

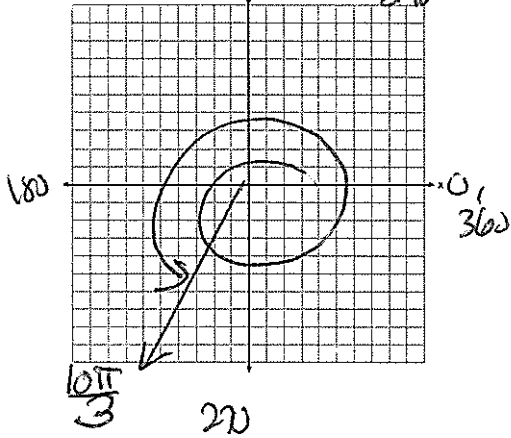
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Date \_\_\_\_\_  
Pre Calculus

## Unit 6: Trigonometry Review Sheet

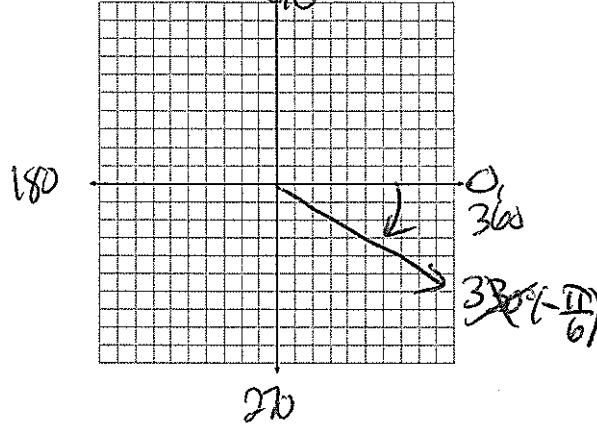
Sketch the following angles on the grid

$$1. \theta = \frac{10\pi}{3} \cdot \frac{180}{\pi} = \frac{600^\circ}{240} = 240^\circ$$



$$2. \theta = \frac{\pi}{6} \cdot \frac{180}{\pi} = -30^\circ$$

$$\begin{array}{r} -30 \\ +360 \\ \hline 330 \end{array}$$



3. If  $\sin \theta = \frac{5}{8}$  and  $\theta$  is in Quadrant II, find:

a)  $\cos \theta$

$$-\frac{\sqrt{39}}{8}$$

b)  $\sin \theta$

$$\frac{5}{8}$$

c)  $\tan \theta$

$$\frac{5}{\sqrt{39}} \cdot \frac{\sqrt{39}}{\sqrt{39}}$$

$$-\frac{5\sqrt{39}}{39}$$

d)  $\sec \theta$

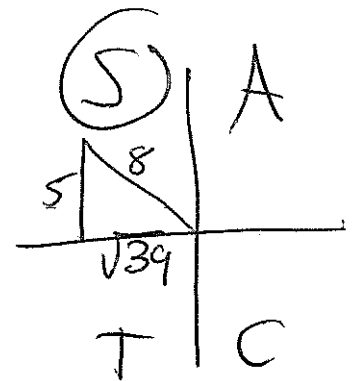
$$\frac{8\sqrt{39}}{\sqrt{39}\sqrt{39}} = \frac{8\sqrt{39}}{39}$$

e)  $\csc \theta$

$$\frac{8}{5}$$

f)  $\cot \theta$

$$-\frac{\sqrt{39}}{5}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + b^2 &= 8^2 \\ 25 + b^2 &= 64 \\ -25 \quad -25 & \\ \hline b^2 &= 39 \\ b &= \sqrt{39} \end{aligned}$$

