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Date ____________
Pre Calculus

Trigonometric Ratios with Identities

Functions of the Double Angle

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Functions of the Sum of Two Angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

Functions of the Difference of Two Angles

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

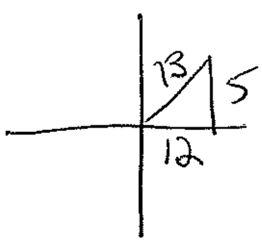
Functions of the Half Angle

$\sin\frac{1}{2}A = \pm\sqrt{\dfrac{1 - \cos A}{2}}$

$\cos\frac{1}{2}A = \pm\sqrt{\dfrac{1 + \cos A}{2}}$

$\tan\frac{1}{2}A = \pm\sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

1. If $\sin A = \dfrac{5}{13}$, and A is an angle in quadrant I, find the value of $\cos 2A$.



$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{5}{13}\right)^2$

$= 1 - 2\left(\frac{25}{169}\right)$

$\frac{169}{169} - \frac{50}{169} = \frac{119}{169}$

2. If $\cos\theta = \dfrac{\sqrt{7}}{4}$, and $0 < x < \dfrac{\pi}{2}$, find the value of $\cos 2\theta$.

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{\sqrt{7}}{4}\right)^2 - 1$

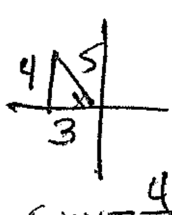
$= 2\left(\frac{7}{16}\right) - 1$

$\frac{14}{16} - \frac{16}{16}$

$-\frac{2}{16}$

$\boxed{-\frac{1}{8}}$

3. If $\sin x = \dfrac{4}{5}$, and $\dfrac{\pi}{2} < x < \pi$, find the value of $\sin 2x$.



$\sin x = \frac{4}{5}$

$\cos x = -\frac{3}{5}$

$\sin 2x = 2\sin x\cos x$

$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= -\frac{24}{25}$

4. If $\sin B < 0$ and $\tan B = -\frac{3}{4}$, find the value of $\tan 2B$.

$$\tan 2B = \frac{2\tan B}{1-\tan^2 B}$$

$$\tan 2B = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$$

$$\frac{-\frac{6}{4}}{\frac{16}{16}-\frac{9}{16}}$$

$$\frac{-\frac{6}{4}}{\frac{7}{16}}$$

$$-\frac{6}{\cancel{4}}\cdot\frac{\cancel{16}^{\,4}}{7}$$

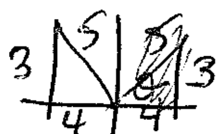
$$\boxed{-\frac{24}{7}}$$

QII $\frac{1}{2}$QII = QI

$\frac{9}{5}\cdot\frac{1}{2}=\frac{9}{10}$

5. If $\sin\theta = \frac{3}{5}$, and $\cos\theta < 0$, find $\sin\frac{1}{2}\theta$.

$$\sin\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}}$$



$\cos\theta = \frac{-4}{5}$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{1+\frac{4}{5}}{2}}$$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}}$$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{\frac{9}{5}}{\frac{2}{1}}}$$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{9}{10}}$$

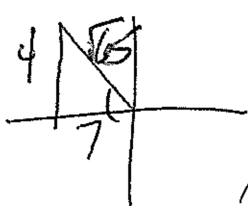
$$= \frac{3}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}}$$

