Variable Exponential Equations with a Common Base

1) Find a common exponential base
2) Convert each side to the common base (multiply when raising a power to a power)
3) Solve equation

1. $2^4 = 16^x$

$2^4 = (2^4)^x$

$2^4 = 2^{4x}$

$\frac{4}{4} = \frac{4x}{4}$

$1 = x$

2. $9^{3x} = 3^{3x+1}$

$(3^2)^{3x} = 3^{3x+1}$

$3^{6x} = 3^{3x+1}$

$6x = 3x + 1$

$-3x \quad -3x$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

3. $5^{x+1} = 125^{2x}$

$5^{x+1} = (5^3)^{2x}$

$5^{x+1} = 5^{6x}$

$x + 1 = 6x$

$-x \quad -x$

$\frac{1}{5} = \frac{5x}{5}$

$\frac{1}{5} = x$

4. $27^x = 9^{x+2}$

$(3^3)^x = (3^2)^{x+2}$

$3^{3x} = 3^{2(x+2)}$

$3x = 2(x+2)$

$3x = 2x + 4$

$-2x \quad -2x$

$x = 4$

5. $4^{2b-3} = 8^{1-b}$

$(2^2)^{2b-3} = (2^3)^{1-b}$

$2(2b-3) = 3(1-b)$

$4b - 6 = 3 - 3b$

$+3b \quad +3b$

$7b - 6 = 3$

$+6 \quad +6$

$\frac{7b}{7} = \frac{9}{7}$

$b = \frac{9}{7}$

6. $64^{x-2} = 256^{2x}$

$(4^3)^{x-2} = (4^4)^{2x}$

$3(x-2) = 8x$

$3x - 6 = 8x$

$-3x \quad -3x$

$\frac{-6}{5} = \frac{5x}{5}$

$x = \frac{-6}{5}$

7. $3^{m+1} - 5 = 22$

$+5 \quad +5$

$3^{m+1} = 27$

$3^{m+1} = 3^3$

$m + 1 = 3$

$-1 \quad -1$

$m = 2$

8. $27 - 3^x = 0$

$+3^x \quad +3^x$

$27 = 3^x$

$3^3 = 3^x$

$3 = x$