4. Angle  $\theta$  is in standard position and  $(4, -7)$  is a point on the terminal side of  $\theta$ . Find:

a)  $\cos \theta$

$$\frac{4}{\sqrt{65}}$$

$$\frac{4\sqrt{65}}{65}$$

b)  $\sin \theta$

$$\frac{-7}{\sqrt{65}}$$

$$-\frac{7\sqrt{65}}{65}$$

c)  $\tan \theta$

$$-\frac{7}{4}$$

d)  $\sec \theta$

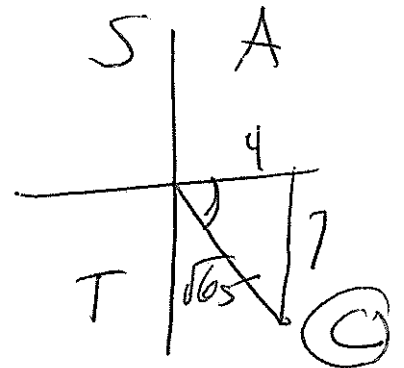
$$\frac{\sqrt{65}}{4}$$

e)  $\csc \theta$

$$-\frac{\sqrt{65}}{7}$$

f)  $\cot \theta$

$$-\frac{4}{7}$$



$$a^2 + b^2 = c^2$$

$$4^2 + 7^2 = c^2$$

$$16 + 49 = c^2$$

$$\sqrt{65} = \sqrt{c^2}$$

$$\sqrt{65} = c$$

Q1

5. If  $\cos \theta = \frac{\sqrt{7}}{4}$ , and  $0 < \theta < \frac{\pi}{2}$ , find the value of  $\cos 2\theta$ .

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 2\left(\frac{\sqrt{7}}{4}\right)^2 - 1$$

$$2\left(\frac{7}{16}\right) - 1$$

$$\frac{14}{16} - \frac{16}{16}$$

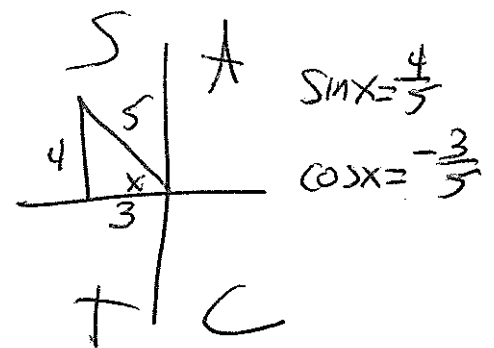
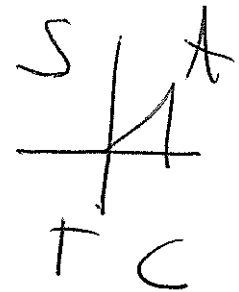
$$-\frac{2}{16}$$

$$-\frac{1}{8}$$

6. If  $\sin x = \frac{4}{5}$ , and  $\frac{\pi}{2} < x < \pi$ , find the value of  $\sin 2x$ .

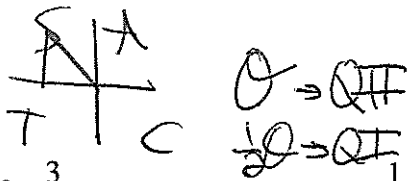
$$\sin 2x = 2\sin x \cos x$$

$$\sin 2x = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$



$$\sin x = \frac{4}{5}$$

$$\cos x = -\frac{3}{5}$$



7. If  $\sin \theta = \frac{3}{5}$ , and  $\cos \theta < 0$ , find  $\sin \frac{1}{2} \theta$ .

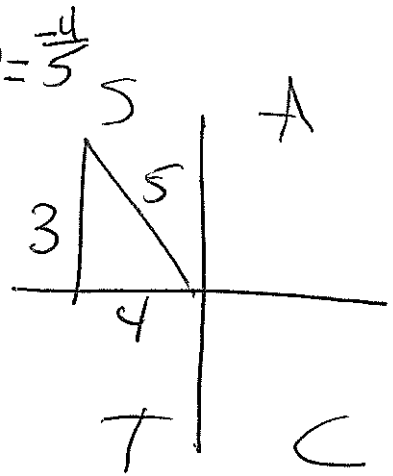
$$\sin \frac{1}{2} \theta = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin \frac{1}{2} \theta = \pm \sqrt{\frac{1 + \frac{4}{5}}{2}}$$

