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Date _____
Algebra II

Finding k in a Polynomial Equation

1. Consider the polynomial $p(x) = x^3 + kx^2 + x + 6$. Find a value of k so that $x+1$ is a factor of P .
Find all the zeros of P .

$$\frac{x^3 - 4x^2 + x + 6}{x+1}$$

$$0 = (-1)^3 + k(-1)^2 + -1 + 6$$

$$p(-1) = 0$$

$$0 = -1 + k - 1 + 6$$

$$p(x) = x^3 - 4x^2 + x + 6$$

$$0 = k + 4$$

$$-4 = k$$

$$\begin{array}{r} | 1 & -4 & 1 & 6 \\ -1 & | & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$$p(x) = (x+1)(x^2 - 5x + 6)$$

$$0 = (x+1)(x-3)(x-2)$$

$$\boxed{x=-1 \quad x=3 \quad x=2}$$

2. Consider the polynomial $p(x) = x^3 + kx - 30$. Find a value of k so that $x+3$ is a factor of P .
Find all the zeros of P .

$$\frac{x^3 - 10x - 30}{x+3}$$

$$0 = (-3)^3 + k(-3) - 30$$

$$p(-3) = 0$$

$$0 = -27 - 3k - 30$$

$$p(1) = x^3 - 10x - 30$$

$$0 = -3k - 57$$

$$+57 \qquad +57$$

$$\begin{array}{r} | 1 & 0 & -10 & -30 \\ -3 & | & -3 & 9 & 30 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$$\frac{57}{3} = \frac{-3k}{3}$$

$$-19 = k$$

$$p(x) = (x+3)(x^2 - 3x - 10)$$

$$p(x) = (x+3)(x-5)(x+2)$$

$$\boxed{x=-3 \quad x=5 \quad x=2}$$

3. Given $p(x) = 6x^3 + 31x^2 + kx - 12$, and $p(-4) = 0$, algebraically determine all the zeros of $p(x)$.

$$\begin{aligned} p(x) &= (x+4)(6x^2 + 7x - 3) \\ &= (x+4)(x^2 + 7x - 18) \\ &\quad \frac{(x+9)(x-2)}{6} \\ &= (x+3)(6x-1) - 100 \\ &= (x+4)(2x+3)(3x-1) - 100 = \frac{-4K}{25} \\ &\quad \begin{array}{l|l} x+4=0 & 2x+3=0 \\ x=-4 & x=-\frac{3}{2} \\ x=-4 & x=\frac{1}{3} \end{array} \\ &25 = K \end{aligned}$$

$$p(x) = 6x^3 + 31x^2 + 25x - 12$$

$$\frac{6x^3 + 31x^2 + 25x - 12}{x+4}$$

$$\begin{array}{r} -4 \\ \hline 6 & 31 & 25 & -12 \\ & -24 & -28 & 12 \\ \hline 6 & 7 & -3 & 0 \end{array}$$

$$p(x) = (x+4)(6x^2 + 7x - 3)$$

4. Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

$$\begin{aligned} 0 &= b(-5)^3 + b(-5)^2 - 52(-5) + 15 \\ 0 &= -75b + 25b + 260 + 15 \\ 0 &= 25b - 475 \\ &\quad +475 \\ \frac{475}{25} &= \frac{25b}{25} \\ 19 &= b \end{aligned}$$

$$\begin{array}{r} x+5 \text{ is a factor} \\ z(x) = (x+5)(6x^2 - 11x + 3) \\ z(x) = (x+5)(x^2 - 11x + 18) \\ z(x) = (x+5)(x^2 - 9x - 2x + 18) \\ z(x) = (x+5)(x-3)(x-6) \\ z(x) = (x+5)(2x-3)(3x-1) \end{array}$$

$$\begin{array}{r} -5 \\ \hline 6 & 19 & -52 & 15 \\ & -30 & 55 & -15 \\ \hline 6 & -11 & 3 & 0 \end{array}$$

$$\begin{array}{r} x+5=0 \\ -5=5 \\ x=-5 \end{array} \quad \begin{array}{r} 2x-3=0 \\ 2x=3 \\ x=\frac{3}{2} \end{array} \quad \begin{array}{r} 3x-1=0 \\ 3x=1 \\ x=\frac{1}{3} \end{array}$$

5. Given $p(x) = x^3 + 5x^2 + kx - 24$, and $x+3$ is a factor, algebraically determine all the zeros of $p(x)$.

$$p(-3) = 0$$

$$\begin{aligned} 0 &= (-3)^3 + 5(-3)^2 + k(-3) - 24 \\ 0 &= -27 + 45 - 3k - 24 \\ 0 &= -3k - 6 \\ &\quad +6 \\ \frac{6}{3} &= \frac{-3k}{-3} \\ -2 &= k \end{aligned}$$

$$\frac{x^3 + 5x^2 + 2x - 24}{x+3}$$

$$\begin{array}{r} -3 \\ \hline 1 & 5 & -2 & -24 \\ & -3 & -6 & 24 \\ \hline 1 & 2 & -8 & 0 \end{array}$$

$$p(x) = (x+3)(x^2 + 2x - 8)$$

$$\begin{array}{r} x+3=0 \\ x=-3 \end{array} \quad \begin{array}{r} x+4=0 \\ x=-4 \end{array} \quad \begin{array}{r} x-2=0 \\ x=2 \end{array}$$