Name:

Common Core Algebra II

Unit 13

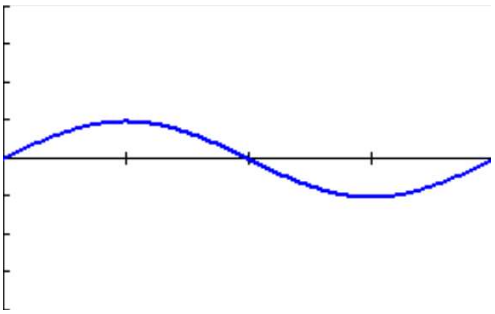
Graphing Trigonometric Functions

Mr. Schlansky

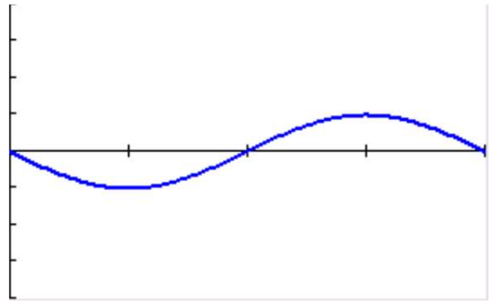
Lesson 1: I can graph sinusoidal functions with different amplitudes using AMPSINFREQXSHIFT **and knowing the four different waves.**

Know what your waves look like!

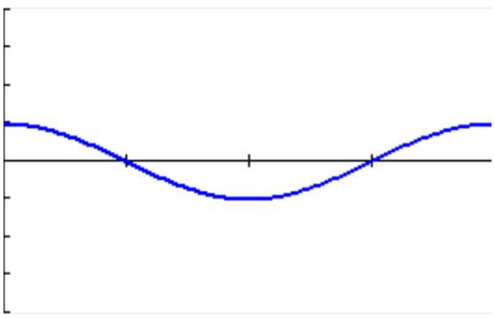
$f(x) = \sin x$



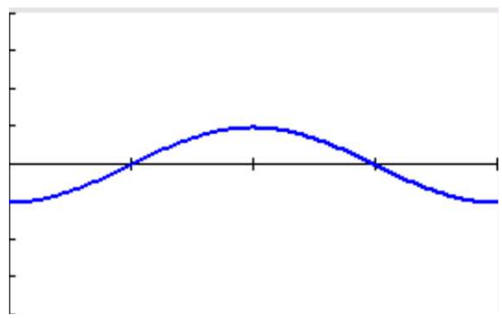
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT

Amplitude: Distance from the midline to minimum or maximum

Frequency: How many waves from 0 to 2π

Period: (Wavelength): How long it takes to make one full cycle

Shift: y value of the midline. The average value of the function.

$$Period = \frac{2\pi}{frequency}, Frequency = \frac{2\pi}{period}$$

To graph:

1) Use AMPSINFREQXSHIFT to determine the amplitude, sin or cos (+ or -), frequency, and shift/midline.
2) Use $Period = \frac{2\pi}{frequency}$ to find the period.
3) Draw in midline and make dashes for max and min on the y axis.
4) Make 4 dashes and put the period at the end of the 4^{th} dash on the x axis.
5) Draw in the appropriate curve.

Lesson 2: I can graph sinusoidal functions with different amplitudes and midlines using a AMPSINFREQXSHIFT **and knowing the four different waves.**

Same notes as Lesson 1.

Lesson 3: I can graph sinusoidal functions with different amplitudes, midlines, and frequencies using AMPSINFREQXSHIFT**, knowing the four different waves, and using** $period = \frac{2\pi}{frequency}$

Same notes as lesson 1

Lesson 4: I can graph sinusoidal functions with given domains by stretching or shrinking the waves.

Same notes as lesson 1. If the domain is bigger than the period, you may need to make multiple waves and/or compress them.

Lesson 5: I can write the equation of sinusoidal functions using AMPSINFREQXSHIFT**,** $midline = \frac{\min + \max}{2}$, and $frequency = \frac{2\pi}{period}$.

To write the equation:

1) Find midline ($\frac{\min + \max}{2}$)
2) Determine the period (the x value where the first full wave ends)
3) Use $Frequency = \frac{2\pi}{period}$ to determine the frequency
4) Substitute all values into AMPSINFREQXSHIFT

Lesson 6: I can graph sinusoidal models using AMPSINFREQXSHIFT **and drawing a little picture.**

Draw a little picture that contains the midline, min, max, and period to give you structure.
Same notes as Lessons 1 and 5.

Lesson 7: I can apply the components of sinusoidal functions in different ways by drawing a little picture.

Draw a little picture that contains the midline, min, max, and period to give you structure.
Same notes as Lessons 1 and 5.

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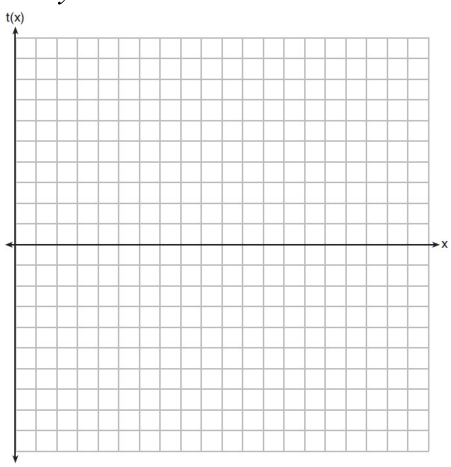
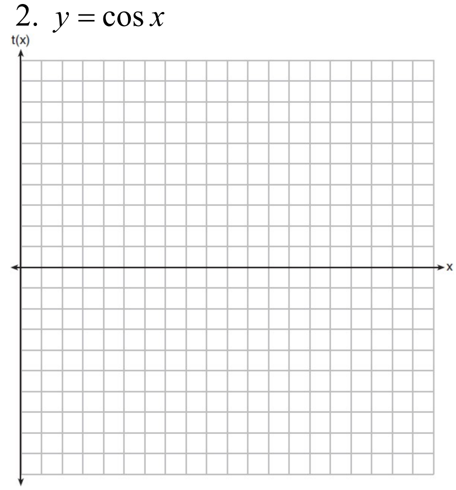
Graphing Sinusoidal Curves