$$\sin \frac{1}{2} \theta = \pm \sqrt{\frac{\frac{5}{5} + \frac{4}{5}}{2}}$$

$$\sin \frac{1}{2} \theta = \pm \sqrt{\frac{9}{5 \cdot 2}}$$

$$\sin \frac{1}{2} \theta = \pm \frac{3\sqrt{10}}{10}$$



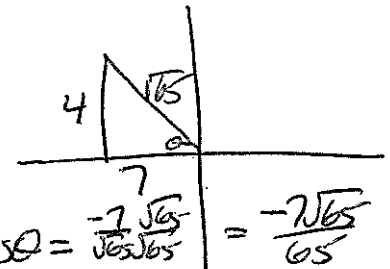
8. If  $\tan \theta = -\frac{4}{7}$ , and  $\sin \theta > 0$ , find  $\cos \frac{1}{2} \theta$ .

$$\cos \frac{1}{2} \theta = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\cos \frac{1}{2} \theta = \pm \sqrt{\frac{1 - \frac{7\sqrt{5}}{65}}{2}}$$

$$\cos \frac{1}{2} \theta = \pm \sqrt{\frac{65 - 7\sqrt{5}}{130}}$$

$$\cos \frac{1}{2} \theta = \frac{\sqrt{65 - 7\sqrt{5}}}{\sqrt{130}}$$



$$\cos \theta = \frac{-7\sqrt{5}}{7\sqrt{5} \cdot \sqrt{5}} = \frac{-7\sqrt{5}}{65}$$

$$a^2 + b^2 = c^2$$

$$4^2 + 3^2 = c^2$$

$$16 + 9 = c^2$$

$$\sqrt{25} = c$$

$$5 = c$$

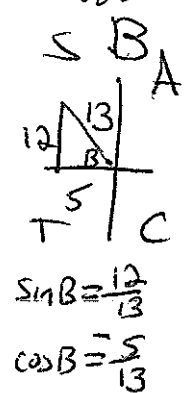
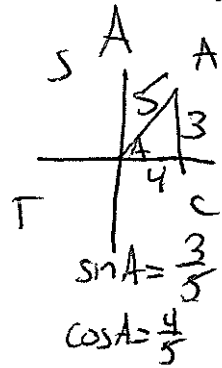
9. If  $\sin A = \frac{3}{5}$  and  $\cos B = -\frac{5}{13}$ , find  $\cos(A - B)$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A - B) = \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-20}{65} + \frac{36}{65}$$

$$= \frac{16}{65}$$



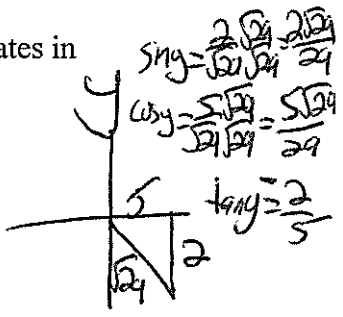
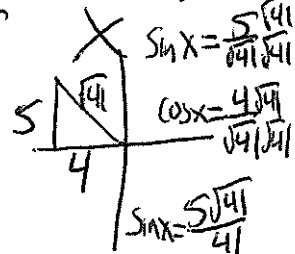
10. If  $\tan x = -\frac{5}{4}$  and  $\cos y = \frac{5}{\sqrt{29}}$ , and  $x$  terminates in quadrant II and  $y$  terminates in quadrant IV, find the value of  $\tan(x - y)$ .

