

- Isolate ~~at least~~ 1 variable in at least 1 equation
- Substitute one equation into the other
- Solve equation
- Substitute into 1 of original equations to find other variable

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Algebra II

## Solving Non-Linear Systems Algebraically

Solve each of the following systems of equations for all values of x and y

1.  $y = -2x + 1$   
 $y = -2x^2 + 3x + 1$

$-2x + 1 = -2x^2 + 3x + 1$   
 $+2x^2 - 3x - 1 + 2x^2 - 3x - 1$   
 $2x^2 - 5x = 0$   
 $x(2x - 5) = 0$   
 $x = 0$  or  $2x - 5 = 0$   
 $x = \frac{5}{2}$

3.  $x^2 - y = 5$   
 $y = 3x - 1$

$x^2 - y = 5$   
 $+y + y$   
 $x^2 = y + 5$   
 $-5 -5$   
 $x^2 - 5 = y$

$x^2 - 5 = 3x - 1$   
 $-3x + 3x + 1$

$x^2 - 3x - 4 = 0$   
 $(x - 4)(x + 1) = 0$   
 $x = 4$  or  $x = -1$

$x = 4$  or  $x = -1$   
 $y = 3x - 1$   
 $y = 3(4) - 1 = 11$   
 $y = 3(-1) - 1 = -4$   
 $(4, 11)$  or  $(-1, -4)$

5.  $y^2 - x^2 + 32 = 0$   
 $3y - x = 0$   
 $+x + x$   
 $3y = x$

$y^2 - (3y)^2 + 32 = 0$

$y^2 - 9y^2 + 32 = 0$

$-8y^2 + 32 = 0$   
 $-8y^2 = -32$   
 $y^2 = 4$   
 $(y + 2)(y - 2) = 0$

$y = -2$   
 $x = 3y$   
 $x = 3(-2) = -6$   
 $(-6, -2)$   
 $y = 2$   
 $x = 3y$   
 $x = 3(2) = 6$   
 $(6, 2)$

2.  $y = x^2 - 4x + 3$   
 $y + 1 = x$   
 $+1 -1$   
 $y = x - 1$   
 $x - 1 = x^2 - 4x + 3$   
 $-x + 1$   
 $0 = x^2 - 5x + 4$   
 $0 = (x - 4)(x - 1)$   
 $x = 4$  or  $x = 1$

$x = 4$  or  $x = 1$   
 $y = x - 1$   
 $y = 4 - 1 = 3$   
 $y = 1 - 1 = 0$   
 $(4, 3)$  or  $(1, 0)$

4.  $5 = y - x$   
 $4x^2 = -17x + y + 4$

$5 = y - x$   
 $+x +x$   
 $5 + x = y$   
 $4x^2 = -17x + y + 4$   
 $+17x - 4 + 17x - 4$   
 $4x^2 + 17x - 4 = y$

$5 + x = -17x + y + 4$   
 $-x - 5$   
 $0 = 4x^2 + 16x - 9$   
 $x^2 + 4x - \frac{9}{4}$   
 $(x + \frac{9}{2})(x - \frac{1}{2})$

$(2x + 9)(2x - 1) = 0$   
 $2x + 9 = 0$  or  $2x - 1 = 0$   
 $-9 -9$  or  $\frac{1}{2} \frac{1}{2}$   
 $\frac{2x}{2} = \frac{-9}{2}$  or  $\frac{2x}{2} = \frac{1}{2}$   
 $x = -4.5$  or  $x = 0.5$

6.  $x^2 + y^2 = 10$   
 $x = y - 4$

$(y - 4)^2 + y^2 = 10$

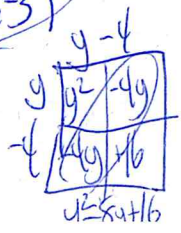
$y^2 - 8y + 16 + y^2 = 10$

$2y^2 - 8y + 16 = 10$   
 $-10 -10$

$2y^2 - 8y + 6 = 0$   
 $\frac{2y^2}{2} - \frac{8y}{2} + \frac{6}{2} = \frac{0}{2}$

$x = -4.5$  or  $x = 0.5$   
 $y = x + 5$   
 $y = -4.5 + 5 = 0.5$   
 $y = 0.5 + 5 = 5.5$   
 $(-4.5, 0.5)$  or  $(0.5, 5.5)$