Graph one full wave of the following trigonometric functions

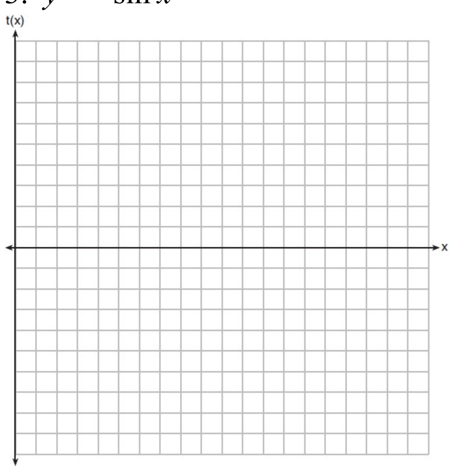
1. $y = \sin x$



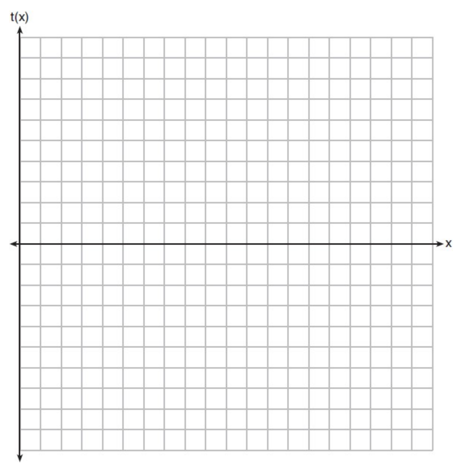
2. $y = \cos x$



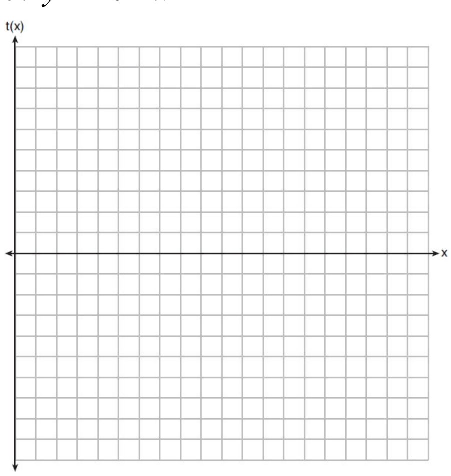
3. $y = -\sin x$



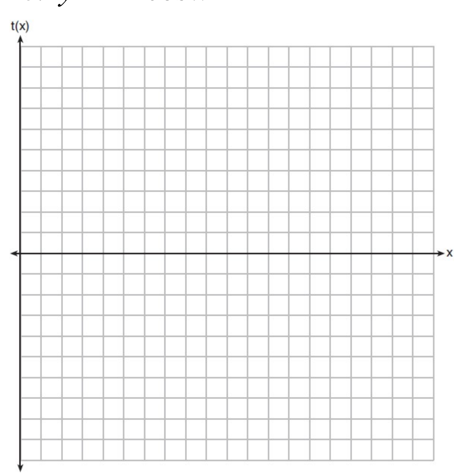
4. $y = -\cos x$



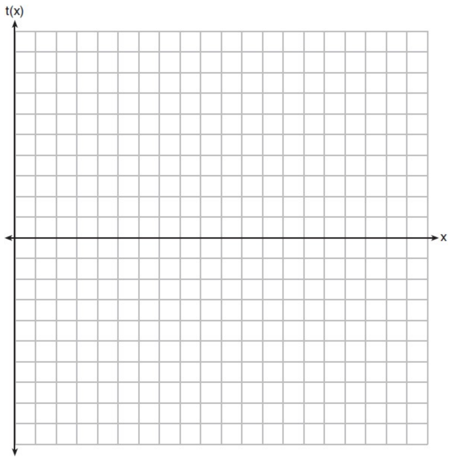
5. $y = 2\sin x$



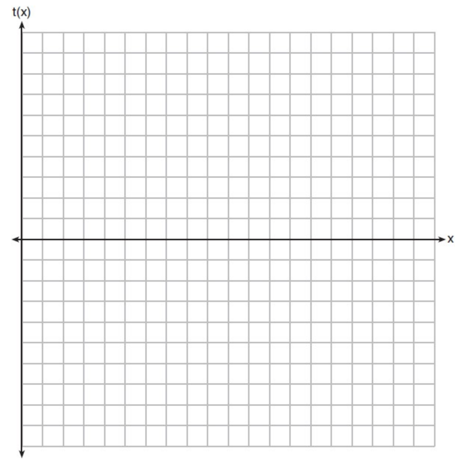
6. $y = -2\cos x$



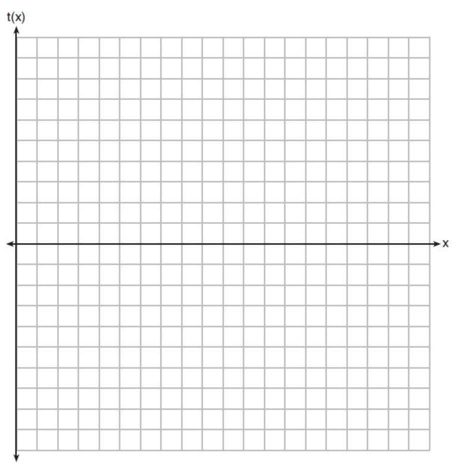
7. $y = -4\sin x$



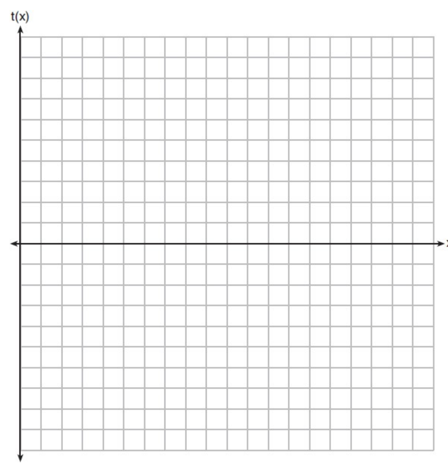
8. $y = \frac{1}{2}\cos x$



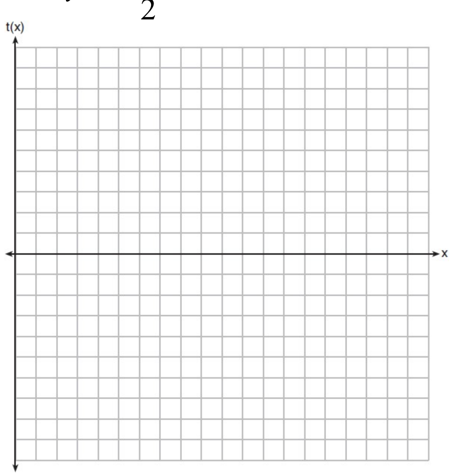
9. $y = 3\sin x$



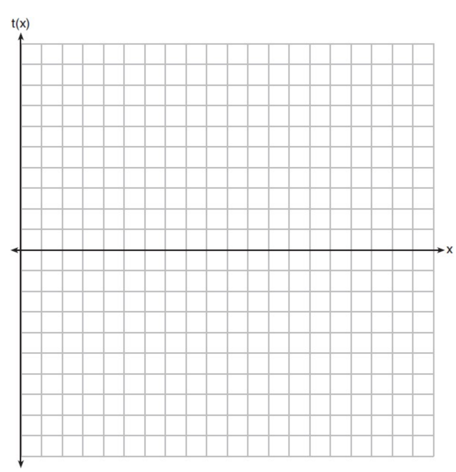
10. $y = -3\cos x$



11. $y = -\frac{1}{2}\sin x$



12. $y = 5\cos x$



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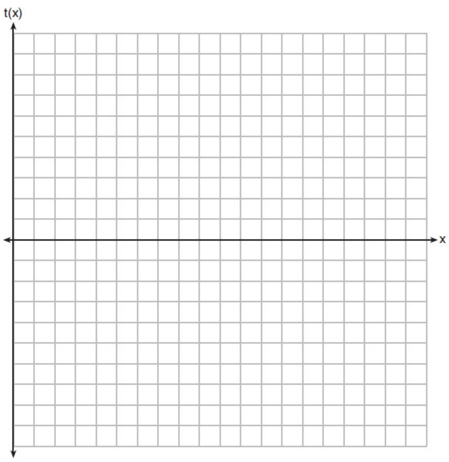
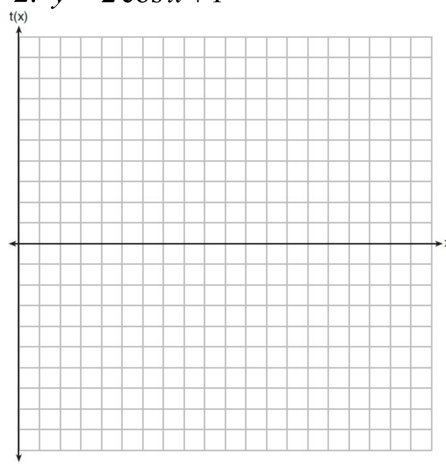
Graphing Sinusoidal Curves with Vertical Shifts

Graph one full wave of the following trigonometric functions and state the domain and range.