$$\sin\tfrac{1}{2}\theta = \frac{3\sqrt{10}}{10}$$

θ = QII $\frac{1}{2}\theta$ = QI

6. If $\tan\theta = -\frac{4}{7}$, and $\sin\theta > 0$, find $\cos\frac{1}{2}\theta$.

$$\cos\tfrac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}}$$



$a^2+b^2=c^2$

$4^2+7^2=c^2$

$16+49=c^2$

$\sqrt{65}=\sqrt{c^2}$

$\sqrt{65}=c$

$$\cos\theta = \frac{-7}{\sqrt{65}}\frac{\sqrt{65}}{\sqrt{65}}$$

$$\cos\theta = \frac{-7\sqrt{65}}{65}$$

$$+\sqrt{\frac{\frac{1}{1}-\frac{7\sqrt{65}}{65}}{2}}$$

$$\pm\sqrt{\frac{\frac{65}{65}-\frac{7\sqrt{65}}{65}}{2}}$$

$$\pm\sqrt{\frac{\frac{65-7\sqrt{65}}{65}}{\frac{2}{1}}}$$

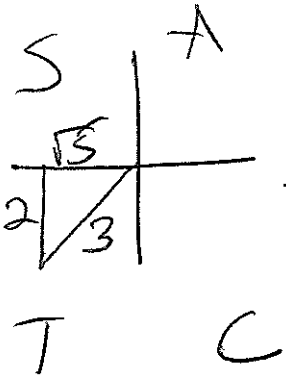
$$\pm\sqrt{\frac{65-7\sqrt{65}}{65}\cdot\frac{1}{2}}$$

$$+\sqrt{\frac{65-7\sqrt{65}}{130}}$$

$\theta = QIII \quad \frac{1}{2}\theta = QII$

7. If $\sin\theta = -\frac{2}{3}$, and $\pi < \theta < \frac{3\pi}{2}$, find $\tan\frac{1}{2}\theta$.

$\tan\frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$



$2^2 + b^2 = 3^2$

$4 + b^2 = 9$

$-4 \quad -4$

$\sqrt{b^2} = \sqrt{5}$

$b = \sqrt{5}$

$\cos\theta = \frac{-\sqrt{5}}{3}$

$\tan\frac{1}{2}\theta = -\sqrt{\frac{1+\frac{\sqrt{5}}{3}}{1-\frac{\sqrt{5}}{3}}}$

$= -\sqrt{\frac{\frac{3}{3}+\frac{\sqrt{5}}{3}}{\frac{3}{3}-\frac{\sqrt{5}}{3}}}$

$-\sqrt{\frac{\frac{3+\sqrt{5}}{3}}{\frac{3-\sqrt{5}}{3}}}$

$-\sqrt{\frac{3+\sqrt{5}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{5}}}$

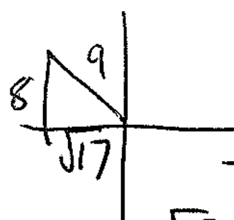
$-\sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}}$

$\theta = QII \quad \frac{1}{2}\theta = QI$

8. If $\csc\theta = \frac{9}{8}$, and $\cos\theta < 0$, find $\cos\frac{1}{2}\theta$.

$\sin\theta = \frac{8}{9}$

$\cos\frac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}}$



$8^2 + b^2 = 9^2$

$64 + b^2 = 81$

$-64 \quad -64$

$b^2 = 17$

$b = \sqrt{17}$

$\cos\theta = \frac{-\sqrt{17}}{9}$

$+\sqrt{\frac{1-\frac{\sqrt{17}}{9}}{2}}$

$+\sqrt{\frac{\frac{9}{9}-\frac{\sqrt{17}}{9}}{2}}$

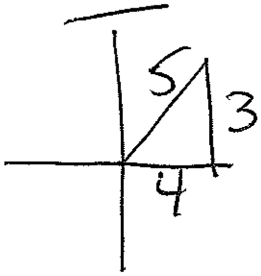
$+\sqrt{\frac{\frac{9-\sqrt{17}}{9}}{\frac{2}{1}}}$

$+\sqrt{\frac{9-\sqrt{17}}{9} \cdot \frac{1}{2}}$

$+\sqrt{\frac{9-\sqrt{17}}{18}}$

9. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, and angles A and B are positive acute angles, find $\cos(A - B)$

