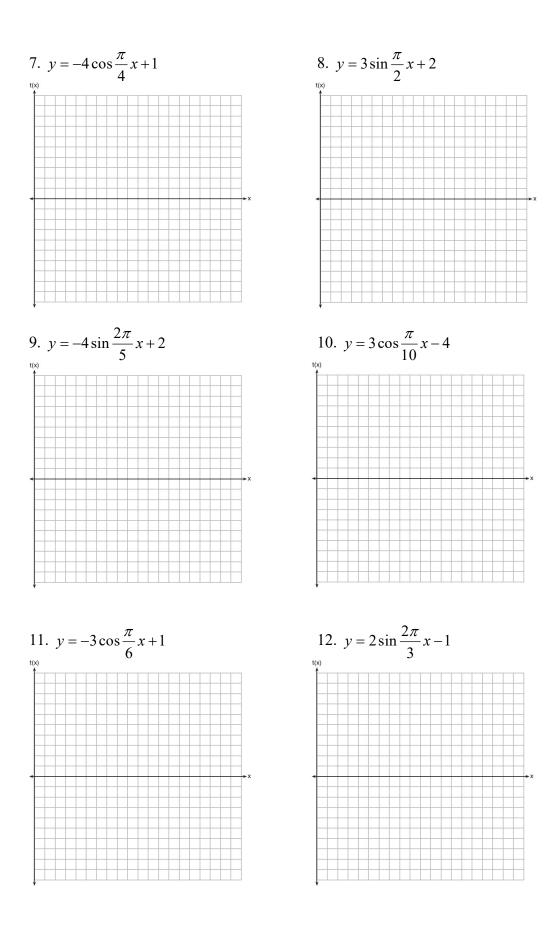


Date _____ Algebra II

Graphing Sinusoidal Curves Practice

Graph one full wave of the following trigonometric functions and state the domain and range.



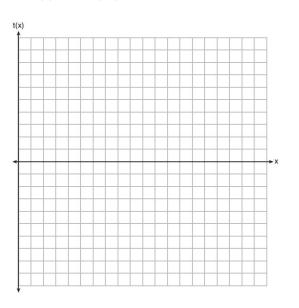


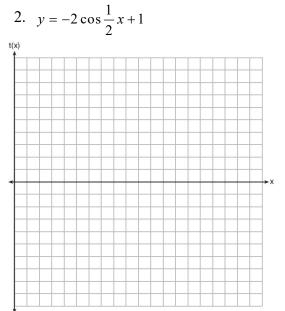
Date _____ Algebra II

Graphing Sinusoidal Curves Over Given Domains

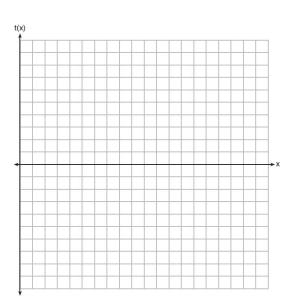
Graph the following two functions over the domain $[0, 2\pi]$ on the set of axes below.