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan x = -\frac{5}{4}$$

$$\tan y = \frac{2}{5}$$

$$\frac{-\frac{5}{4} - \frac{2}{5}}{1 + \left(-\frac{5}{4}\right)\left(\frac{2}{5}\right)} = \frac{-\frac{25}{20} - \frac{8}{20}}{\frac{20}{20} - \frac{10}{20}} = \frac{-\frac{33}{20}}{\frac{10}{20}} = -\frac{33}{10}$$



$$\frac{\left(\frac{5}{5}\right) - \frac{2(4)}{5(4)} - \frac{25}{20} + \frac{8}{20}}{1 + \left(-\frac{5}{4}\right)\left(\frac{2}{5}\right)} = \frac{-\frac{33}{20}}{\frac{10}{20}} = -\frac{33}{10}$$

$$a^2 + b^2 = c^2$$

$$4^2 + 3^2 = c^2$$

$$16 + 9 = c^2$$

$$\sqrt{25} = c$$

$$5 = c$$

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = \sqrt{29}^2$$

$$25 + b^2 = 29$$

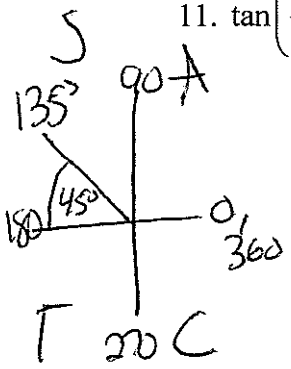
$$-25 -25$$

$$\sqrt{4} = b$$

$$b = 2$$

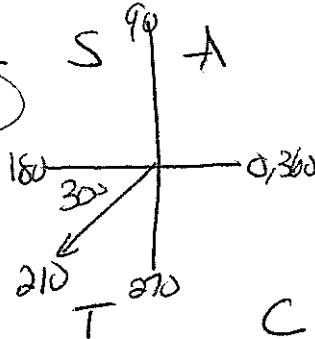
Find the exact value of the following

11.  $\tan\left(\frac{3\pi}{4}\right) \cdot \frac{180}{\pi} = 135^\circ$



Q S F R  
 II  $-\tan 45$   
 -1

12.  $\cos\left(\frac{7\pi}{6}\right) \cdot \frac{180}{\pi} = 210^\circ$

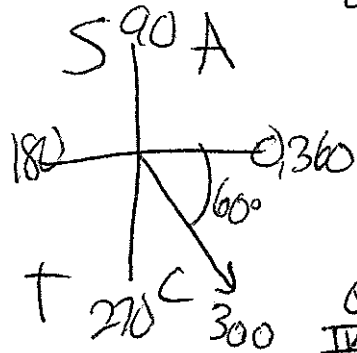


Q S F R  
 III  $-\cos 30$   
 $-\frac{\sqrt{3}}{2}$

	30	45	60
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

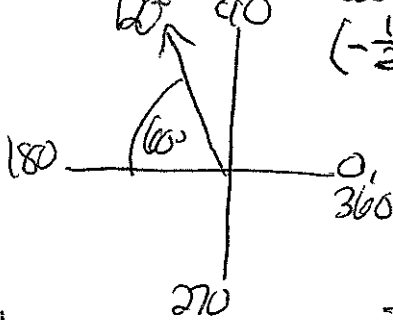
Find the point on the unit circle that terminates with the following angles:

13.  $\theta = 300^\circ$   $\cos 300, \sin 300$



$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$   
 $\cos 300$   
 Q S F R  
 IV  $+\cos 60$   $\frac{1}{2}$

14.  $\theta = 120^\circ$   $\cos 120, \sin 120$



$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$   
 $\cos 120$   
 Q S F R  
 II  $-\cos 60$   $-\frac{1}{2}$   
 $\sin 120$   
 Q S F R  
 II  $+\sin 60$   $\frac{\sqrt{3}}{2}$

Find the exact value of the following

15.  $\sin 75^\circ$

$\sin(45+30)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$   
 $\sin(45+30) = \sin 45 \cos 30 + \cos 45 \sin 30$   
 $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$   
 $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$

16.  $\tan 15^\circ$

$\tan(45-30)$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$   
 $\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$   
 $= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)(\frac{\sqrt{3}}{3})}$   
 $\frac{3 - \sqrt{3}}{3} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$   
 $\frac{3 - \sqrt{3}}{3 + \sqrt{3}}$