$y = 3$  or  $y = 1$   
 $x = y - 4$   
 $x = 3 - 4 = -1$   
 $x = 1 - 4 = -3$   
 $(-1, 3)$  or  $(-3, 1)$



y	+2
y <sup>2</sup>	+2y
2y	+4

7.  $x^2 + y^2 = 2$   
 $y + 2 = x$

$y^2 + 2y + 1 = 0$   
 $(y+1)(y+1) = 0$   
 $y = -1$   
 $x = y + 2 = 1$   
 $(1, -1)$

9.  $(x+2)^2 + (y-4)^2 = 40$   
 $y = x + 2$

x	+2
x <sup>2</sup>	+2x
2x	+4

x	-2
x <sup>2</sup>	-2x
-2x	+4

$x^2 + 4x + 4 + x^2 - 4x + 4 = 40$   
 $2x^2 + 8 = 40$   
 $2x^2 - 32 = 0$   
 $x^2 - 16 = 0$   
 $(x+4)(x-4) = 0$   
 $x = -4$  or  $x = 4$   
 $y = x + 2$   
 $(-4, -2)$  and  $(4, 6)$

11.  $x + y = 5$   
 $(x+3)^2 + (y-3)^2 = 53$

-x	+2
x <sup>2</sup>	-2x
2x	+4

x	+3
x <sup>2</sup>	+3x
3x	+9

$(x+3)^2 + (5-x-3)^2 = 53$   
 $(x+3)^2 + (-x+2)^2 = 53$   
 $x^2 + 6x + 9 + x^2 - 4x + 4 = 53$   
 $2x^2 + 2x + 13 = 53$   
 $2x^2 + 2x - 40 = 0$   
 $x^2 + x - 20 = 0$   
 $(x+5)(x-4) = 0$   
 $x = -5$  or  $x = 4$   
 $y = 5 - x$   
 $(-5, 10)$  and  $(4, 1)$

$2x^2 + 2x - 40 = 0$   
 $x^2 + x - 20 = 0$   
 $(x+5)(x-4) = 0$   
 $x = -5$  or  $x = 4$   
 $y = 5 - x$   
 $(-5, 10)$  and  $(4, 1)$

8.  $3x^2 + y^2 = 13$   
 $y = x - 3$

x	-3
x <sup>2</sup>	-3x
-3x	+9

$3x^2 + (x-3)^2 = 13$   
 $3x^2 + x^2 - 6x + 9 = 13$   
 $4x^2 - 6x - 4 = 0$   
 $2x^2 - 3x - 2 = 0$   
 $(x-4)(x+1) = 0$   
 $x = 4$  or  $x = -1$   
 $y = x - 3$   
 $(4, 1)$  and  $(-1, -4)$

10.  $x^2 + (y+4)^2 = 109$   
 $y = x + 3$

x	+7
x <sup>2</sup>	+7x
7x	+49

$x^2 + (x+3+4)^2 = 109$   
 $x^2 + (x+7)^2 = 109$   
 $x^2 + x^2 + 14x + 49 = 109$   
 $2x^2 + 14x - 60 = 0$   
 $x^2 + 7x - 30 = 0$   
 $(x+10)(x-3) = 0$   
 $x = -10$  or  $x = 3$   
 $y = x + 3$   
 $(-10, -7)$  and  $(3, 6)$

12.  $(x-3)^2 + (y+2)^2 = 16$

-x	+2
x <sup>2</sup>	-2x
-2x	+4

x	-3
x <sup>2</sup>	-3x
-3x	+9

-x	+7
x <sup>2</sup>	-7x
-7x	+49

$2x + 2y = 10$   
 $2y = -2x + 10$   
 $y = -x + 5$   
 $(x-3)^2 + (-x+5+2)^2 = 16$   
 $(x-3)^2 + (-x+7)^2 = 16$   
 $x^2 - 6x + 9 + x^2 - 14x + 49 = 16$   
 $2x^2 - 20x + 58 = 16$   
 $2x^2 - 20x + 42 = 0$   
 $x^2 - 10x + 21 = 0$   
 $(x-7)(x-3) = 0$   
 $x = 7$  or  $x = 3$   
 $y = -x + 5$   
 $(7, -2)$  and  $(3, 2)$