1. $t(x) = 3\sin(2x) + 2$

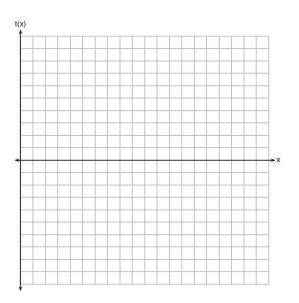


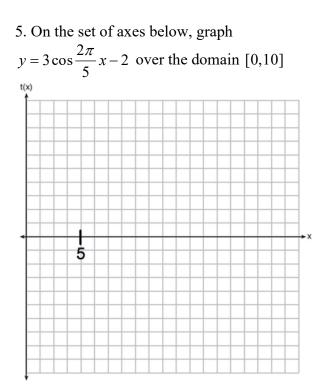


3. $y = -2\sin 4x + 3$

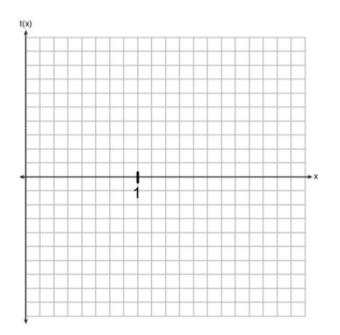


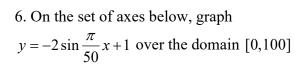
4.
$$y = 3\cos\frac{1}{2}x - 5$$

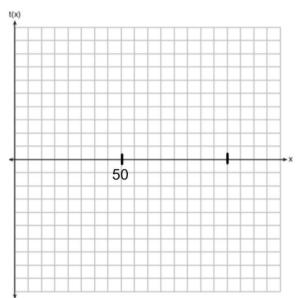




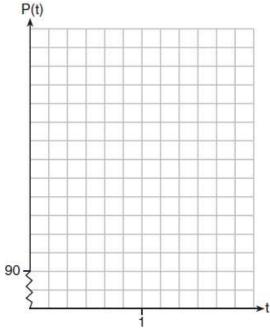
7. On the set of axes below, graph $y = 3 \sin 4\pi x - 2$ over the domain [0,2]







8. On the set of axes below, graph $P(t) = 24 \cos(3\pi t) + 120$ over the domain $0 \le t \le 2$.

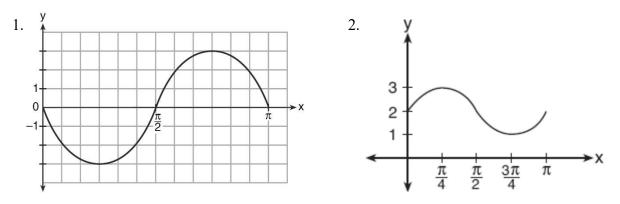


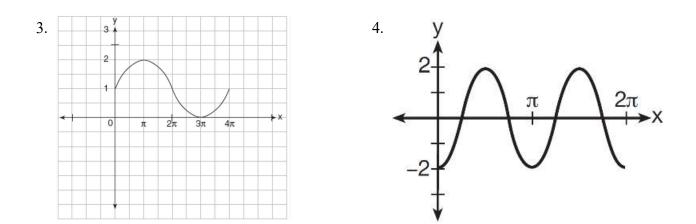


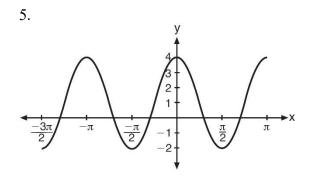
Date _____ Algebra II

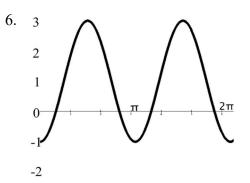
Writing Equations of Sinusoidal Graphs

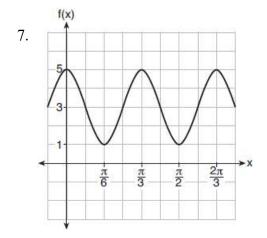
Write an equation for the graph of the trigonometric functions shown below.

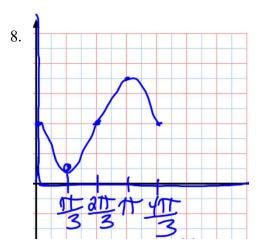


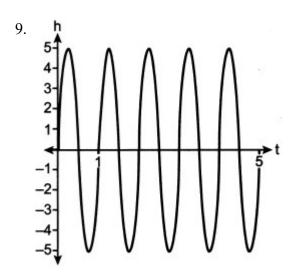


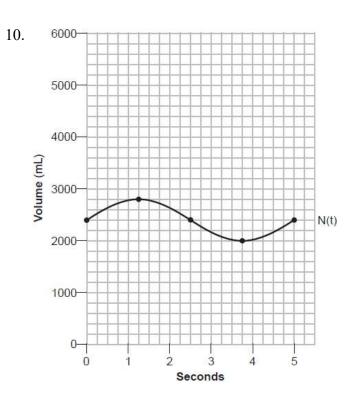












11. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

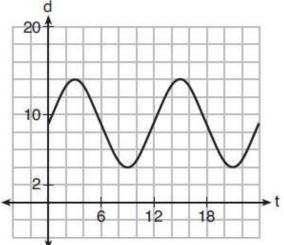
If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker? d