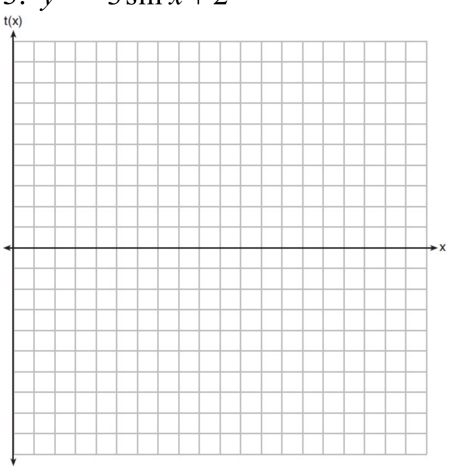
2. $y = \sin x - 3$



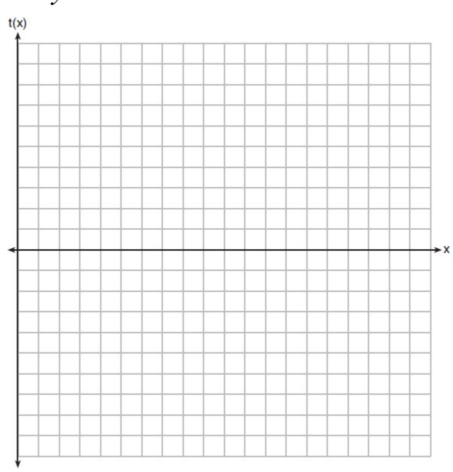
2. $y = 2\cos x + 1$



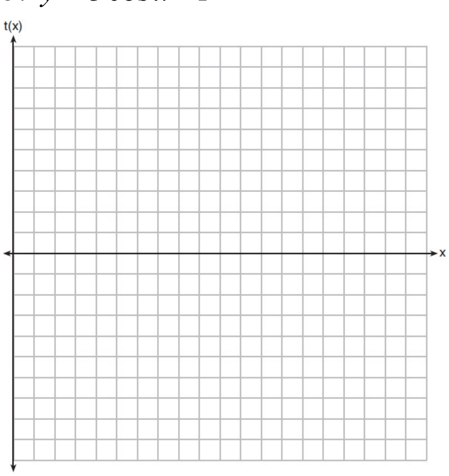
3. $y = -3\sin x + 2$



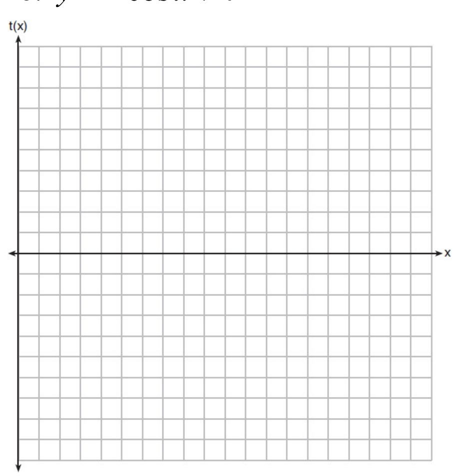
4. $y = -2\cos x - 3$



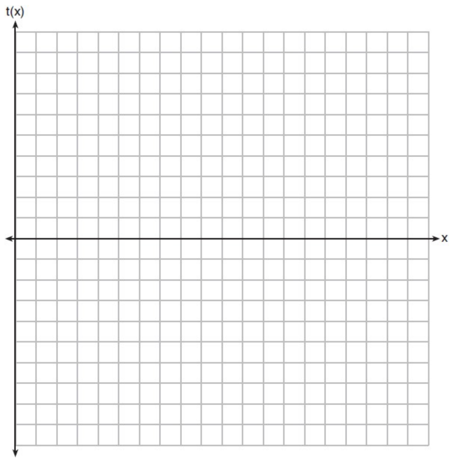
5. $y = 3\cos x - 1$



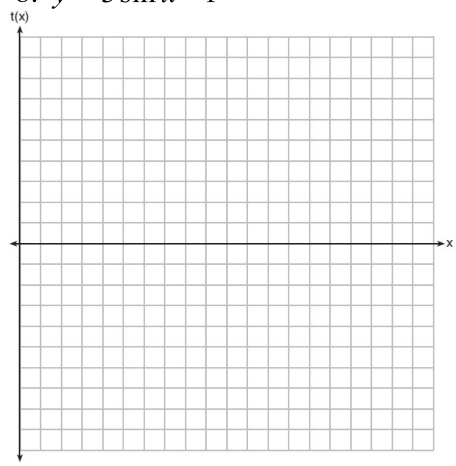
6. $y = -\cos x + 4$



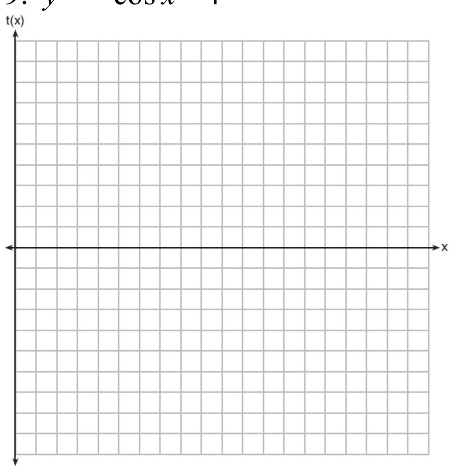
7. $y = -2\sin x + 2$



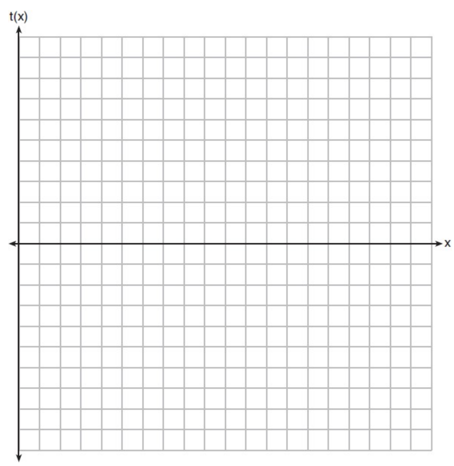
8. $y = 3\sin x - 1$



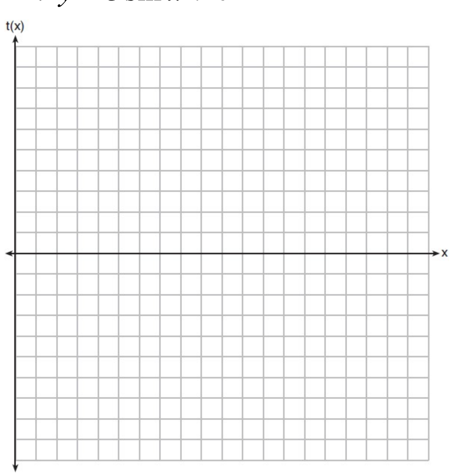
9. $y = -\cos x - 4$



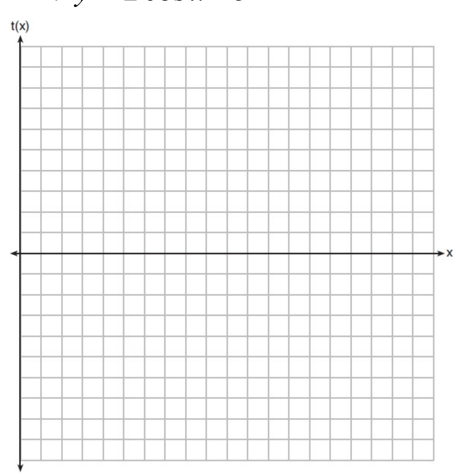
10. $y = -4\sin x + 2$



11. $y = 3\sin x + 4$



12. $y = 2\cos x - 5$



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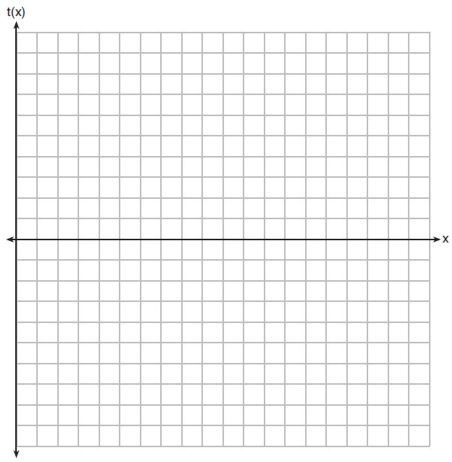
Mr. Schlansky Algebra II

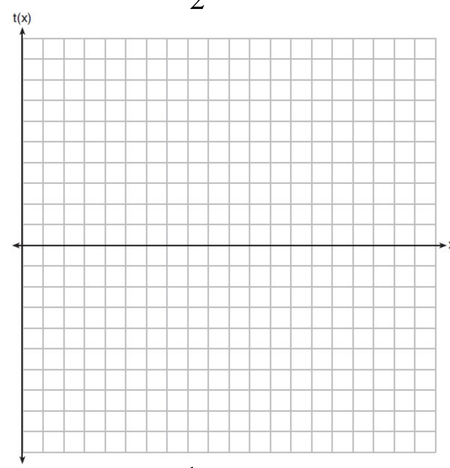
Graphing Sinusoidal Curves with Frequency

Graph one full wave of the following trigonometric functions and state the domain and range.