Express the following as a single trigonometric functions

17.  $\sec \theta \csc \theta \cos \theta$

$$\frac{1}{\cancel{\cos \theta}} \cdot \frac{1}{\sin \theta} \cdot \cancel{\cos \theta}$$

$$\frac{1}{\sin \theta} = \csc \theta$$

18.  $\csc \theta \tan \theta \cos \theta$

$$\frac{1}{\cancel{\sin \theta}} \cdot \frac{\cancel{\sin \theta}}{\cancel{\cos \theta}} \cdot \cancel{\cos \theta}$$

$$1$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

19.  $\sec^2 \theta (1 - \cos^2 \theta)$

$$\frac{1}{\cos^2 \theta} \frac{(\sin^2 \theta)}{1} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

20.  $\tan^2 \theta + (\sin^2 \theta + \cos^2 \theta)$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Solve the following equations for all values of  $\theta$  such that  $0 \leq \theta < 360$

21.  $2 \sin \theta + 1 = 0$

$$x = \sin \theta$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

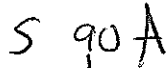
$$\sin^{-1} \sin \theta = \theta = \sin^{-1} \left(-\frac{1}{2}\right)$$

$$\theta = 210^\circ \text{ and } 330^\circ$$

$$R\theta = \sin^{-1} \left(\frac{1}{2}\right)$$

$$R\theta = 30^\circ$$

$$\theta = 210^\circ \text{ and } 330^\circ$$



22.  $3 \cos \theta + 1 = 1$

$$3x + 1 = 1$$

$$3x = 0$$

$$x = 0$$

$$\cos^{-1} \cos \theta = \theta = \cos^{-1}(0)$$

$$\theta = 90^\circ \text{ and } 270^\circ$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ \text{ and } 270^\circ$$

$$x = \cos \theta$$

	30	45	60
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

	0	90	180	270
sin	0	1	0	-1
cos	1	0	-1	0
tan	0	U	0	U

$$\cos 2\theta = 2\cos^2\theta - 1$$

~~$$\cos 2\theta = 1 - 2\cos^2\theta$$~~

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$X = \sin\theta$$

$$X = \cos\theta$$

23.  $3\cos 2\theta + 2\sin\theta = -1$

24.  $\cos 2\theta + \cos\theta = -1$

$$3(1 - 2\sin^2\theta) + 2\sin\theta = -1$$

$$3(1 - 2x^2) + 2x = -1$$

$$3 - 6x^2 + 2x = -1$$

$$-6x^2 + 2x + 4 = 0$$

$$\frac{-2 \pm \sqrt{4 + 96}}{-12}$$

$$3x^2 - x - 2 = 0$$

$$3x^2 - 3x + 2x - 2 = 0$$

$$3x(x-1) + 2(x-1) = 0$$

$$(3x+2)(x-1) = 0$$

$$3x+2=0 \quad x-1=0$$

$$x = -\frac{2}{3} \quad x = 1$$

$$\sin\theta = 1 \quad \theta = 0, 180$$

$$\sin\theta = -\frac{2}{3} \quad \theta = 0, 180, 220, 318$$

$$2\cos^2\theta - 1 + \cos\theta = -1$$

$$2x^2 + x = 0$$

$$x(2x+1) = 0$$

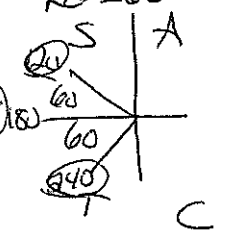
$$x=0 \quad 2x+1=0$$

$$x = -\frac{1}{2}$$

$$\cos\theta = 0 \quad \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}(0) \quad \theta = \cos^{-1}(-\frac{1}{2})$$