1)
$$d = 5\cos\left(\frac{\pi}{6}t\right) + 9$$

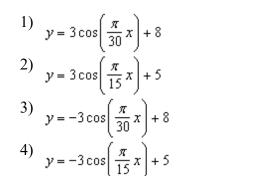
2)
$$d = 9\cos\left(\frac{\pi}{6}t\right) + 5$$

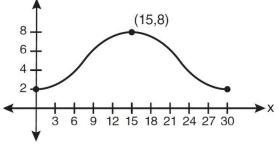
3)
$$d = 9\sin\left(\frac{\pi}{6}t\right) + 5$$

4)
$$d = 5\sin\left(\frac{\pi}{6}t\right) + 9$$



12. Which equation is graphed in the diagram below?

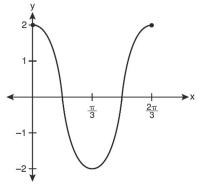




13. Which equation is represented by the graph below?

1) $y = 2\cos 3x$ 2) $y = 2\sin 3x$ 3) $y = 2\cos \frac{2\pi}{2}x$

$$(4) \quad y = 2\sin\frac{2\pi}{3}x$$



Date _____ Algebra II



Sinusoidal Applications

1. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7?$

- 1) The period is 6.
- 2) The amplitude is 3.
- 3) The range is [4,10].
- 4) The midline is y = -4.

2. Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) $y = -4\cos\left(\frac{\pi}{4}x\right) - 3$ 2) $y = -4\cos\left(\frac{\pi}{4}x\right) + 5$ 3) $y = -4\cos(8x) - 3$ 4) $y = -4\cos(8x) + 5$

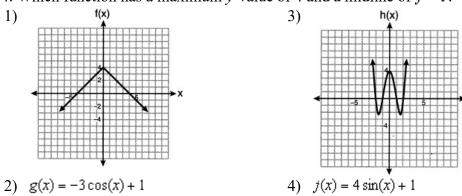
3. The equation below can be used to model the height of a tide in feet, H(t), on a beach at t hours.

$$H(t) = 4.8 \sin\left(\frac{\pi}{6} (t+3)\right) + 5.1$$

Using this function, the amplitude of the tide is

 1) $\frac{\pi}{6}$ 3) 3

 2) 4.8
 4) 5.1



4. Which function has a maximum *y*-value of 4 and a midline of y = 1?

5. The depth of the water, d(t), in feet, on a given day at Thunder Bay, *t* hours after midnight is modeled by $d(t) = 5 \sin\left(\frac{\pi}{6}(t-5)\right) + 7$. Which statement about the Thunder Bay tide is *false*? 1) A low tide occurred at 2 a.m. 3) The water depth at 9 a.m. was

- 2) The maximum depth of the water was 4 12 feet.
- approximately 11 feet.4) The difference in water depth between high tide and low tide is 14 feet.

6. A person's lung capacity can be modeled by the function $C(t) = 250 \sin\left(\frac{2\pi}{5}t\right) + 2450$, where

C(t) represents the volume in mL present in the lungs after *t* seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

7. Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation

 $B(x) = 23.914 \sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238 \sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models *x* months after starting data collection?

- 1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
- 2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
- The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
- 4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

8. The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

- 1) amplitude
- 2) horizontal shift

3) period

9. Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the time of the next high tide, the most important value to have would be the

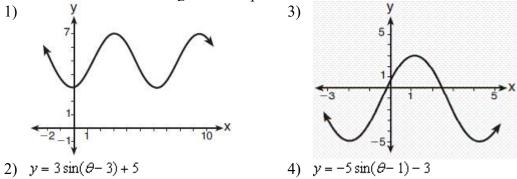
1) time between consecutive low tides

- 3) average depth of water over a 24-hour period
- 2) time when the tide height is 20 feet
- 4) difference between the water heights at low and high tide

10. Consider the function $h(x) = 2\sin(3x) + 1$ and the function *q* represented in the table below. Determine which function has the *smaller* minimum value for the domain [-2, 2]. Justify your answer.