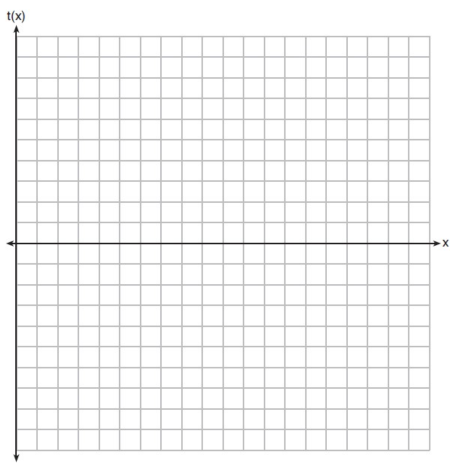
1. $y = \sin 2x$



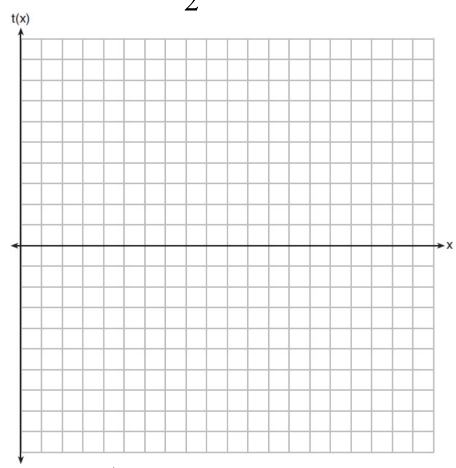
2. $y = -2\cos\frac{1}{2}x$



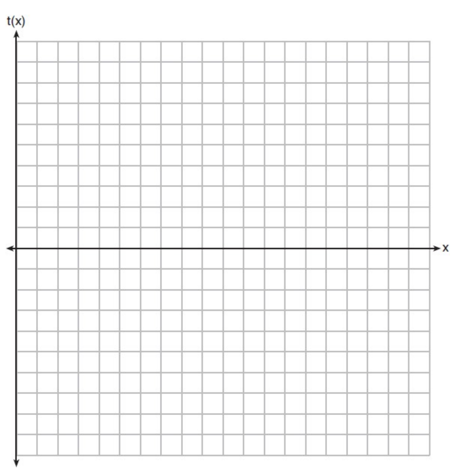
3. $y = 3\cos 4x + 2$



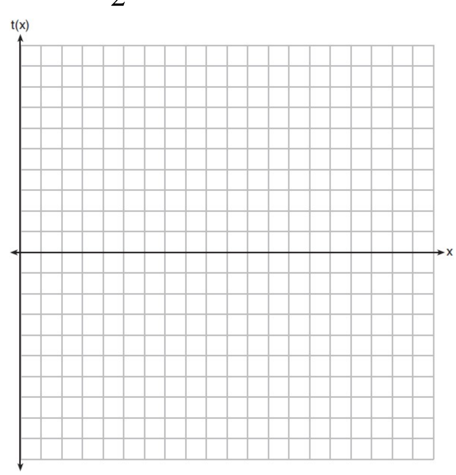
4. $y = -2\sin\frac{1}{2}x - 1$



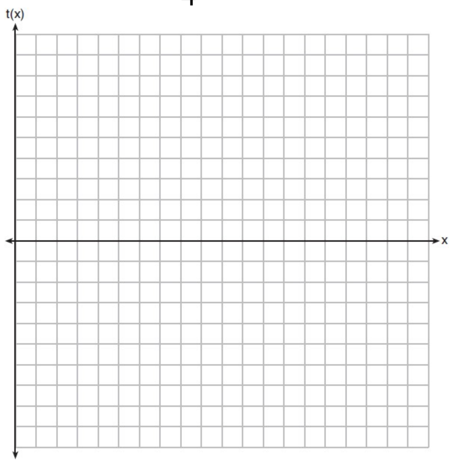
5. $y = 2\cos 4x - 3$



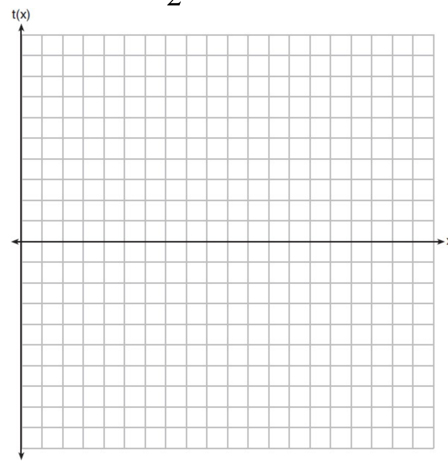
6. $y = \frac{1}{2}\sin 2x - 4$



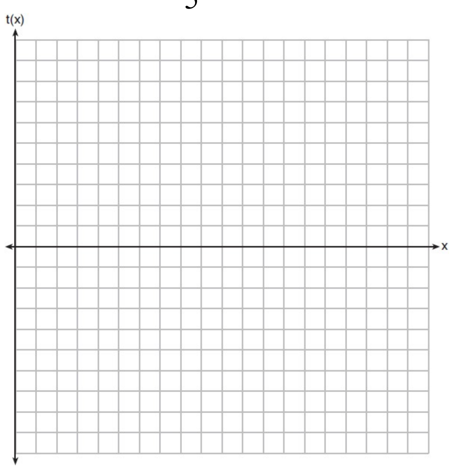
7. $y = -4\cos\frac{\pi}{4}x + 1$



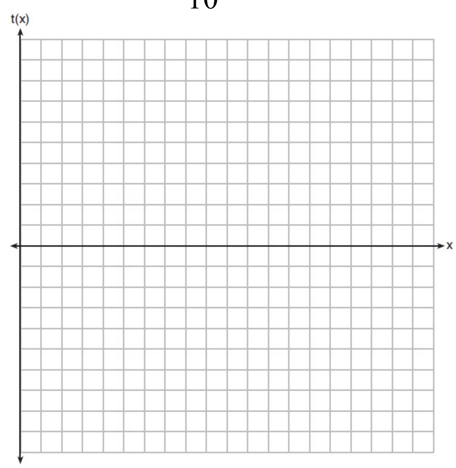
8. $y = 3\sin\frac{\pi}{2}x + 2$



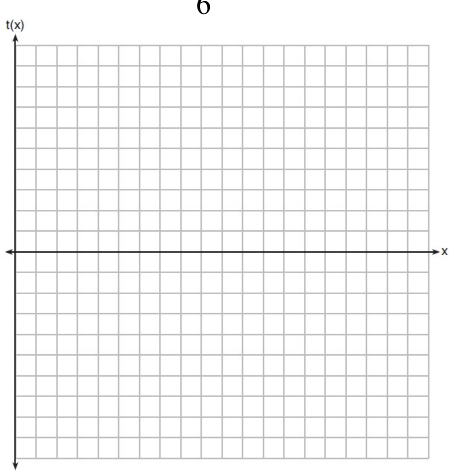
9. $y = -4\sin\frac{2\pi}{5}x + 2$



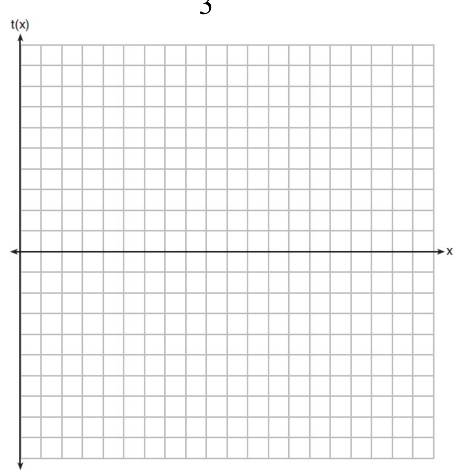
10. $y = 3\cos\frac{\pi}{10}x - 4$



11. $y = -3\cos\frac{\pi}{6}x + 1$



12. $y = 2\sin\frac{2\pi}{3}x - 1$



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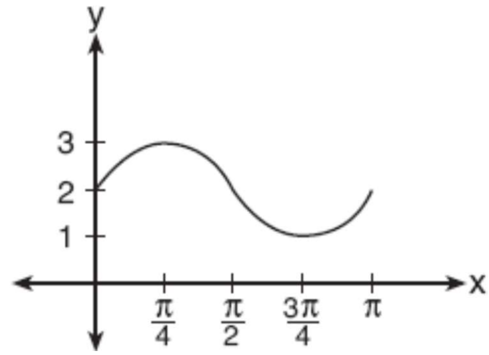
Mr. Schlansky Algebra II

Writing Equations of Sinusoidal Graphs

1. The accompanying graph represents a portion of a sound wave.

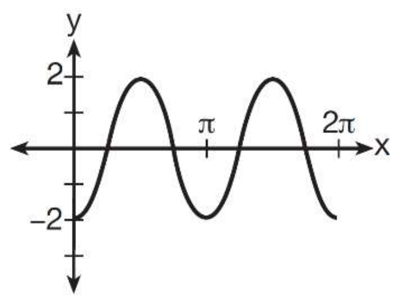


Which equation best represents this graph?