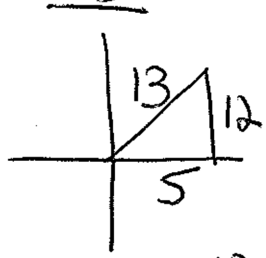
A



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

B



$\sin B = \frac{12}{13}$

$\cos B = \frac{5}{13}$

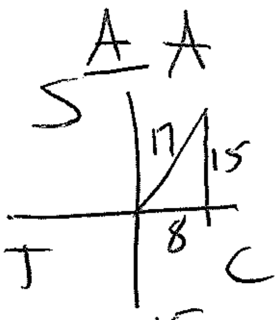
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A-B) = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{20}{65} + \frac{36}{65}$

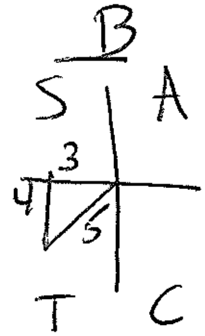
$= \frac{56}{65}$

10. If angle A terminates in quadrant I with $\cos A = \frac{8}{17}$ and angle B terminates in quadrant III with $\tan B = \frac{4}{3}$, find the value of $\sin(A + B)$.



$\sin A = \frac{15}{17}$

$\cos A = \frac{8}{17}$



$\sin B = \frac{-4}{5}$

$\cos B = \frac{-3}{5}$

$\sin A \cos B + \cos A \sin B$

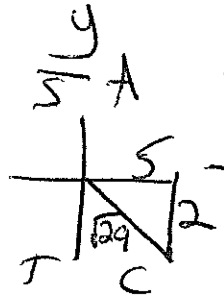
$\left(\frac{15}{17}\right)\left(\frac{-3}{5}\right) + \left(\frac{8}{17}\right)\left(\frac{-4}{5}\right)$

$\frac{-45}{85} - \frac{32}{85}$

$-\frac{77}{85}$

11. If $\tan x = -\frac{5}{4}$ and $\cos y = \frac{5}{\sqrt{29}}$, and x terminates in quadrant II and y terminates in quadrant IV, find the value of $\tan(x - y)$.

$\tan X = \frac{-5}{4}$



$a^2 + b^2 = c^2$

$5^2 + b^2 = \sqrt{29}^2$

$25 + b^2 = 29$

$-25 \quad -25$

$\sqrt{b^2} = \sqrt{4}$

$b = 2$

$\tan y = -\frac{2}{5}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\tan(x-y) = \frac{-\frac{5}{4} + \frac{2}{5}}{1 + \left(-\frac{5}{4}\right)\left(-\frac{2}{5}\right)}$

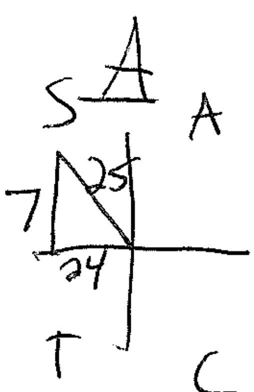
$\tan(x-y) = \frac{-\frac{25}{20} + \frac{8}{20}}{\frac{20}{20}\ 1 + \frac{10}{20}}$

$\frac{\frac{-17}{20}}{\frac{30}{20}}$

$\frac{-17}{\cancel{20}} \cdot \frac{\cancel{20}}{30}$

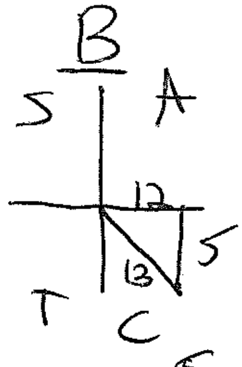
$\frac{-17}{30}$

12. If $\frac{\pi}{2} < A < \pi$ with $\sin A = \frac{7}{25}$ and $\frac{3\pi}{2} < B < 2\pi$ with $\sin B = -\frac{5}{13}$, find the value of $\sin(A - B)$.



$\sin A = \frac{7}{25}$

$\cos A = -\frac{24}{25}$



$\sin B = \frac{5}{13}$

$\cos B = \frac{12}{13}$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \left(\frac{7}{25}\right)\left(\frac{12}{13}\right) - \left(\frac{-24}{25}\right)\left(-\frac{5}{13}\right)$

$\frac{84}{325} - \frac{120}{325}$

$\frac{-36}{325}$