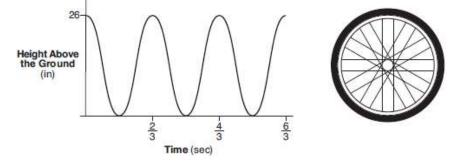
x	q(x)
-2	-8
-1	0
0	0
1	-2
2	0

11. Which sinusoid has the greatest amplitude?

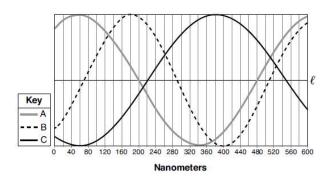


12. The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.

Identify the period of the graph and describe what the period represents in this context.



13. Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled ℓ . Based on the graph, which light wave has the longest period? Justify your answer.



14. The Sea Dragon, a pendulum ride at an amusement park, moves from its central position at rest according to the trigonometric function $P(t) = -10\sin(\frac{\pi}{3}t)$, where *t* represents time, in seconds. How many seconds does it take the pendulum to complete one full cycle? 1) 5 3) 3 2) 6 4) 10

15. A wave displayed by an oscilloscope is represented by the equation $y = 3 \sin x$. What is the period of this function?

1) 2π	3) 3
2) 2	4) 3π

16. The height above ground for a person riding a Ferris wheel after *t* seconds is modeled by $h(t) = 150 \sin\left(\frac{\pi}{45}t + 67.5\right) + 160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel? 1) 10 3) 90 2) 45 4) 150

17. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos \theta$ will1) decrease from 1 to 03) increase from -1 to 02) decrease from 0 to -14) increase from 0 to 1

18. Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p 1) decreases, then increases

- 2) increases, then decreases
- 3) decreases throughout the interval
- 4) increases throughout the interval
- 19. As x increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y = 2 \tan x$ will 1) increase from 0 to 2 3) increase without limit
- 2) decrease from 0 to -24) decrease without limit
- 20. As x increases from $\frac{3\pi}{2}$ to 2π , the graph of $y = \csc x$ will 1) increase without limit 3) increase to -1 2) decrease without limit 4) decrease to 1

21. As x increases from
$$-\frac{\pi}{2}$$
 to 0, the graph of $y = \sec x$ will
1) increase without limit
3) increase to -1
2) decrease without limit
4) decrease to 1



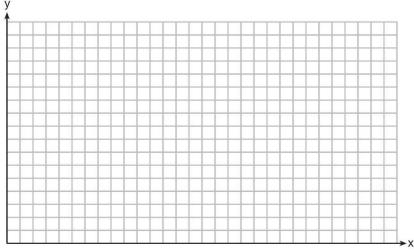
Date _____ Algebra II

Graphing Sinusoidal Models

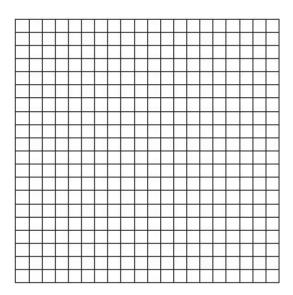
1. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. A passenger's height, in feet, above the ground after t minutes can be modeled by the equation

 $h(t) = -260\cos\left(\frac{\pi}{15}t\right) + 290$. Graph one full cycle of h(t) on the axes provided. Identify the period

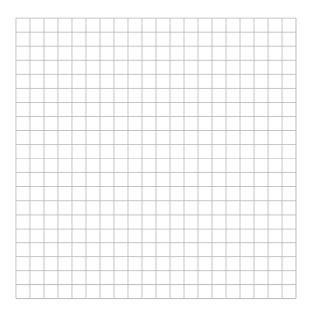
and state its meaning in the context of the problem. To the *nearest tenth of a second*, after how much time will the passenger first reach a height of 500 feet?



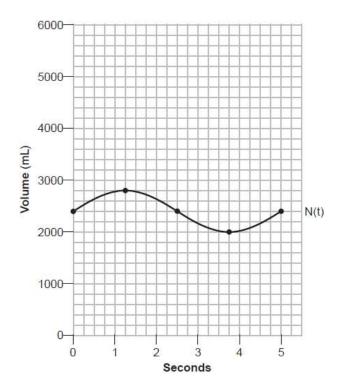
2. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8 \pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of f(t). Interpret what the period represents in this context. On the grid below, graph *at least one* cycle of f(t) that includes the y-intercept of the function. Does the height of the nail ever reach 30 inches above the ground? Justify your answer.