(1) $y = 2\sin\frac{1}{2}x$ (3) $y = \sin 2x$

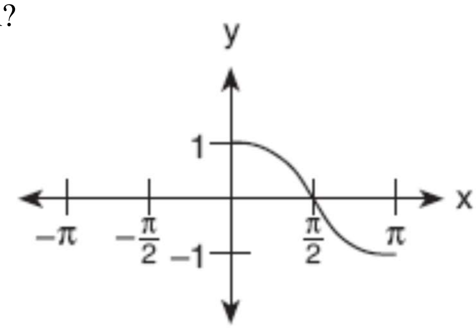
(2) $y = \sin\frac{1}{2}x + 2$ (4) $y = \sin 2x + 2$

2. Which equation represents the graph below?



1) $y = -2\sin 2x$
2) $y = -2\sin\frac{1}{2}x$
3) $y = -2\cos 2x$
4) $y = -2\cos\frac{1}{2}x$

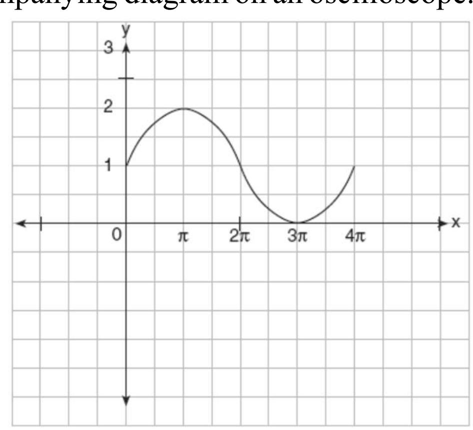
3. Which equation is represented by the accompanying graph?



(1) $y = \cos x$ (3) $y = \cos 2x$

(2) $y = \cos\frac{1}{2}x$ (4) $y = \frac{1}{2}\cos x$

4. In physics class, Eva noticed the pattern shown in the accompanying diagram on an oscilloscope.

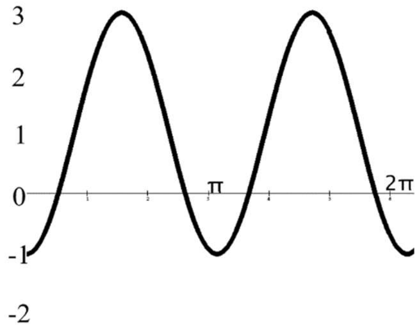


Which equation best represents the pattern shown on this oscilloscope?

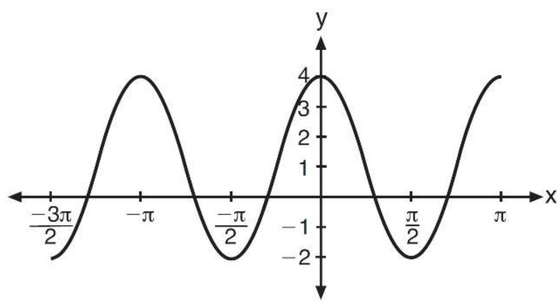
(1) $y = \sin(\frac{1}{2}x) + 1$ (3) $y = 2\sin x + 1$

(2) $y = \sin x + 1$ (4) $y = 2\sin(-\frac{1}{2}x) + 1$

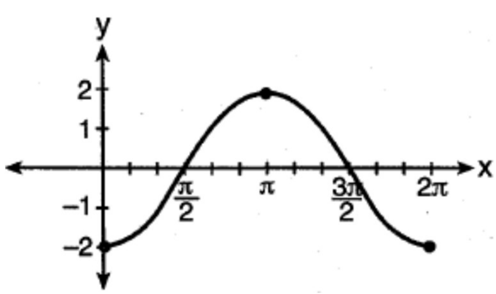
5. Write the equation of the sinusoidal function shown below:



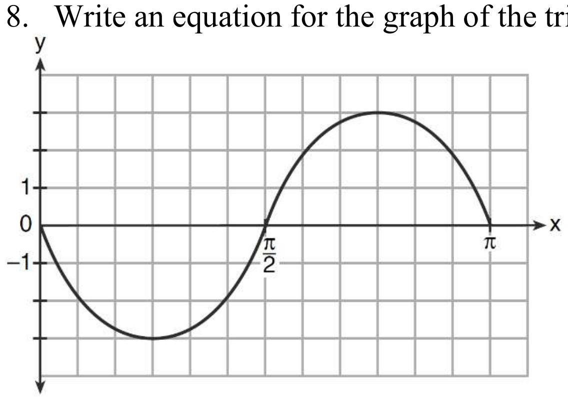
6. The periodic graph below can be represented by the trigonometric equation $y = a\cos bx + c$ where a, b, and c are real numbers.
State the values of a, b, and c, and write an equation for the graph.



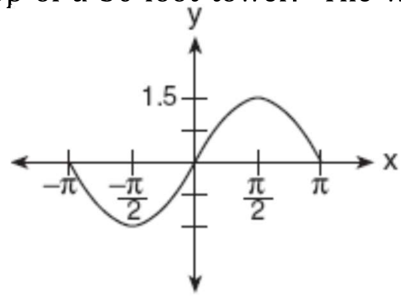
7. The accompanying graph shows a trigonometric function. State an equation of this function.



8. Write an equation for the graph of the trigonometric function shown below.



9. A radio transmitter sends a radio wave from the top of a 50-foot tower. The wave is represented by the accompanying graph.



What is the equation of this radio wave?

(1) $y = \sin x$ (3) $y = \sin 1.5x$

(2) $y = 1.5 \sin x$ (4) $y = 2 \sin x$

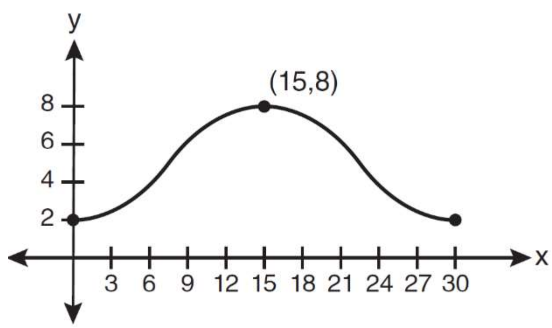
10. Which equation is graphed in the diagram below?

1) $y = 3\cos\left(\frac{\pi}{30}x\right) + 8$

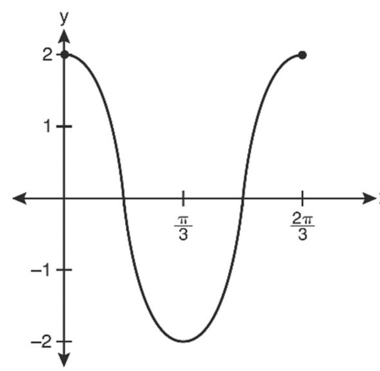
2) $y = 3\cos\left(\frac{\pi}{15}x\right) + 5$

3) $y = -3\cos\left(\frac{\pi}{30}x\right) + 8$

4) $y = -3\cos\left(\frac{\pi}{15}x\right) + 5$



11. Which equation is represented by the graph below?



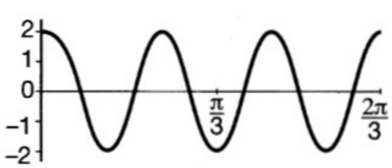
1) $y = 2\cos 3x$

2) $y = 2\sin 3x$

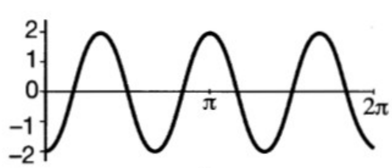
3) $y = 2\cos\frac{2\pi}{3}x$

4) $y = 2\sin\frac{2\pi}{3}x$

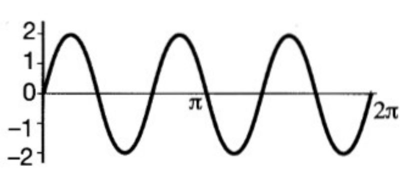
12 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of $\frac{2\pi}{3}$?