$$\theta = 90, 270 \quad \theta = 120, 240$$



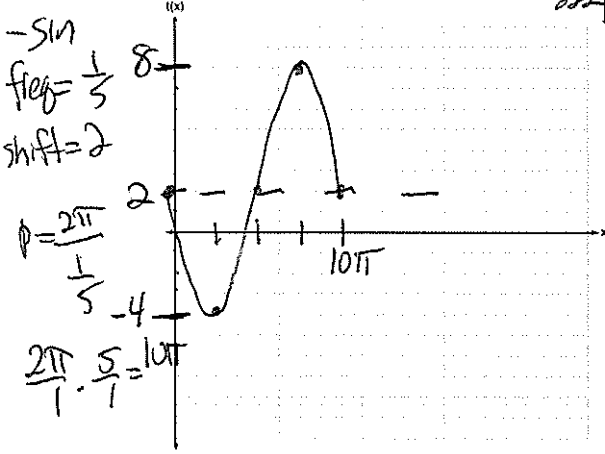
Graph one full cycle of each of the following trig equations

amp sin freq x shift

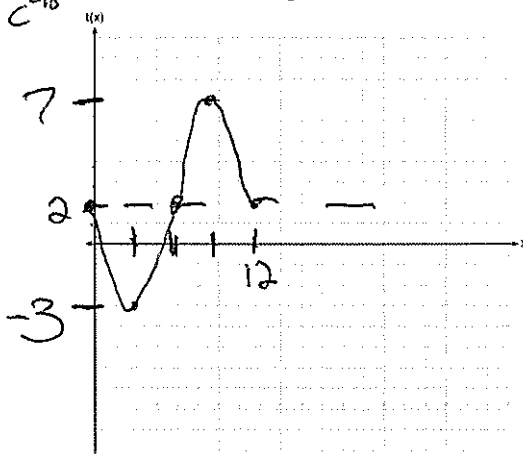
RO = 42°

amp sin freq x shift

25.  $y = -6\sin\frac{1}{5}x + 2$



26.  $y = -5\sin\frac{\pi}{6}x + 2$

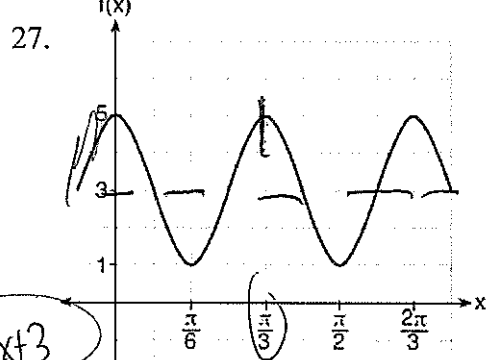


amp = 5  
-sin  
freq =  $\frac{\pi}{6}$   
shift = 2

$$P = \frac{2\pi}{\frac{\pi}{6}} = 12$$

Write the equation of each of the following trig graphs.

amp = 2  
+cos  
freq = 6  
shift = 3



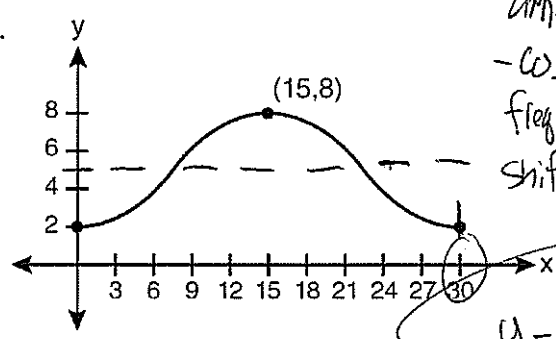
$$y = 2\cos 6x + 3$$

$$f = \frac{2\pi}{p}$$

$$f = \frac{2\pi}{\frac{\pi}{3}} = 6$$

$$\text{midline} = \frac{\text{min} + \text{max}}{2} = \frac{1 + 5}{2} = 3$$

28.



amp = 3  
-cos  
freq =  $\frac{\pi}{15}$   
shift = 5

$$y = -3\cos\frac{\pi}{15}x + 5$$

$$f = \frac{2\pi}{p}$$

$$f = \frac{2\pi}{30}$$

$$f = \frac{\pi}{15}$$

$$\text{midline} = \frac{\text{min} + \text{max}}{2} = \frac{2 + 8}{2} = 5$$