3. The ocean tides near Carter Beach follow a repeating pattern over time, which can be modeled by the equation $h(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ where h(t) represents height above sea level and *t* represents hours after 8:30 AM. On the grid below, graph one cycle of this function. Determine the period and state its meaning in the context of the problem. People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.



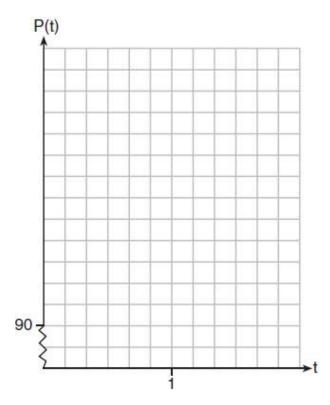
4. The volume of air in an average lung during breathing can be modeled by the graph below. Using the graph, write an equation for N(t), in the form $N(t) = A \sin(Bt) + C$. That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where E(t) is volume in mL and t is time in seconds. Graph *at least one* cycle of E(t) on the same grid as N(t). How many times during the 5-second interval will N(t) = E(t)?



5. The resting blood pressure of an adult patient can be modeled by the function P below, where P(t) is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

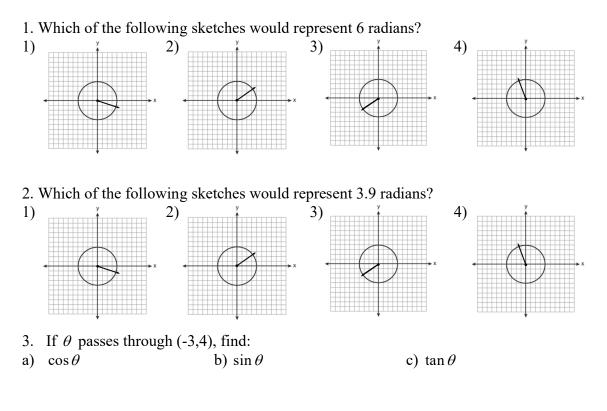
On the set of axes below, graph y = P(t) over the domain $0 \le t \le 2$.



Determine the period of *P*. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.



Trigonometry Review Sheet



d) $\sec \theta$ e) $\csc \theta$ f) $\cot \theta$

4. If θ passes through (2,-7), sec θ must be: 1) $\frac{\sqrt{53}}{7}$ 2) $\frac{\sqrt{53}}{2}$ 3) $-\frac{\sqrt{53}}{7}$ 4) $-\frac{\sqrt{53}}{2}$

5. If $\sin\theta = \frac{5}{6}$	and θ is in Quadrant II, find:
a) $\cos \theta$	b) $\sin \theta$

d)
$$\sec \theta$$
 e) $\csc \theta$ f) $\cot \theta$

6. If
$$\cos \theta = -\frac{5}{4}$$
 and θ is in Quadrant III, then $\sin \theta$ is equivalent to
1) $-\frac{\sqrt{7}}{4}$
2) $\frac{\sqrt{7}}{4}$
4) $\frac{5}{4}$

7. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ , intercepts the circle in Quadrant III at point *P*. The *x*-coordinate of point *P* is 2. Find all six trigonometric functions.

c) $\tan \theta$

a) $\cos \theta$ b) $\sin \theta$ c) $\tan \theta$

d) $\sec \theta$ e) $\csc \theta$ f) $\cot \theta$

8. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ , intercepts the circle in Quadrant II at point *C*. The *y*-coordinate of point *C* is 8. What is the value of $\cos \theta$?

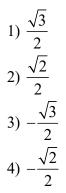
 $\begin{array}{ccc} 1) -\frac{3}{5} & 3) \frac{3}{5} \\ 2) -\frac{3}{4} & 4) \frac{4}{5} \end{array}$

9. What is the exact value of $\cos\left(\frac{5\pi}{6}\right)$?

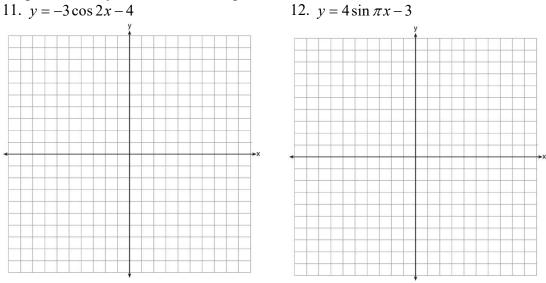
1)
$$\frac{\sqrt{3}}{2}$$

2) $\frac{1}{2}$
3) $-\frac{\sqrt{3}}{2}$
4) $-\frac{1}{2}$

10. What is the exact value of $\tan\left(\frac{3\pi}{4}\right)$?



Graph one full cycle of the following sinusoidal functions:



13. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

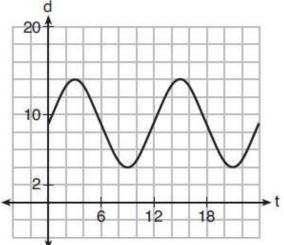
If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker? d

1)
$$d = 5\cos\left(\frac{\pi}{6}t\right) + 9$$

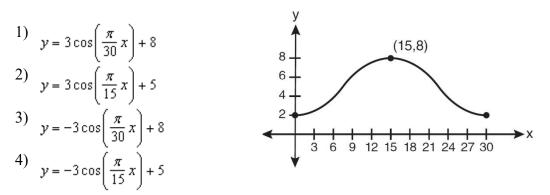
2)
$$d = 9\cos\left(\frac{\pi}{6}t\right) + 5$$

3)
$$d = 9\sin\left(\frac{\pi}{6}t\right) + 5$$

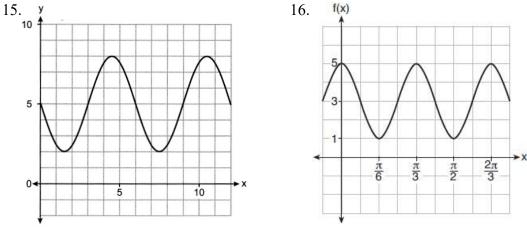
4)
$$d = 5\sin\left(\frac{\pi}{6}t\right) + 9$$



14. Which equation is graphed in the diagram below?



Write the equations of the sinusoidal functions given below.



17. A person's lung capacity can be modeled by the function $C(t) = 250 \sin\left(\frac{2\pi}{5}t\right) + 2450$, where C(t) represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

18. The function $d(t) = 2\cos\left(\frac{\pi}{6}t\right) + 5$ models the water depth, in feet, at a location in a bay, t hours since the last high tide. Determine the *minimum* water depth of the location, in feet, and justify your answer.

- 19. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos \theta$ will
- 1) decrease from 1 to 03) increase from -1 to 0
- 2) decrease from 0 to -1 4) increase from 0 to 1

20.	Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the	interval $-\pi < \theta < \pi$, the function p
1)	decreases, then increases	3) decreases throughout the interval
2)	increases, then decreases	4) increases throughout the interval

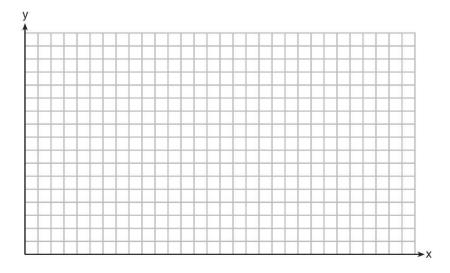
21. The monthly high temperature (°F) in Buffalo, New York can be modeled by $B(m) = 24.9 \sin(0.5m - 2.05) + 55.25$, where *m* is the number of the month and January = 1. Find the average rate of change in the monthly high temperature between June and October, to the *nearest* hundredth.

22. The height, h(t) in cm, of a piston, is given by the equation $h(t) = 12 \cos\left(\frac{\pi}{3}t\right) + 8$, where t

represents the number of seconds since the measurements began. Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \le t \le 2$.

23. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. A passenger's height, in feet, above the ground after *t* minutes can be modeled by the equation $h(t) = -260 \cos\left(\frac{\pi}{15}t\right) + 290$. Graph one full cycle of h(t) on the axes provided. Identify the period

and state its meaning in the context of the problem.



24. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8 \pi t) + 13$, where *t* represents the time (in seconds) since the nail first became caught in the tire. Determine the period of f(t). Interpret what the period represents in this context. On the grid below, graph *at least one* cycle of f(t) that includes the *y*-intercept of the function.