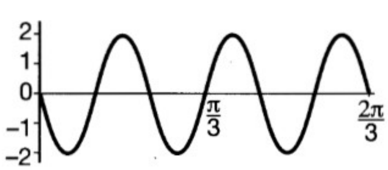
(1)



(3)

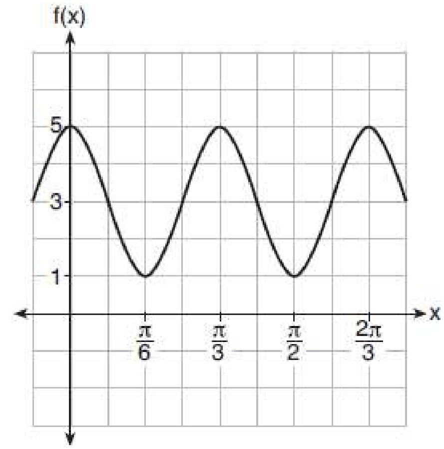


(2)



(4)

13. The function $f(x) = a\cos bx + c$ is plotted on the graph shown below.



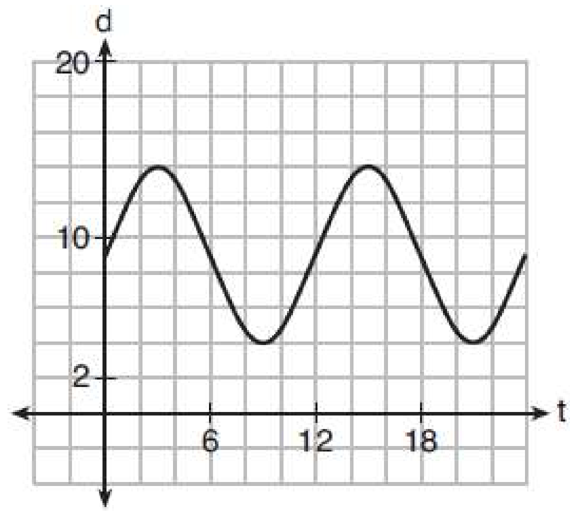
What are the values of *a*, *b*, and *c*?

1) $a = 2, b = 6, c = 3$
2) $a = 2, b = 3, c = 1$
3) $a = 4, b = 6, c = 5$
4) $a = 4, b = \frac{\pi}{3}, c = 3$

14. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, *d*, is measured in feet and time, *t*, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$
2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$
3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$
4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

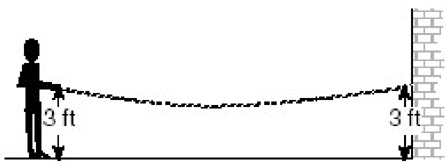


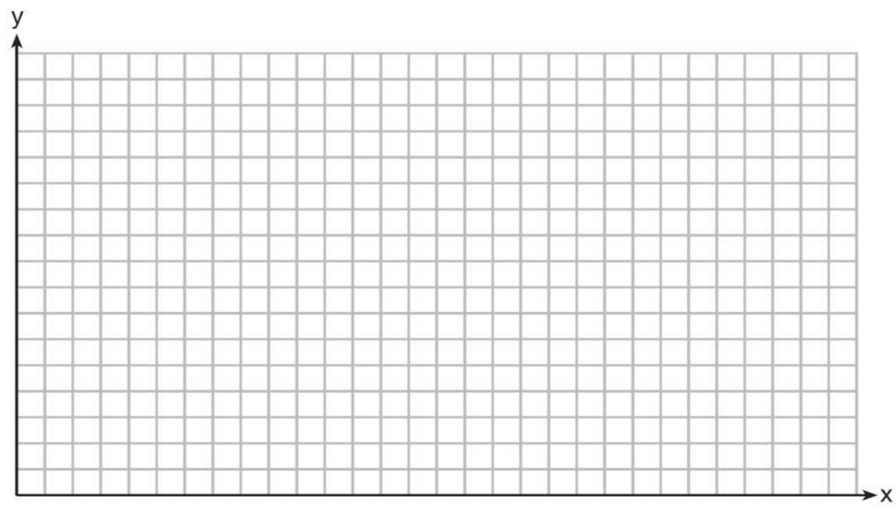
Name ____________ Date _____________

Mr. Schlansky Algebra I

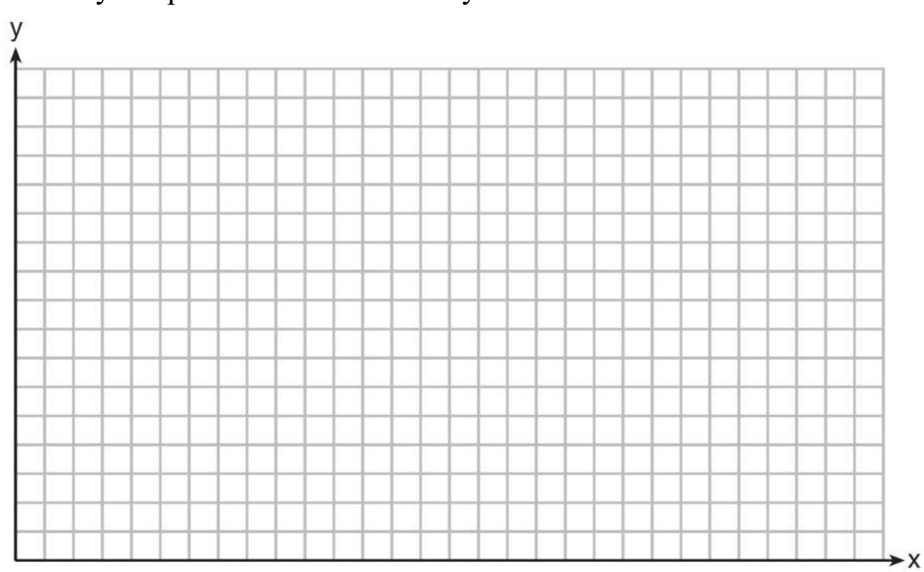
Graphing Sinusoidal Models

1. A student attaches one end of a rope to a wall at a fixed point 3 feet above the ground, as shown in the accompanying diagram, and moves the other end of the rope up and down, producing a wave described by the equation $y = a \sin bx + c$. The range of the rope's height above the ground is between 1 and 5 feet. The period of the wave is 4π. Graph one full wavelength and write the equation that represents this wave.

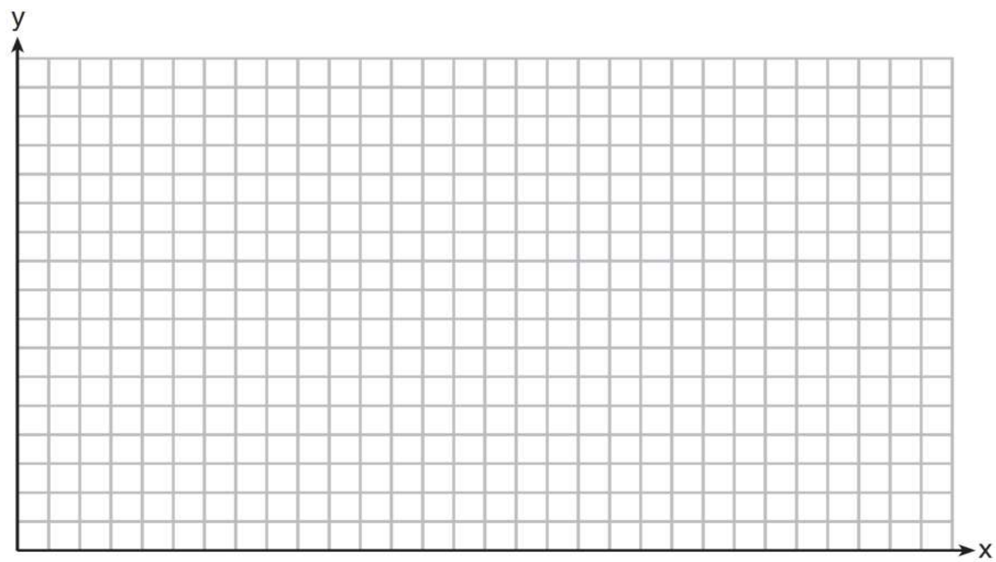




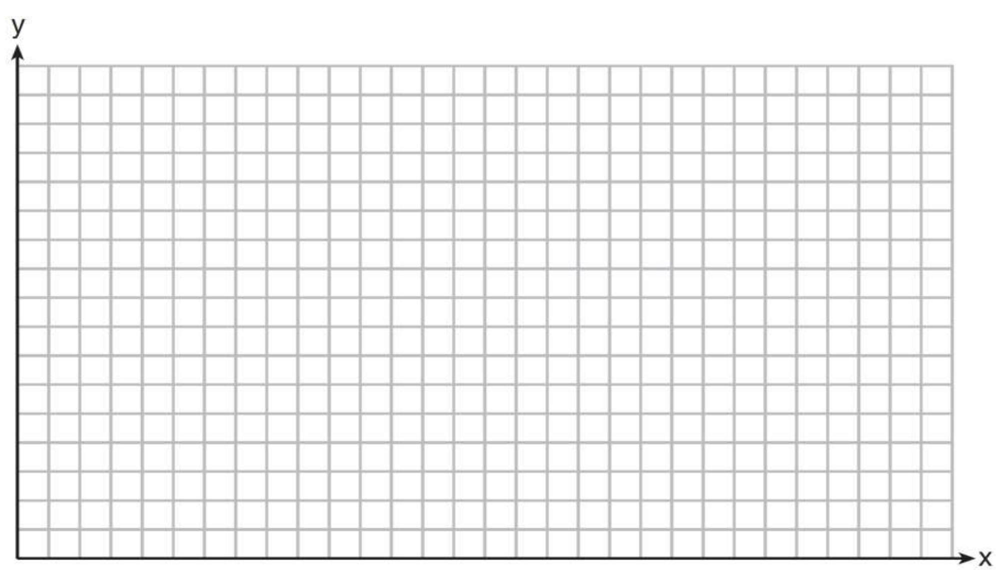
2. In Johannesburg in June, the daily low temperature is usually around 4 degrees Celcius at 4AM, and the daily high temperature is around 16 degrees Celcius at 4PM. Write the equation of a sinusoidal function that models the temperature T in Johannesburg t hours after 4AM. Graph the daily temperature for one full day.



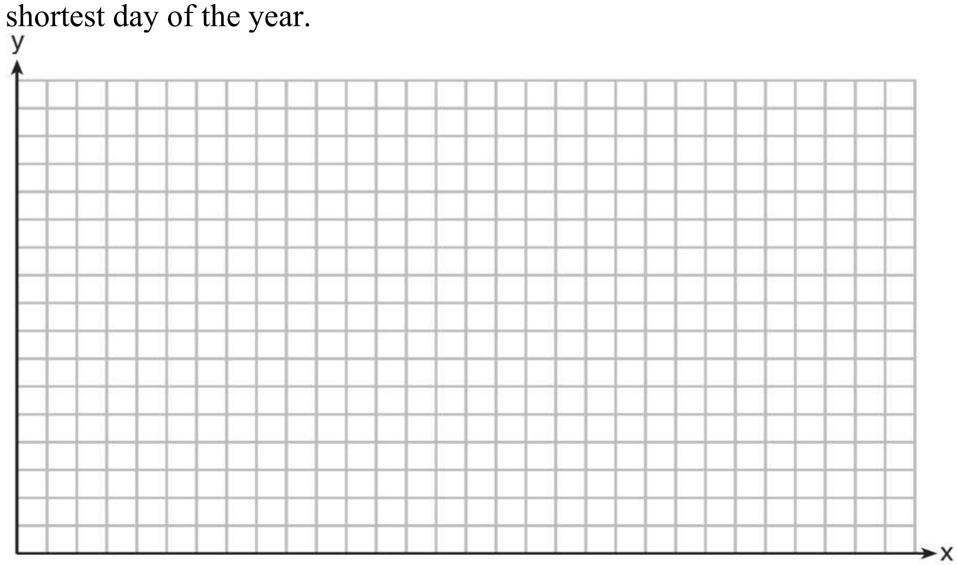
3. The hottest day of the year in Santiago, Chile, on average is January 7 when the average high temperature is 30 degrees Celcius. Six months later, the coolest day of the year has an average high temperature of 14 degrees Celcius. Use a trigonometric function to model the temperature in Santiago, Chile, where t represents months since January 7. Graph your trigonometric function on the grid below.



4. Aditya's dog Sparky routinely eats Aditya's leftovers, which vary seasonally. As a result, his weight fluctuates throughout the year.
The dog's weight $W(t)$ as a function of time t (in months) over the course of a year can be modeled by a sinusoidal expression of the form $a\cos(bt)+d$. At $t=0$ months, the start of the year, he is at his maximum weight of 10 kg. Six months later, when $t=6$ months, he is at his minimum weight of 6.0 kg. Write an equation for $W(t)$, the weight of Sparky after t months. Sketch the graph of Sparky's weight over one full year.



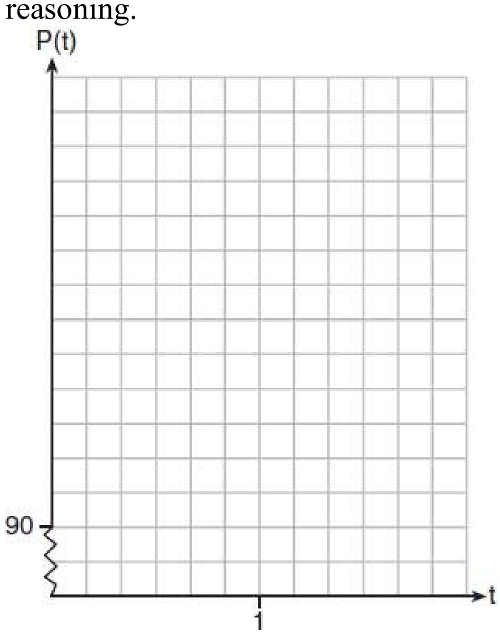
5. The shortest day of the year in Town A is 10 hours and the longest day of the year is 14 hours. Create a cosine function to represent this situation. Graph the cosine function on the grid below to represent the length of the days in Town A for one full year. Let t represents months since the shortest day of the year.



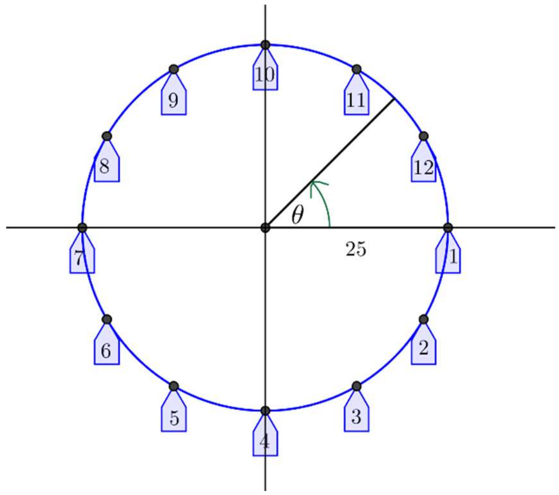
6. The resting blood pressure of an adult patient can be modeled by the function *P* below, where $P(t)$ is the pressure in millimeters of mercury after time *t* in seconds.

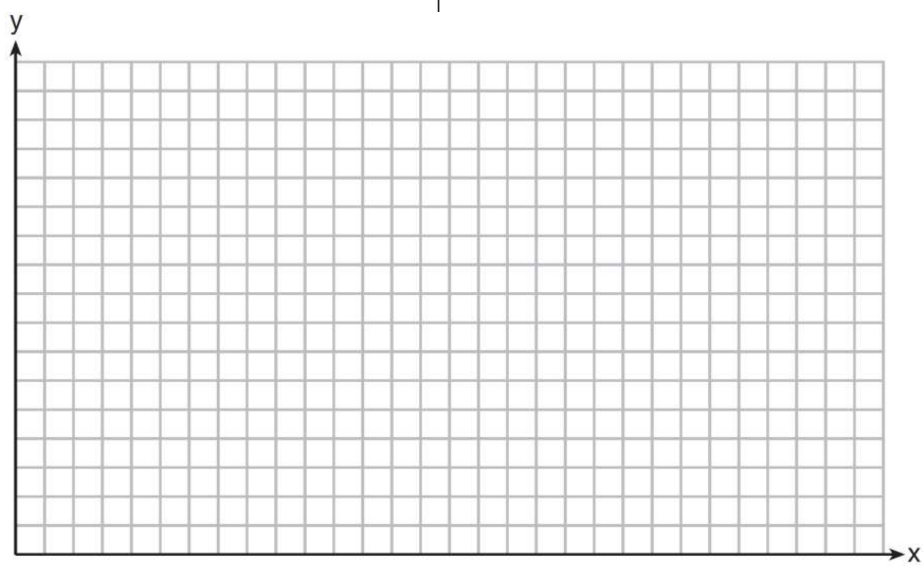
$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \le t \le 2$.
Determine the period of *P*. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.



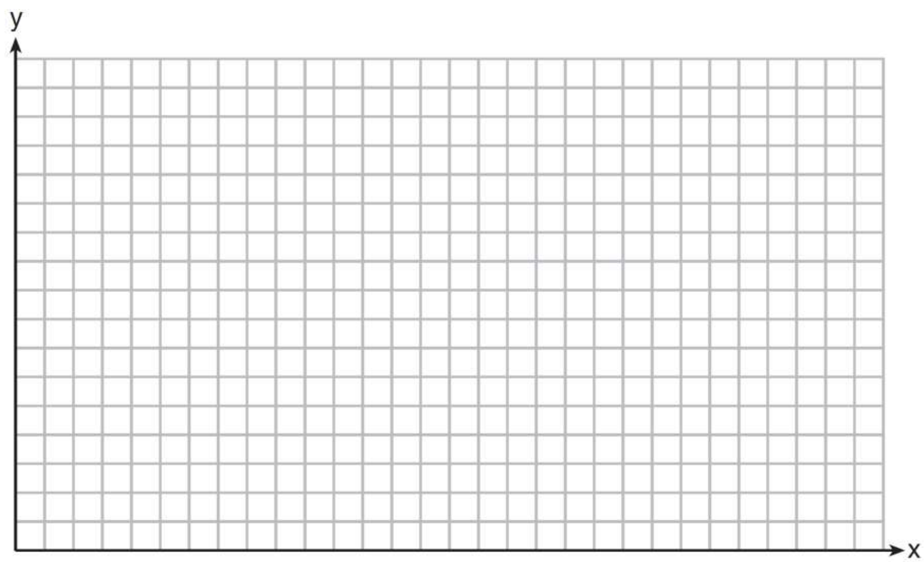
7. A carnival has a Ferris wheel that is 50 feet in diameter with 12 passenger cars. When viewed from the side where passengers board, the Ferris wheel rotates counterclockwise and makes two full turns each minute. Riders board the Ferris wheel from a platform that is 40 feet above the ground. Create a graph of the height of car 1 as a function of time in seconds. Write an equation for your graph.



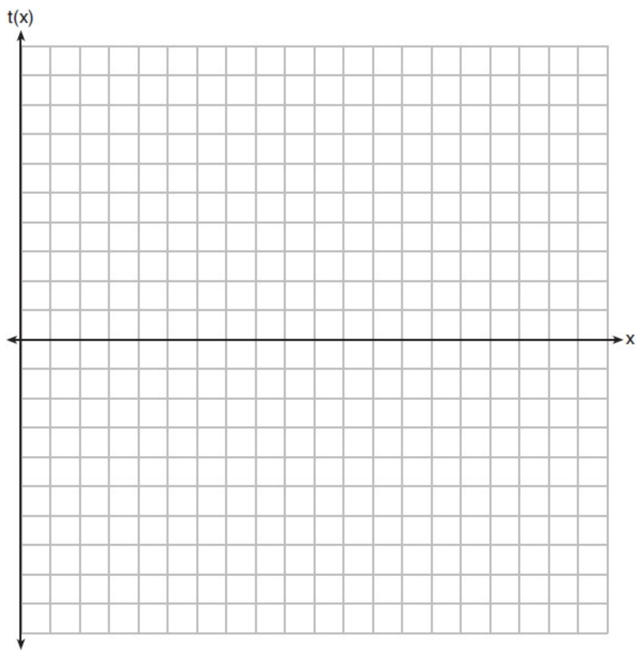


8. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. The 550 ft. tall wheel has a diameter of 520 feet. A ride on one of its 28 passenger cars lasts 30 minutes, the time it takes the wheel to complete one full rotation. Riders board the passenger cars at the bottom of the wheel. Assume that once the wheel is in motion, it maintains a constant speed for the 30-minute ride and is rotating in a counterclockwise direction.

a) Sketch a graph of the height of a passenger car on the High Roller as a function of the time the ride began.
b) Write a sinusoidal function H that represents the height of a passenger car t minutes after the ride begins.
c) Identify the amplitude, midline, frequency, and period and explain how they relate to the situation.
d) If you were on this ride, how high would you be above the ground after 22.5 minutes?



9. The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.



People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

Name ______________ Date _____________

Mr. Schlansky Algebra II

Sinusoidal Applications

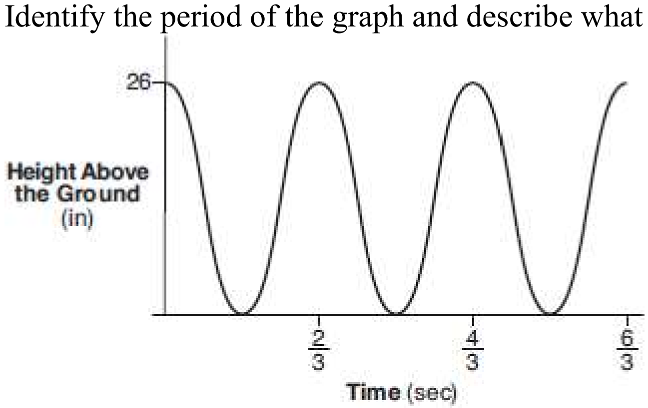
1. The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where *t* is time in seconds?

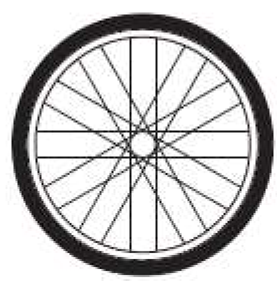
1) $V = 120\sin(t)$
2) $V = 120\sin(60t)$
3) $V = 120\sin(60\pi t)$
4) $V = 120\sin(120\pi t)$

2. A sine wave passes through a point 440 times every second. The wave is centered 2 feet from the ground fluctuates up 6 inches and down 6 inches. Write an equation for H(t), the height in feet the wave is above the ground after t seconds.

3. The graph below represents the height above the ground, *h*, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, *t*, in seconds.

Identify the period of the graph and describe what the period represents in this context.





4. The height above ground for a person riding a Ferris wheel after *t* seconds is modeled by $h(t) = 150\sin\left(\frac{\pi}{45}t + 67.5\right) + 160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

1) 10
2) 45
3) 90
4) 150

5. A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?

1) $(0, 200)$
2) $(100, 300)$
3) $(200, 400)$
4) $(300, 400)$

6. A cosine function decreasing through the origin has a frequency of $\frac{\pi}{100}$. What is the first positive interval where the wave is increasing?

7. As *x* increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y = 2\tan x$ will

1) increase from 0 to 2
2) decrease from 0 to -2
3) increase without limit
4) decrease without limit

8. As x increases from $\frac{3\pi}{2}$ to 2π, the graph of $y = \csc x$ will

1) increase without limit
2) decrease without limit
3) increase to -1
4) decrease to 1

9. As x increases from $-\frac{\pi}{2}$ to 0, the graph of $y = \sec x$ will

1) increase without limit
2) decrease without limit
3) increase to -1
4) decrease to 1

10. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

1) The period is 6.
2) The amplitude is 3.
3) The range is [4,10].
4) The midline is $y = -4$.

11. Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) $y = -4\cos\left(\frac{\pi}{4}x\right) - 3$
2) $y = -4\cos\left(\frac{\pi}{4}x\right) + 5$
3) $y = -4\cos(8x) - 3$
4) $y = -4\cos(8x) + 5$

12. Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models *x* months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

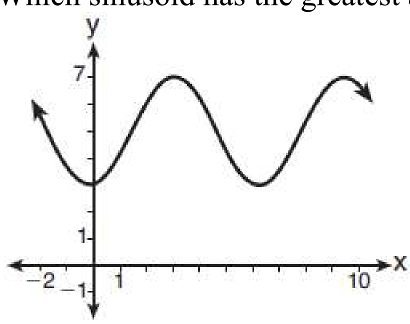
13. Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the time of the next high tide, the most important value to have would be the

1) time between consecutive low tides
2) time when the tide height is 20 feet
3) average depth of water over a 24-hour period
4) difference between the water heights at low and high tide

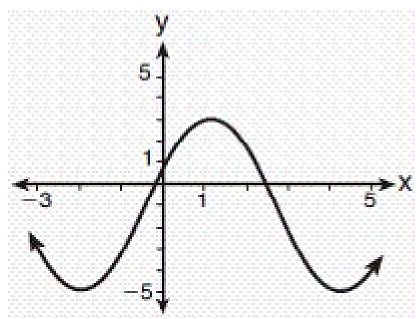
14. Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below. Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

x	$q(x)$
−2	−8
−1	0
0	0
1	−2
2	0

15. Which sinusoid has the greatest amplitude?

1)


2) $y = 3\sin(\theta - 3) + 5$

3)


4) $y = -5\sin(\theta - 1) - 3$

16. The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.